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DICE: Discrete inverse continuity equation for learning population dynamics

**Abstract:** We introduce the Discrete Inverse Continuity Equation (DICE) method, a generative modeling approach that learns the evolution of a stochastic process from given sample populations at a finite number of time points. Models learned with DICE capture the typically smooth and well-behaved population dynamics, rather than the dynamics of individual sample trajectories that can exhibit complex or even chaotic behavior. The DICE loss function is developed specifically to be invariant, even in discrete time, to spatially constant but time-varying spurious constants that can emerge during training; this invariance increases training stability and robustness. Generating a trajectory of sample populations with DICE is fast because samples evolve directly in the time interval over which the stochastic process is formulated, in contrast to approaches that condition on time and then require multiple sampling steps per time step. DICE is stable to train, in situations where other methods for learning population dynamics fail, and DICE generates representative samples with orders of magnitude lower costs than methods that have to condition on time. Numerical experiments on a wide range of problems from random waves, Vlasov-Poisson instabilities and high-dimensional chaos are included to justify these assertions.