MP-SPDZ – A Versatile Framework for Multi-Party Computation

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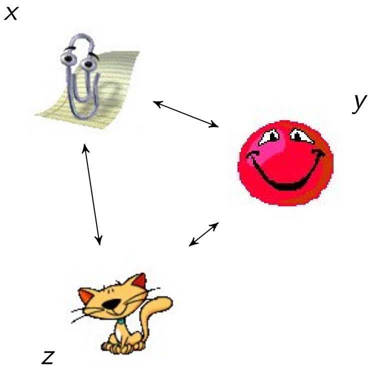
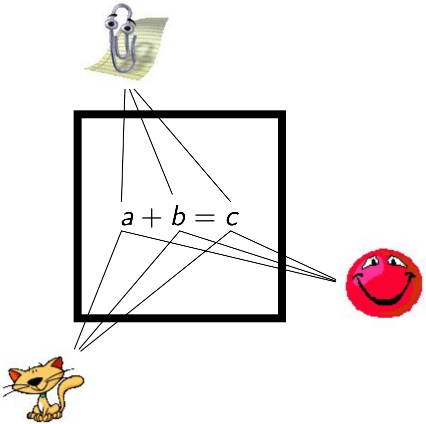
Secure Multiparty Computation



Wanted: $f(x, y, z)$

- Computation on secret inputs
- Replace trusted third party
- Central questions in MPC
  - How many honest parties?
  - Dishonest parties still follow the protocol?
- MP-SPDZ supports $>$ 30 protocols across all properties

Unifying MPC: Black Box



Parties

- Have handles to values
- Don't know the values
- Can input values
- Can agree on computations creating new values
- Can agree on outputting values

Unifying MPC: Basic Operations

	Communication
Addition	✗
Multiplication	✓

Multiplication with Random Triple (Beaver Randomization)

Have: $\boxed{x}$, $\boxed{y}$, addition in black box

Want: $\boxed{x \cdot y}$

Multiplication with Random Triple (Beaver Randomization)

Have: $\boxed{x}$, $\boxed{y}$, addition in black box

Want: $\boxed{x \cdot y}$

$$\begin{aligned} x \cdot y &= (x + a - a) \cdot (y + b - b) \\ &= (x + a) \cdot (y + b) - (y + b) \cdot a - (x + a) \cdot b + a \cdot b \end{aligned}$$

Multiplication with Random Triple (Beaver Randomization)

Have: $\boxed{x}$, $\boxed{y}$, addition in black box, ($\boxed{a}$, $\boxed{b}$, $\boxed{a \cdot b}$ for random a, b)

Want: $\boxed{x \cdot y}$

$$\begin{aligned}\boxed{x \cdot y} &= (x + a - a) \cdot (y + b - b) \\ &= (x + a) \cdot (y + b) - (y + b) \cdot \boxed{a} - (x + a) \cdot \boxed{b} + \boxed{a \cdot b}\end{aligned}$$

Masked and revealed
(one-time pad)

Random secret triple
(preprocessed)

Scaling Up: I/O Parallelization

$$z = x \cdot y$$
$$u = z \cdot w$$

$$z = x \cdot y$$
$$u = v \cdot w$$

Scaling Up: I/O Parallelization

$$z = x \cdot y$$
$$u = z \cdot w$$

1. Compute z
2. Compute u

$$z = x \cdot y$$
$$u = v \cdot w$$

1. Compute z and u

Goal: Automatize I/O Parallelization

Manual parallelization is tedious:

$x_{10} = x_2 \cdot x_3$	$x_{20} = x_4 + x_6$	$x_{30} = x_{19} \cdot x_1$	$x_{40} = x_{12} \cdot x_{39}$	$x_{50} = x_{28} + x_{16}$
$x_{11} = x_8 + x_4$	$x_{21} = x_{16} + x_2$	$x_{31} = x_{16} + x_{26}$	$x_{41} = x_{34} + x_7$	$x_{51} = x_{15} + x_{38}$
$x_{12} = x_{10} \cdot x_1$	$x_{22} = x_0 + x_{12}$	$x_{32} = x_0 \cdot x_{10}$	$x_{42} = x_{32} + x_5$	$x_{52} = x_{50} \cdot x_{46}$
$x_{13} = x_7 + x_9$	$x_{23} = x_{22} + x_{14}$	$x_{33} = x_{26} + x_{32}$	$x_{43} = x_{12} + x_{26}$	$x_{53} = x_{19} + x_2$
$x_{14} = x_7 \cdot x_1$	$x_{24} = x_{11} + x_{19}$	$x_{34} = x_7 + x_3$	$x_{44} = x_{43} \cdot x_{38}$	$x_{54} = x_{20} \cdot x_{13}$
$x_{15} = x_9 + x_{12}$	$x_{25} = x_4 \cdot x_{19}$	$x_{35} = x_9 \cdot x_{29}$	$x_{45} = x_{38} + x_{14}$	$x_{55} = x_{21} + x_{22}$
$x_{16} = x_{13} \cdot x_{14}$	$x_{26} = x_{23} \cdot x_9$	$x_{36} = x_{33} + x_{22}$	$x_{46} = x_{44} \cdot x_{27}$	$x_{56} = x_{19} \cdot x_6$
$x_{17} = x_0 + x_{11}$	$x_{27} = x_7 \cdot x_5$	$x_{37} = x_{29} \cdot x_{24}$	$x_{47} = x_{22} + x_{24}$	$x_{57} = x_{46} + x_1$
$x_{18} = x_{11} \cdot x_{15}$	$x_{28} = x_{13} + x_{21}$	$x_{38} = x_{16} + x_{23}$	$x_{48} = x_{39} \cdot x_{38}$	$x_{58} = x_{38} \cdot x_{55}$
$x_{19} = x_{13} \cdot x_7$	$x_{29} = x_{14} + x_{27}$	$x_{39} = x_{15} + x_{37}$	$x_{49} = x_{21} \cdot x_3$	$x_{59} = x_{47} + x_{29}$

Use Case: Parallel Maximum

```
from util import max

M = sint.Matrix(n_rows, n_cols)
res = sint.Array(n_rows)

# populate M
...

for i in range(n_rows):
  res[i] = M[i][0]
  for j in range(1, n_cols):
    res[i] = max(res[i], M[i][j])
```

Want

Maximum of every row

Without optimization

`n_rows * (n_cols - 1)` rounds of `max`

MP-SPDZ optimization

`n_cols - 1` rounds of `max`

Use Case: Parallel Maximum

```
from util import max, tree_reduce

M = sint.Matrix(n_rows, n_cols)
res = sint.Array(n_rows)

# populate M
...

for i in range(n_rows):
  res[i] = tree_reduce(max, M[i])
```

Want

Maximum of every row

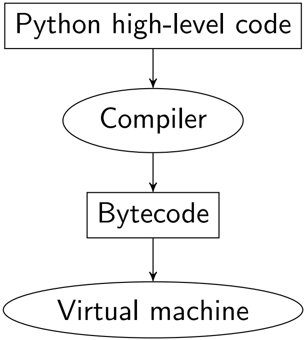
Without optimization

`n_rows * log(n_cols)` rounds of `max`

MP-SPDZ optimization

`log(n_cols)` rounds of `max`

Toolchain Overview



Compiler

- Implemented in Python
- Optimization (reduce network rounds)
- Library for various arithmetic: integer, fractional, mathematical
- Machine learning functionality

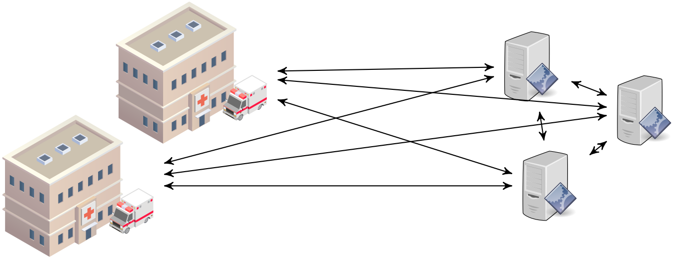
Virtual machine

- One per protocol
- Implemented in C++
- Optimized for speed

Section 2

Machine Learning

Privacy-Preserving Machine Learning



Outsourced training

- Data owners share their inputs among computing parties
- Computing parties train a model securely using MPC
- Output model OR use it for secure inference

Deep Learning

- Established supervised machine learning concept
  (known input-output combinations)
- Computation as chain of functions (layers)
- Some functions have parameters to be changed during training
- Function quantifying quality (loss)
- Chain rule allows changing of parameters toward minimizing loss
  (backward propagation)

Secure Deep Learning Building Blocks

Quantization

Represent x as $\lfloor x \cdot 2^f \rceil$ to use integer computation for fractional numbers.

Mathematical functions

- Comparison
- Division
- Exponentiation
- Logarithm
- Square root

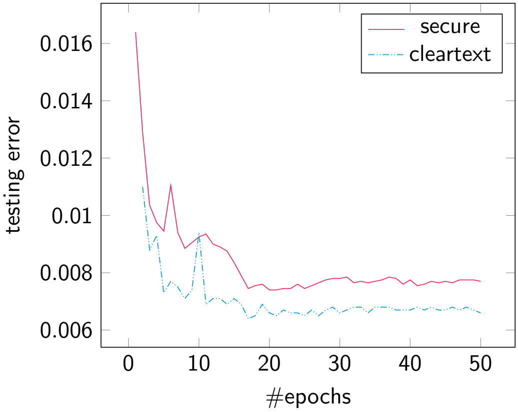
MNIST – Handwritten Digit Recognition

By Josef Steppan - Own work, CC BY-SA 4.0,

https://commons.wikimedia.org/w/index.php?curid=64810040

- "Hello world" of machine learning
- Input: 28x28 gray-scale
- Output: 0–9
- Demonstrates utility of convolution (local linear function)

Results for LeNet



- Convolutional neural network by LeCun et al.
- 4 linear layers
- AMSgrad optimizer (improved stochastic gradient descent)
- Co-located AWS c5.9xlarge
- Time per epoch: 9 minutes
- 1 hour for 99% accuracy

Section 3

Outlook

Secure Computation Suitability

More suitable

- Small input/output: e.g. mathematical functions
- Predictable computation path: e.g. matrix multiplication

Less suitable: data-dependent computation path

- Graph algorithms
- Dictionary data structure

Opinion Page

- More utility, less mystery
- Beware of lower bounds
- Tell me

Links

https://github.com/data61/MP-SPDZ

https://mp-spdz.readthedocs.io

https://ia.cr/2020/521

https://twitter.com/mkskeller