125 years of inventory management, 100 years of EOQ and 40 years of vLm

In this paper, we want to dwell on the fact that Harris' formula was published in 1913 . This formula, which was later given a name by Camp, Wilson and in the Netherlands by Goudriaan, seems to mark the beginning of a scientific approach to inventory management. A scientific approach that can also be found in Goudriaan's inaugural address (1926) entitled "Business theory as Theoretical and as Applied Science". Much has been achieved in the past 100 years, in which Dutch professionals and scientists have made important contributions. What is special about the Dutch situation is that we can observe an almost continuous cross-pollination between professional development and application of inventory management concepts and models and scientific development of inventory management concepts. Just like Goudriaan, there are still scientists active in the field of inventory management, who have applied and developed inventory management concepts during a previous professional career and then continued to work on deepening and generalizing models and concepts, leading to implementations that demonstrate the importance of a scientifically sound analysis of inventory management concepts.

This contribution aims on the one hand to provide a historical overview of the most important milestones in inventory management, on the other hand we hope to make it clear that the problem of the inventory management of one item in one location with one supplier and one "homogeneous" market is scientifically ready. The demarcation of the problem indicated immediately indicates where scientific questions still lie in the field of inventory management: multiple items, multiple locations, multiple suppliers and non-homogeneous markets. There are strong indications that for all these extensions of the basic situation, finding an optimal strategy or solution within a formulated model is impossible due to the so-called "curses of dimensionality". We implicitly assume that all extensions relate to stochastic demand, With this fact the way is open for further cooperation between professionals and scientists: professionals who, based on concrete problems, develop ideas regarding effective management strategies, which can then be carefully tested and, if possible, generalized and refined by scientists.

Below we first define the basic model indicated above: inventory management of one item at one location with one supplier and one "homogeneous" market. Then we walk more or less chronologically through milestones, starting with an article by Edgeworth from 1888, i.e. 125 years ago, about what has come to be called the newsboy problem. We pay attention to the impact of technological and social developments on the applicability of the models and associated formulas from stock theory. Technological developments in production and distribution have led to economically justified reductions in batch sizes, while market developments have led to greater product diversity, making it increasingly difficult to predict demand. Finally, globalisation has led to longer lead times at certain links in the chain, especially at continental distribution centres and final assembly of electronic products. We show that these three partly parallel developments have led to well-known formulas for safety stocks losing their validity. Relatively recent scientific results show that these formulas can be replaced by algorithms, which do have general validity and can be built into standard software.

Basic inventory control model

We specify the basic model as follows. We consider a question process defined by

T_n	Time between registration (n-1)e customer demand and n^e customer demand
D_n	Quantity n^e recorded customer demand
Q_k	Quantity k^e order quantity
L_k	Delivery time k^e order quantity
r	Cost of capital per unit of money per unit of time
p	Shortage cost per unit of product in stock per unit of time
A	Cost per order
v	(Cost) price per unit of product
P_1	Chance of non-negative stock just before order entry
P_2	Long-term fraction of demand delivered directly from stock.
P_3	Probability of non-negative stock at any registration point

This defines the operational processes. The operational performance of an inventory point is determined by the *net inventory*, the control of the inventory point is determined by the inventory *position*.

$X(t)$	Net stock at time t, physical stock minus backorders at time *t*
$Y(t)$	Inventory position at time *t*, the sum of net stock and outstanding orders

The control of the stock point is determined by the following parameters:

R	Time between consecutive potential order moments, review period
s	If the stock position at a potential order moment is below the order *point s* , an order will be placed
S	If the control of the stock point uses "*Order-up-to*" *level* S, then at the time of ordering the stock position is made equal to *S*
Q	If the control of the stock point uses the order quantity Q, the order quantity must be a multiple of *Q*

If the stock is controlled in real-time, *then R*=0, and we omit *R*. This creates the following possible strategies: (s,S), (s,nQ), (R,s,S) and (*R,s,nQ*). The *n* for the Q indicates that, after the stock position falls below s, Q is ordered several times, until the stock position is back above the order point *s*. The notation used is that of Peterson and Silver (1979). Unfortunately, other notations have since been used by other authors, causing some confusion in the inventory management world.

Edgeworth (1888), Newsboy problem

The famous statistician Edgeworth was the first to publish on the so-called newsboy problem, albeit in the context of cash stock management (Edgeworth (1888)). The corresponding basic model is described by

$$R = 1, E[T_n] = 1, \sigma(T_n) = 0, E[L_k] = 0, A = 0 .$$

In the newsboy problem, the product to be delivered is only saleable for one day (period), while the demand per period is uncertain. The optimal order size *S* is determined by solving the following equation,

$$P\{D \le S\} = P\{X \ge 0\} = P_3 = \frac{p}{p+h}.$$

Here *D* stands for the demand per period, *X* for the net stock at the end of the period, *p* for the penalty costs per product unit shortage, *h* for the inventory costs per product unit excess and P_3 for the non-stockout probability at the end of the period (i.e. at any stock registrationmoment). This result appears to be much more widely applicable than is usually indicated in the literature. It turns out that for all known ordering strategies ((s,S), (s,nQ), (R,s,S) and (*R,s,n*Q), the strategy that minimizes the sum of order costs, inventory costs and shortage costs satisfies the requirement

$$P_3 = \frac{p}{p+h}.$$

It even appears that this requirement continues for "multi-item multi-echelon" stock systems, i.e. for networks of stock systems, although the ordering strategies for these systems must meet additional requirements (De Kok and Fransoo (2003)).

This result underscores the importance of the P_3 as a service measure. The non-stockout probability at any given registration moment is not only the key to optimal management strategies, it is also much easier and more accurate to determine than the fill rate P_2, the fraction delivered directly from stock, for which one must be able to record both the total demand and the directly delivered demand. This seems simple, but in many cases the real question is not known. For example, in retail, where no-sales are not registered, but also in companies that sell standard products, where a potential customer asks several suppliers for a price.

Among other things, there is no other reason for the lack of a case for the P_3 as a service measure than that simple mathematical expressions for this service measure were not available before 1980. This is now the case and they are simple enough to be built into the software (De Kok (1991)).

Harris (1913), economic order size

Harris (1913) determined the optimal order quantity, minimizing the sum of inventory costs and order costs in the situation where the demand per unit of time *D* is constant. Harris' assumptions are

$$R = 0, E[T_n] = 0, \tfrac{E[D_n]}{E[T_n]} = D.$$

We assume again that *h* stands for the cost per product unit in stock. The cost per order is *A.* Then the optimal (economic) order quantity Q^*, the Economic Order Quantity (EOQ),

$$Q^* = \sqrt{\frac{2AD}{h}}.$$

From this result follows the optimal order frequency, $f^* = \dfrac{D}{Q^*}$

$$f^* = \sqrt{\frac{r}{2A}}\sqrt{Dv}, \text{ met } h = vr \, .$$

We assume that inventory costs are primarily determined by interest expense on capital *r* and value/cost per unit *of product v*. With this spelling we see that the optimal ordering frequency shows a more appealing result: as the *turnover* of a product is *higher*, theproduct must be ordered *more often*. With this, the ABC principle immediately follows from the optimal order frequency, that A-articles, the fast runners, should receive more attention than B and C articles. After all, paying attention in inventory control is the same as considering an order. ABC is not just *common sense*, Harris' model provides its scientific underpinnings.

Now, 100 years later, we can see that Harris' result is still applicable. Research has shown that the EOQ can also be used in situations where demand is stochastic. Also in so-called multi-echelon stock systems, i.e. systems consisting of a network of interrelated stock points, as is customary in reality, the EOQ per stock point can be used as a first step in determining the order frequency. Given these order frequency(s), one can then determine safety stocks that lead to the desired operational performance.

Oddly enough, there has been a brief period of time when the EOQ has been denounced. This took place in the 70s and 80s of the last century: the EOQ would be the *cause* of the large order series, which would put Western industry at a competitive disadvantage compared to the emerging Japanese industry. I suspect Shigeo Shingo would have laughed in his fist if he had heard this. After all, he worked on shortening changeover time in order to become more flexible. Harris' formula indicates this immediately, because the optimal order quantity becomes smaller, as the order costs (conversion costs) become lower. Shingo's pragmatic methods led to both shorter changeover times and lower changeover costs. Harris' formula should be used as a management tool!

In the 60s to the present, countless variants of Harris' model have been studied. The question is whether all this work has added much. Concrete applications usually involve determining the relevant costs, the costs that are influenced by the decision derived from the model. If we assume constant demand per unit of time, finding expressions for these relevant costs is easy. With tools such as Excel, an optimal order series can then be determined. When the cost structure is correctly determined, the result found will have the same robustness as the EOQ.

Whitin (1953), Safety stock

In Hadley and Whitin (1963) the authors give a brief historical overview of the state of affairs up to that point. They note that Whitin (1953) is the first English-language book to consider stochastic stock models. That's why I hereby give the *credit* for the safety stock to Tom Whitin. The security stock is the *average net stock just before order entry*. Hadley and Whitin (1963) derive optimal parameter values for various stock models, from which expressions for the safety stock can be directly derived. Nevertheless, it seems that in the course of the 60s, 70s and 80s of the last century authors of books on inventory management, including Van Hees and Monhemius (1970) and Fogarty and Hoffmann (1983) have passed on to each other a special result, which I believe has taken on a life of its own to this day:

safety stock = $k\sqrt{L}\sigma$,

where the *safety factor k* is defined by

$$k = \Phi^{-1}(P_1),$$

with Φ the probability distribution of the standard normal distribution, P_1 the probability of positive stock just before the arrival of an order, L the (constant) delivery time of the product and σ the standard deviation of the demand per unit of time.

Presumably, this formula has become so important for the safety stock, because it is simple and allows for discussion of almost all important aspects of stochastic inventory management: the safety stock increases as

- the service level must be higher
- the delivery time is getting longer
- the variability in demand increases.

The safety stock formula is (just like the EOQ formula) an important *didactic* tool of logistics. As mentioned, it has taken on a life of its own, because it can be found in almost all textbooks since 1970, but is also used in countless software packages. This in itself would not be objectionable, but it is underexposed that the formula is only valid under *very strict assumptions*. And it is precisely these assumptions that have lost their validity over the course of the past 60 years, since Whitin (1953). Let's break down the assumptions:

1. The stock is recorded periodically and sometimes per unit of time (e.g. day, week).
2. The delivery time L is constant and a whole number of time units.
3. The demand per unit of time is normally distributed with mean μ and standard deviation σ
4. Questions in different time units are mutually independent.
5. The stock is controlled with a (R,s,Q) rule, with R=1.
6. At the time of ordering, the stock position is exactly the same as order point *s.*
7. The service criterion is the P_1 measure.
8. Not immediately available demand is backlogged.

Perhaps the most important common mistake in applying the formula is the assumption (7) that the service measure is the P_1 measure. Because many books talk about the service measure or degree without a very precise definition of this concept, the formula is also used when referring to the fill rate P_2. Many graduation theses contain this error, which makes the quantitative analysis, the business case one would now say, completely incorrect. Often the P_1 is also referred to as the non-stockout probability, with many believing that this corresponds to the probability that a customer will find stock on the shelf. This is not the case at all, as this non-stockout probability is defined from the perspective of the incoming order, and therefore **not** from the perspective of *the incoming customer*. Especially with large order quantities, it may be that the P_1 is close to zero, while the customer almost never misses out. After all, with a large order quantity, say six months of demand, there is always stock for five and a half months, which means at least 90% service level for the customer, while at an order point equal to zero one is always out of stock when the order arrives.

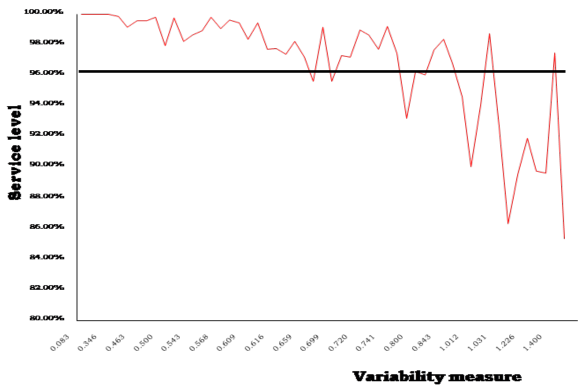
Among the above assumptions, there is also a simple expression that determines the safety stock for the fill rate P_2 by another expression for the safety factor k:

$$k = G_u^{-1}\left(\frac{Q(1-P_2)}{\sigma\sqrt{L}}\right),$$

where G u is the "normal loss function" (Peterson and Silver (1979)), which, like the standard normal distribution in most inventory control books, is included in the form of a table. We now see that the batch size does play a role in determining the safety stock, which in practice already means a huge (quantitative) improvement. After all, it is precisely from the above example that it becomes clear that the incorrect use of the P_1 measure leads to far too high stocks. Unfortunately, the commonly used formula for the k-factor is only valid if the above assumptions are valid and, in addition, the following assumption:

9. The net stock after receiving an order is positive.

This assumption (9) may have been valid in the fifties, where orders were still placed in large quantities, but nowadays it often happens that a temporary large demand per unit of time leads to a shortage that is only eliminated by several successive incoming small orders.



Thefigure above shows the *realized* service level at a company that has always been in the Supply Chain top 10 of Gartner (AMR) for the past 10 years, where they had entered 95% as a target value in the inventory module of their ERP system. It turned out that the empirical results found are *fully* explained by the use of the no longer valid formula for the safety factor for the P_2 measure.

This brings us to another assumption (3), which has lost its validity over time. Due to increasing product diversity, but also increasing order frequencies, which makes the relevant time unit shorter and shorter, the variability of demand per unit of time has increased significantly. With high variability, the normal distribution relative to its mean becomes too wide, causing too much of the probability mass to enter the negative half-plane. The shortening of the unit of time also makes the assumption (4) of independence between demand in different periods problematic. After all, if demand comes in on average once a week, with some variation, and the unit of time is a day, then the entry of a customer order implies that in the next three days the occurrence of the demand becomes unlikely. This implies dependence. If the demand per unit of time or the demand per customer can be more than one, the assumption (6) is also in question: at the time of ordering, the stock position will usually be below *s*.

Does this mean that everything that has been said about the safety stockpile in the last 60 years is false? Of course not!. We have begun to point out that there is no need to comment on the literature in a qualitative sense. In fact, books such as those by Fogarty and Hoffmann (1983) and certainly also Van Hees and Monhemius (1970) have made crucial contributions to the *understanding* of inventory management problems and the transfer of management *rules* to practice. In *quantitative* terms, there is a lot wrong, as indicated above. But there is indeed an extensive literature, in which all the problems outlined above have been addressed and practically solved (see, for example, De Kok (1991), Axsater (2000) and Zipkin (2000))). However, this literature is much less accessible to most professionals, as it is very mathematical in nature. But in essence, this is completely irrelevant. The developed results have now been converted into lightning-fast algorithms, which have been extensively tested (empirically and mathematically) in all kinds of inventory management situations. The management rules (s,S), (s,nQ), (R,s,S) and (*R,s,nQ*) are still equally relevant, but correct analysis is done via these mathematical algorithms.

We summarize the above with a concrete example. We assume that a stock point is controlled by an (*R,s,nQ*) rule. In the table below we see the major differences between P_1 and P_2. We see that the actual value for P_1 determined by computer simulation differs significantly from the target value. We also see that the P_1 and P_2 value is close to the target value according to the correct formula.

Target value P_1: 95%		classic formula		simulation		correct formula	
R=1, Q=200, $\mu = 100$, $\sigma = 50$, L=5	*s*	P_1	P_2	P_1	P_2	P_1	P_2
$\text{veiligheidsvoorraad} = k\sqrt{L}\sigma$	684	95.0%	98.8%	***84.0%***	***95.0%***	84.2%	94.2%
Correct formula	777	99.3%	99.9%	***95.0%***	***98.0%***	95.0%	98.4%

As long as the correct formulas are not built into software packages such as SAP as standard, and are not applied as standard in hbo and wo, companies will never be able to develop an effective control loop for their stock systems. Given the publication years of the sources mentioned for these correct algorithms, it is about time to start implementing after 100, and actually 125 of inventory management.

Inventory management in the future

Above, we argued and substantiated that the control of the one-product-one-location-a-supplier inventory control problem has been solved. That means we can focus on controlling stock point networks. The MRP I logic deserves a first point of attention. This is too simplistic, so planners have to constantly make manual adjustments to released orders, because the materials are not there: MRP I logic does not check for material availability, it only passes orders upstream. If only life were that simple! Over the past 10 years, we have trained hundreds of students who are aware of this deficiency of the MRP I logic and what alternative methods are now available to generate order releases tested against material availability . The biggest hurdle to large-scale implementation of these new concepts is the APICS philosophy that has been built around MRP I and MRP II. This has become paradigmatic, so that a discussion of the above quickly becomes a battle of faith. So perhaps we should start a new crusade. If we do this now and from the Netherlands, it will contribute to strengthening the position of the Netherlands as a logistics knowledge country.

Credentials

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