Optimal Strategies in Sequential Bidding

(Extended Abstract)

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1. INTRODUCTION

We are interested in mechanisms that maximize the final social welfare. In [1] this problem was studied for multi-unit auctions with unit demand bidders and for the public project problem, and in each case social welfare undominated mechanisms in the class of feasible and incentive compatible mechanisms were identified.

One way to improve upon these optimality results is by relaxing the assumption of simultaneity and allowing the players to move sequentially. With this in mind, we study here sequential versions of two feasible Groves mechanisms used for single item auctions: the Vickrey auction and the Bailey-Cavallo mechanism.

Because of the absence of dominant strategies in this sequential setting, we focus on a weaker concept of an optimal strategy. For each mechanism, we introduce natural optimal strategies and observe that in each mechanism these strategies exhibit different behaviour. However, we then show that among all optimal strategies, the one we introduce for each mechanism maximizes the social welfare when each player follows it. The resulting social welfare can be larger than the one obtained in the simultaneous setting.

2. PRELIMINARIES

Assume that there is a finite set of possible outcomes or ***decisions*** D, a set $\{1,\ldots,n\}$ of players where $n \geq 2$, and for each player i a set of ***types*** Θ_i and an (***initial***) ***utility function*** v_i. A ***decision rule*** is a function $f : \Theta \rightarrow D$, where $\Theta := \Theta_1 \times \cdots \times \Theta_n$.

A mechanism is given by a pair of functions (f,t), where f is the decision rule and $t = (t_1,...,t_n)$ is the tax function that determines the players' payments. We assume that the (***final***) ***utility function*** for player i is a function u_i defined by $u_i(d, t_1,\ldots,t_n,\theta_i) := v_i(d,\theta_i) + t_i$.

We focus on two special Groves mechanisms (for details on Groves mechanisms see [5]). The first one is the well known ***pivotal mechanism***[1]. The ***Vickrey auction*** is simply the pivotal mechanism for a single item auction. The second is the ***Bailey-Cavallo*** mechanism, in short ***BC auction***. It was originally proposed in [3, 4], where it was also demonstrated that it can be a more appropriate mechanism when one is interested in maximizing the final social welfare.

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3. SEQUENTIAL MECHANISMS

We are interested in sequential mechanisms, in which the players announce their types sequentially. In this section we review the relevant concepts, some of which were introduced in [2]. Without loss of generality, we assume the order to be $1,\ldots,n$. Hence each player i *knows* the types announced by players $1,\ldots,i-1$, and can use this information to decide which type to announce.

A ***strategy*** of player i in a sequential mechanism is a function

$$s_i : \Theta_1 \times \ldots \times \Theta_i \rightarrow \Theta_i.$$

If the vector of types that the players receive is θ and the vector of strategies that they follow is $s(\cdot) := (s_1(\cdot),\ldots,s_n(\cdot))$, the vector of the announced types will be denoted by $[s(\cdot),\theta]$, where $[s(\cdot),\theta]$ is defined inductively by $[s(\cdot),\theta]_1 := s_1(\theta_1)$ and $[s(\cdot),\theta]_{i+1} := s_{i+1}([s(\cdot),\theta]_1,\ldots,[s(\cdot),\theta]_i,\theta_{i+1})$.

A strategy $s_i(\cdot)$ of player i is ***dominant*** in the sequential version of the mechanism (f,t) if there is no incentive for player i to deviate to another strategy no matter what the other players announce.

A weaker notion is that of an optimal strategy. We say that strategy $s_i(\cdot)$ of player i is ***optimal*** if for all $\theta \in \Theta$ and all $\theta'_i \in \Theta_i$

$$u_i((f,t)(s_i(\theta_1,\ldots,\theta_i),\theta_{-i}),\theta_i) \geq u_i((f,t)(\theta'_i,\theta_{-i}),\theta_i).$$

Call a strategy of player j ***myopic*** if it does not depend on the types of players $1,\ldots,j-1$. Then a strategy $s_i(\cdot)$ is optimal if for all $\theta \in \Theta$ it yields a best response to all joint strategies of players $j \neq i$ in which the strategies of players $i+1,\ldots,n$ are myopic. In particular, an optimal strategy is a best response to the truth-telling by players $j \neq i$. Each dominant strategy is optimal. For player n the concepts of dominant and optimal strategies coincide.

There are two natural ways of maximizing players' utilities. The first one calls for a simultaneous maximization of other players' utilities. That is, we say that strategy $s_i(\cdot)$

[1]This is sometimes referred to as the VCG mechanism.

of player i is ***socially maximal*** if it is optimal and for all optimal strategies $s'_i(\cdot)$ of player i, all $\theta \in \Theta$ and all $j \neq i$

$u_j((f,t)(s_i(\theta_1,\ldots,\theta_i),\theta_{-i}),\theta_j) \geq$
$u_j((f,t)(s'_i(\theta_1,\ldots,\theta_i),\theta_{-i}),\theta_j)$.

So a socially maximal strategy of player i simultaneously guarantees the maximal utility to every other player, under the assumption that players $i+1,\ldots,n$ use myopic strategies.

The second option is to maximize the social welfare. We say that strategy $s_i(\cdot)$ is ***socially optimal*** if it is optimal and for all optimal strategies $s'_i(\cdot)$ of player i and all $\theta \in \Theta$

$\sum_{j=1}^{n} u_j((f,t)(s_i(\theta_1,\ldots,\theta_i),\theta_{-i}),\theta_j) \geq$
$\sum_{j=1}^{n} u_j((f,t)(s'_i(\theta_1,\ldots,\theta_i),\theta_{-i}),\theta_j)$.

Hence a socially optimal strategy of player i yields the maximal social welfare among all optimal strategies, under the assumption that players $i+1,\ldots,n$ use myopic strategies. Note that each socially maximal strategy is socially optimal. The converse does not hold.

For a mechanism where each player i receives a type $\theta_i \in \Theta_i$ and follows a strategy $s_i(\cdot)$, we denote the resulting social welfare by $SW(\theta, s(\cdot))$. We are interested in finding a sequence of optimal players' strategies for which the resulting social welfare is always maximal.

3.1 Sequential Vickrey auctions

In this and the next Section all proofs are omitted. First, we have the following negative result.

THEOREM 3.1. *Consider a sequential Vickrey auction. For $i \in \{1,\ldots,n-1\}$ no dominant strategy exists for player i.*

So we shall focus on the weaker notion of optimal strategy. The following natural strategy for player i is an example of an optimal strategy that deviates from truth-telling:

$$s_i(\theta_1,\ldots,\theta_i) := \begin{cases} \theta_i & \text{if } \theta_i > \max_{j\in\{1,\ldots,i-1\}} \theta_j, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

When we limit our attention to optimal strategies we get the following result.

THEOREM 3.2. *Consider a sequential Vickrey auction. For all $\theta \in \Theta$, all vectors $s(\cdot)$ of optimal players' strategies and all optimal strategies $s'_i(\cdot)$ of player i*

$$u_i((f,t)([s(\cdot),\theta],\theta_i) = u_i((f,t)([(s'_i(\cdot),s_{-i}(\cdot)),\theta],\theta_i).$$

This can be interpreted as a statement that each optimal strategy is dominant within the universe of optimal strategies. However, optimal strategies may differ when the players take into account the utility of other players, in particular, the social welfare.

The following result shows that strategy $s_i(\cdot)$ defined in (1) plays a special role.

THEOREM 3.3. *In the sequential Vickrey auction strategy $s_i(\cdot)$ defined in (1) is socially maximal for player i.*

Finally, when each player i follows strategy $s_i(\cdot)$ of Theorem 3.3, maximal social welfare is generated.

THEOREM 3.4. *In the sequential Vickrey auction for all $\theta \in \Theta$ and vectors $s'(\cdot)$ of optimal players' strategies,*

$$SW(\theta, s(\cdot)) \geq SW(\theta, s'(\cdot))$$

where $s(\cdot)$ is the vector of strategies $s_i(\cdot)$ defined in (1).

This maximal final social welfare under $s(\cdot)$ is always greater than or equal to the social welfare achieved in a Vickrey auction when players bid truthfully. Additionally, for some inputs, it is strictly greater.

3.2 Sequential BC auctions

We can show that in analogy to sequential Vickrey auctions no dominant strategies exist except for the last player. In fact we can establish this for a wide class of Groves auctions. Details will appear in the full version of this work.

We shall thus focus, as in the case of sequential Vickrey auctions, on the weaker notion of optimal strategy. We have various natural optimal strategies that deviate from truth-telling, such as the following one:

$$s_i(\theta_1,\ldots,\theta_i) := \begin{cases} \theta_i & \text{if } \theta_i > \max_{j\in\{1,\ldots,i-1\}} \theta_j \\ (\theta_1,\ldots,\theta_{i-1})^*_1 & \text{if } \theta_i \leq \max_{j\in\{1,\ldots,i-1\}} \theta_j \\ & \text{and } i \leq n-1 \\ (\theta_1,\ldots,\theta_{i-1})^*_2 & \text{otherwise} \end{cases} \quad (2)$$

In contrast to the Vickrey auction, the analogue of Theorem 3.2 does not hold for the sequential BC auctions. In particular this means that optimal strategies are not dominant within the universe of optimal strategies.

We now turn to the question of existence of socially optimal strategies, which we answer negatively. This is again in contrast to the sequential Vickrey auction.

THEOREM 3.5. *The sequential BC auction does not admit socially optimal strategies except for the first and last player.*

The results established so far show that the sequential Vickrey auctions and BC auctions differ in many ways. We conclude by showing that they do share one property. Namely, within the universe of optimal strategies there exists an optimal strategy $s_i(\cdot)$ such that if all players follow it, then maximal social welfare is generated for all $\theta \in \Theta$.

THEOREM 3.6. *In the sequential BC auction for all $\theta \in \Theta$ and all vectors $s'(\cdot)$ of optimal players' strategies,*

$$SW(\theta, s(\cdot)) \geq SW(\theta, s'(\cdot))$$

where $s(\cdot)$ is the vector of strategies $s_i(\cdot)$ defined in (2).

This maximal final social welfare is always greater than or equal to the final social welfare achieved in a BC auction when players bid truthfully. Additionally, for some inputs, it is strictly greater.

4. REFERENCES

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