Why are blackouts in power grids heavy-tailed?

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Overview

- Background: energy networks, blackouts, heavy tails
- Main insight of this talk
- Mathematical model and analysis
- Case study
- Conclusion and outlook
Blackouts in power grids

$80B$ of annual economic damage to US economy from blackouts
Blackouts in the past fifty years

(source: dnv-gl)
Can we mathematically understand blackouts?

- It took TNO 5 months to figure out the cause of the 2018 Schiphol power outage (the TenneT investigation still has not been concluded)
- “It is not complex, but complicated”
- “It is not possible to come up with a both interesting and useful result”
- To predict and detect anomalies, should we use simple black box methods from machine learning or sophisticated high-dimensional nonlinear models?
- At least one feature of blackouts is not complicated
Pareto laws in power grids (Hines 09)

\[ P(S \geq x) = c x^{-\alpha} \]
\[ P(\text{Blackout size} \geq x) = c x^{-\alpha} \]
\[ P(T \geq x) \]

\[ S = \text{Lost power in MW during blackout} \]
\[ T = \# \text{affected customers during blackout} \]

WHY?
Rare Events depend on “Tail Behaviors”

Light-Tailed Distributions
- Extreme Values are Very Rare
- Normal, Exponential, etc

Heavy-Tailed Distributions
- Extreme Values are Frequent
- Pareto Law, Weibull, etc
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Systemwide rare events arise because EVERYTHING goes wrong.
(Conspiracy Principle)

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Systemwide rare events arise because of A FEW Catastrophes.
(Catastrophe Principle)

Heavy tails are not as well understood as light tails.
Examples and properties of heavy tails

As $x \to \infty$:

- Pareto tails (or power tails): $P(X > x) \approx x^{-\alpha} = e^{-\alpha \log x}$

- Lognormal tails: $P(X > x) \approx e^{-\alpha (\log x)^2}$

- Weibull tails: $P(X > x) \approx e^{-\alpha x^\beta}$, $\beta \in (0, 1)$.

Key properties:

$$E[e^{\varepsilon X}] = \infty, \quad \varepsilon > 0.$$

$$P(X_1 + \ldots + X_n > x) \sim P(\max_{i=1,\ldots,n} X_i > x).$$
Heavy Tails are Everywhere:

Computer systems
- delays, files, ...

Finance
- losses

Social networks
- popularity, contagion

Energy Systems
- blackouts

How do heavy tails occur?
Heavy tails can occur in many ways

- Exogenous factors (e.g. job sizes in queueing)
- Multiplication (e.g. gains or losses in finance)
- Preferential attachment (social, and other networks)
- (Self-organized) criticality
- ...

Existing work on blackouts, based on model simulation output data, show long-range correlations in outages, and attributes this to criticality. Earlier work suggests the usage of critical branching processes.
Our contribution: a different explanation

Let $C$ be the size of a city, in terms of number of people, and let $T$ be the size of a blackout, in terms of number of customers affected. Both have statistically significant, almost identical power law for US:

$$P(C > x) \approx x^{-1.37} \quad P(T > x) \approx x^{-1.31}.$$ 

German city sizes: power law with index 1.28
log-log plots and Hill plots

Intuition

1. Initially the network is fully connected and supply equals demand.

2. A failure occurs. Energy is rerouted according to laws of physics, resulting in additional overloads/failures. A cascade occurs.

3. At some point the network stops being connected. At least one of the network components has a shortage.

4. Demand in each network component is proportional to sum of the cities in that component.

5. Since city sizes are heavy-tailed, the sum roughly equals the maximum. A heavy-tailed number of consumers (in a big city) will have a shortage.

To make this rigorous, we need to show that the mismatch between supply and demand after a network cut is heavy-tailed.
Mathematical model

- Graph with heavy-tailed sinks:
  Demand at node $i$: $X_i$, with $P(X_i > x) \sim cx^{-\alpha}$.
  $X = (X_1, \ldots, X_n)$.

- To model electricity, we use the DC load flow model. Network topology and reactances are all encoded in the load-flow matrix $V$, supply vector equals $g$, leading to power flows $V(g - X)$.

- We consider three stages in our model:
  - Planning
  - Operation
  - Emergency
Model: operational stage (DC-OPF)

Generation $g_i$ in each node $i$ is computed by solving

$$\min \frac{1}{2} \sum_{i=1}^{n} g_i^2$$

$$\sum_{i} g_i = \sum_{i} X_i$$

$$-\bar{f} \leq V(g - X) \leq \bar{f}.$$

This determines the network flows $F = V(g - X)$ which play a role in the cascade.

We determine the line limits $\bar{f}$ in a planning problem.
Model: planning stage

Given $n$ cities with random sizes $X_1, \ldots, X_n$ and given a network topology, we determine line limits $\bar{f}$ by solving an unconstrained OPF problem:

$$\min \frac{1}{2} \sum_{i=1}^{n} g_i^2$$

subject to the balance constraint

$$\sum_i g_i = \sum_i X_i.$$  

The planning problem has solution $g_i = \bar{X}_n$ for $i = 1, \ldots, n$, with $\bar{X}_n = (1/n) \sum_{i=1}^{n} X_i$ the average city size. We now let $\lambda \in (0, 1)$ and set

$$\bar{f} = \lambda V (\bar{X}_n e - X) = -\lambda V X,$$

This vector will be used in the operational stage.
Model: operational stage (DC-OPF)

\[
\min \frac{1}{2} \sum_{i=1}^{n} g_i^2
\]

\[
\sum_i g_i = \sum_i X_i
\]

\[-\bar{f} \leq V(g - X) \leq \bar{f}\]

with

\[
\bar{f} = \lambda V(\bar{X}_n e - X) = -\lambda VX.
\]

This leads to actual line flows \(F = V(g - X)\).
Model: emergency stage

Given: line flows $\mathbf{F} = \mathbf{V}(\mathbf{g} - \mathbf{X})$

- Start with one random line outage.
- Recompute power flows.
- Additional lines fail if new line flow exceeds $\bar{f}_i/\lambda$.
- If islands occur, load or generation is shed proportionally.
- $T$: size of total load shed once cascade is over.
Main result

Let $X$ be a generic city size, with $P(X > x) \sim C_X x^{-\alpha}$. Note that $T$ is the blackout size [in terms of number of customers affected]

$$P(T > x) \sim C_T x^{-\alpha}, \quad x \to \infty,$$  \hspace{1cm} (1)

$$C_T = C_X n \sum_{j=1}^{n} P(|A_1| = j)(1 - j\lambda / n)\alpha.$$  \hspace{1cm} (2)

$A_1$ denotes the (random) set of nodes making up the island with the largest city in the network.

Proof idea: heavy-tailed large deviations theory allows us to consider the case of a single big city, and many small cities, reducing the analysis of the cascade to a single-sink network.

Rigorous proof for almost all $\lambda$ [when $C_T$ is continuous in $\lambda$].
Numerical studies

Our result holds up against several simulation studies

- Generation constraints
- Extending DC to AC
- No heavy tailed blackout size if city sizes are uniformly distributed
- IEEE test networks
- Synthetic scalable networks, tailored to power grids [Wang, Scaglione, and co-authors]

Critical assumption: frozen vs random city sizes [quenched vs annealed]
SciGRID case study - Impact of $\lambda$

Figure: Dissection of biggest blackout for loading factors $\lambda = 0.7$ (left panels), $\lambda = 0.8$ (middle) and $\lambda = 0.9$ (right) in terms of the cumulative number of affected customers at each consecutive stage as displayed in the top charts with the biggest jump colored red.
Concluding remarks

- Main insight
  - Network upgrades only make the pre-factor in front of power law smaller. This provides limited effect in preventing large blackouts.
  - Duration of blackout is light-tailed, and seems independent of size.
  - It therefore seems to make more sense to invest in making cities more resilient, rather than to invest in network upgrades.

- City sizes are not (very) random. Simulation studies suggests Pareto laws occur in frozen networks of $\geq 10^4$ cities [this includes North America]. For smaller networks (like Germany), the principle of one big jump still seems to hold.

- New class of network models, of which topologies may not be scale free, but scale-free phenomena occur due to city sizes. Currently looking at other transportation networks.