The Logic of Email Communication

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May 27, 2011

Modeling Email Communication

We want to model the **knowledge-theoretic** effects of emails.

How does the knowledge of agents change when they receive an email?

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Why is this interesting?

Arnold sends Bob an email stating *p*.

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So, Bob gets to know *p*.

Simple, right?



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 $\mathbf{K}_{B}p$



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 $\mathbf{K}_{B}p, \mathbf{K}_{A}\mathbf{K}_{B}p$



$\mathbf{K}_{B}p$, $\mathbf{K}_{A}\mathbf{K}_{B}p$, $\mathbf{K}_{B}\mathbf{K}_{A}\mathbf{K}_{B}p$

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$\mathbf{K}_{B}\mathbf{p}, \ \mathbf{K}_{A}\mathbf{K}_{B}\mathbf{p}, \ \mathbf{K}_{B}\mathbf{K}_{A}\mathbf{K}_{B}\mathbf{p}, \ \mathbf{K}_{A}\mathbf{K}_{B}\mathbf{K}_{A}\mathbf{K}_{B}\mathbf{p}, \ \dots$



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 $C_{A,B}p$



 $C_{A,B}p$



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 $C_{A,B}p, C_{B,C}p$



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 $C_{A,B}p$, $C_{B,C}p$, $C_{B,C}C_{A,B}p$



 $\mathsf{C}_{A,B}\mathsf{p},\ \mathsf{C}_{B,C}\mathsf{p},\ \mathsf{C}_{B,C}\mathsf{C}_{A,B}\mathsf{p},\ \neg\mathsf{K}_{A}\mathsf{C}_{B,C}\mathsf{C}_{A,B}\mathsf{p}$

Corporate email communication



Pattern of e-mail communication among 436 employees of HP Research Lab (From Adamic and Adar, 2005)

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 $K_1K_2p?$

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- So I forwarded the last email to Loes and Rohit with a BCC to Krzysztof.

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How to formalise this?

Agent *i* sends a note *n* to group of recipients G, with BCC's to group B:

 $s(i, n, G)_B$

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If we send a **forward**, the set of BCC recipients is **not** included Let $s(j, n, H)_C$ be an email with BCC group C.

Agent *i* forwards:

 $f(i, m, G)_B$

The original BCC group C is not in the forward.

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 $m_1 = s(L, n, \{F, R\})$

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Did I rectify my supervisor's mistake?

No! With our framework we can show:

 $\{m_1, m_2, m_3, m_4\} \models \neg K_L K_K K_R m_3$

Loes does not know that in the last email Krzysztof was in the BCC list.

Logical analysis

If Krzysztof sent his forward to all recipients in the first place, there would be common knowledge

 $C_{\{L,R,F,K\}}m_2$

With our framework we prove: common knowledge can only be achieved by an email to the **entire** group.

The only way to correct the mistake is to send a forward with the whole group as CC recipients: $f(F, m_3, \{L, R, K\})$.

Long-term objectives:

- Sound and complete axiomatization
- Proving decidability of the state checking in this framework

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Implementation using a model checker

References

Message Passing in a Dynamic Epistemic Logic Setting, Floor Sietsma and Jan van Eijck. To be presented at TARK, July 2011.

Common Knowledge in Email Exchanges, Floor Sietsma and Krzysztof R. Apt. Under submission.

For more infomation email sietsma@cwi.nl

s(i, 'hello', Floor){Jan,Krzysztof}