Linear Space is Impractical: Constructing Antidictionaries in Output-Sensitive Space

Lorraine A.K. Ayad Golnaz Badkobeh Gabriele Fici
Alice Héliou Solon P. Pissis

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  data compression; repetitive DNA patterns; etc.

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So, we define "Big" relative to the available internal memory (RAM).

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Theorem

1. *A word of length n has $\Theta(n)$ different MAWs.*
2. *All MAWs of a word of length n can be computed in $\mathcal{O}(n)$ time.*
3. *Any word w of length n is reconstructible in $\mathcal{O}(n)$ time from $\mathcal{M}_w$.*

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- For a random word of length n this is $r = \Theta(\log n)$

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e.g. k chromosomes of a genome or a collection of k documents

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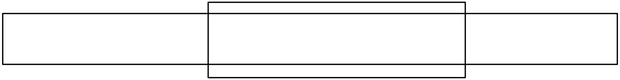

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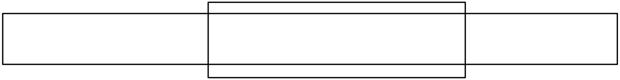

Example

Let $y_1 =$ abaab, $y_2 =$ bbaaab and $\ell = 5$. $y =$ abaab#bbaaab.
$\mathcal{M}^{\ell}_{y_1} = \{$bb,aaa,bab,aaba$\}$ and
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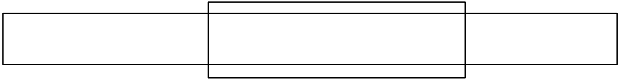

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$$\mathcal{M}^{\ell}_{y} \cap (\mathcal{M}^{\ell}_{y_1} \cup \mathcal{M}^{\ell}_{y_2}) = \{\texttt{aaaa,bab,aaba,abb,bbb}\}.$$

Combinatorial Results: x is not in $\mathcal{M}^{\ell}_{y_1} \cup \mathcal{M}^{\ell}_{y_2}$ (Case 2)

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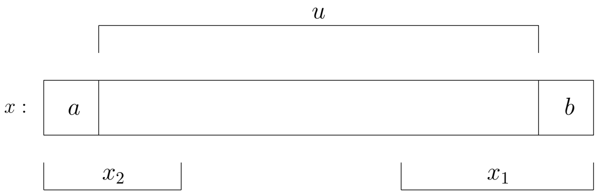

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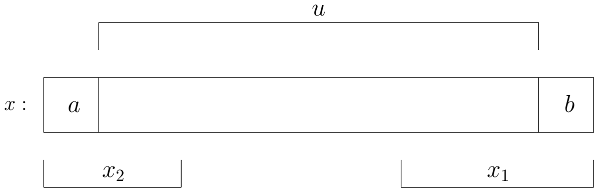

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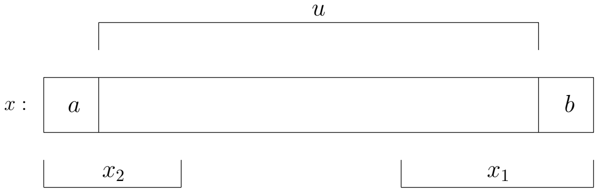

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Consider $x =$ abaaa $\in \mathcal{M}^{\ell}_{y} \setminus (\mathcal{M}^{\ell}_{y_1} \cup \mathcal{M}^{\ell}_{y_2})$ (Case 2 MAW).

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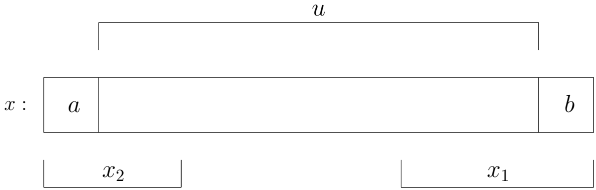

Example

Let $y_1 = \texttt{abaab}$, $y_2 = \texttt{bbaaab}$ and $\ell = 5$. $y = \texttt{abaab\#bbaaab}$.
$\mathcal{R}^{\ell}_{y_1} = \{\texttt{bb},\texttt{aaa}\}$ and $\mathcal{R}^{\ell}_{y_2} = \{\texttt{baab},\texttt{aba}\}$.

Consider $x = \texttt{abaaa} \in \mathcal{M}^{\ell}_{y} \setminus (\mathcal{M}^{\ell}_{y_1} \cup \mathcal{M}^{\ell}_{y_2})$ (Case 2 MAW).

There is an $x_2 \in \mathcal{R}^{\ell}_{y_2}$ that is a prefix of abaa and this is aba.

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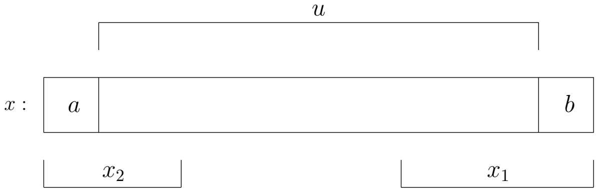

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There is an $x_1 \in \mathcal{R}^{\ell}_{y_1}$ that is a suffix of abaaa and this is aaa.

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Theorem

We compute $\mathcal{M}^{\ell}_{y_1}, \ldots, \mathcal{M}^{\ell}_{y_1\#\ldots\#y_k}$ *in* $\mathcal{O}(kn + \sum_{N=1}^{k} ||\, \mathcal{M}^{\ell}_{y_1\#\ldots\#y_N} \,||)$ *total time using* $\mathcal{O}(\textsc{MaxIn} + \textsc{MaxOut})$ *space.*

Experiments on Human Genome

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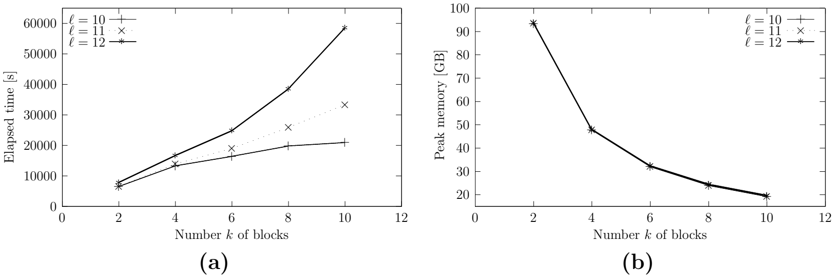

Figure: Time-space tradeoff

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Thanks!