

# Data Assimilation and Machine Learning for Air Quality Forecasts - Emission Inversion

**Hai Xiang Lin**  
**Delft University of Technology &**  
**Leiden University**

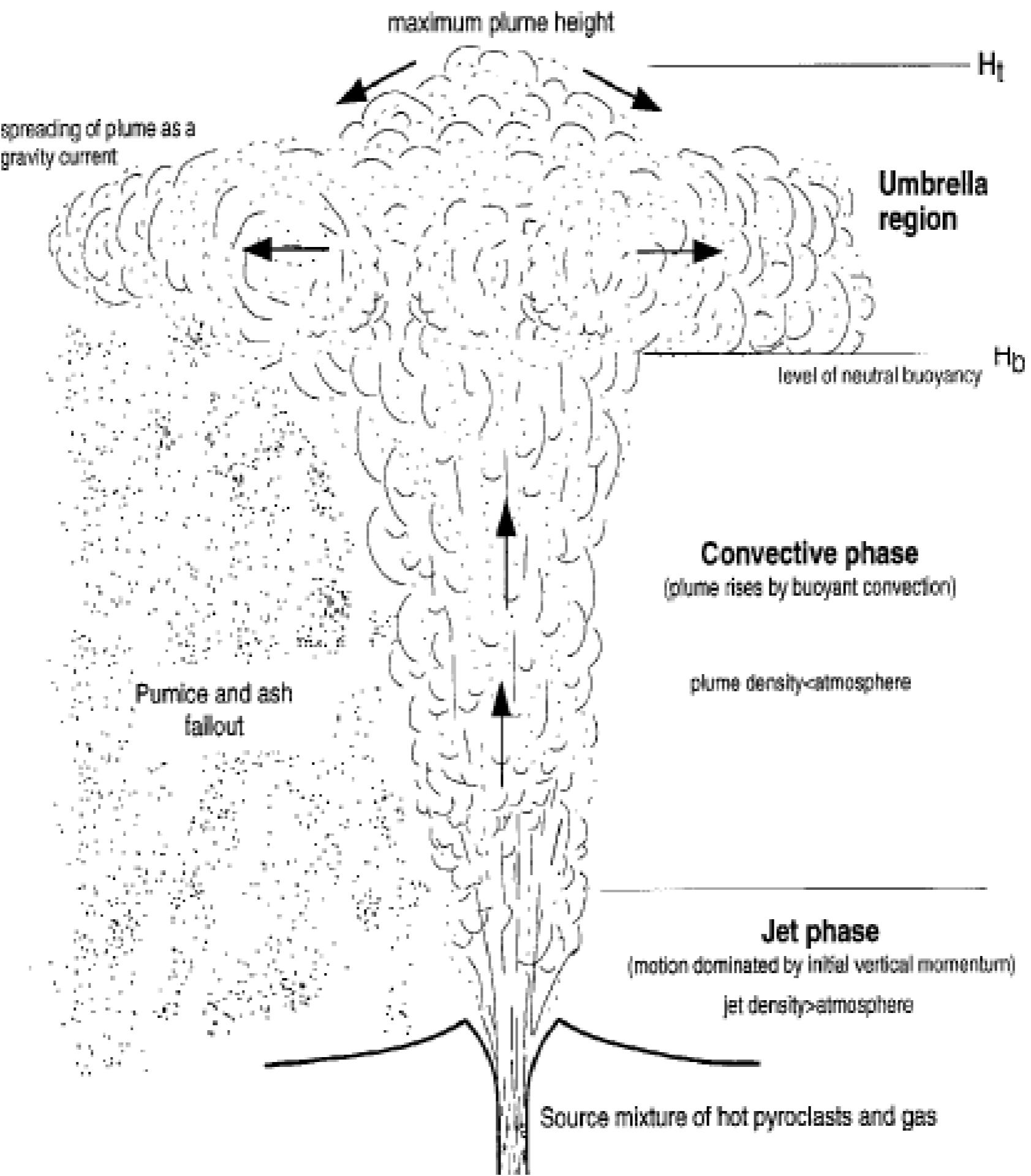
In cooperation with  
G. Fu, A.W. Heemink, J. Jin, S. Lu (TU Delft), A.J. Segers (TNO),  
T. Palsson (Iceland), K. Weber (Germany), A.J. Prata (Norway)



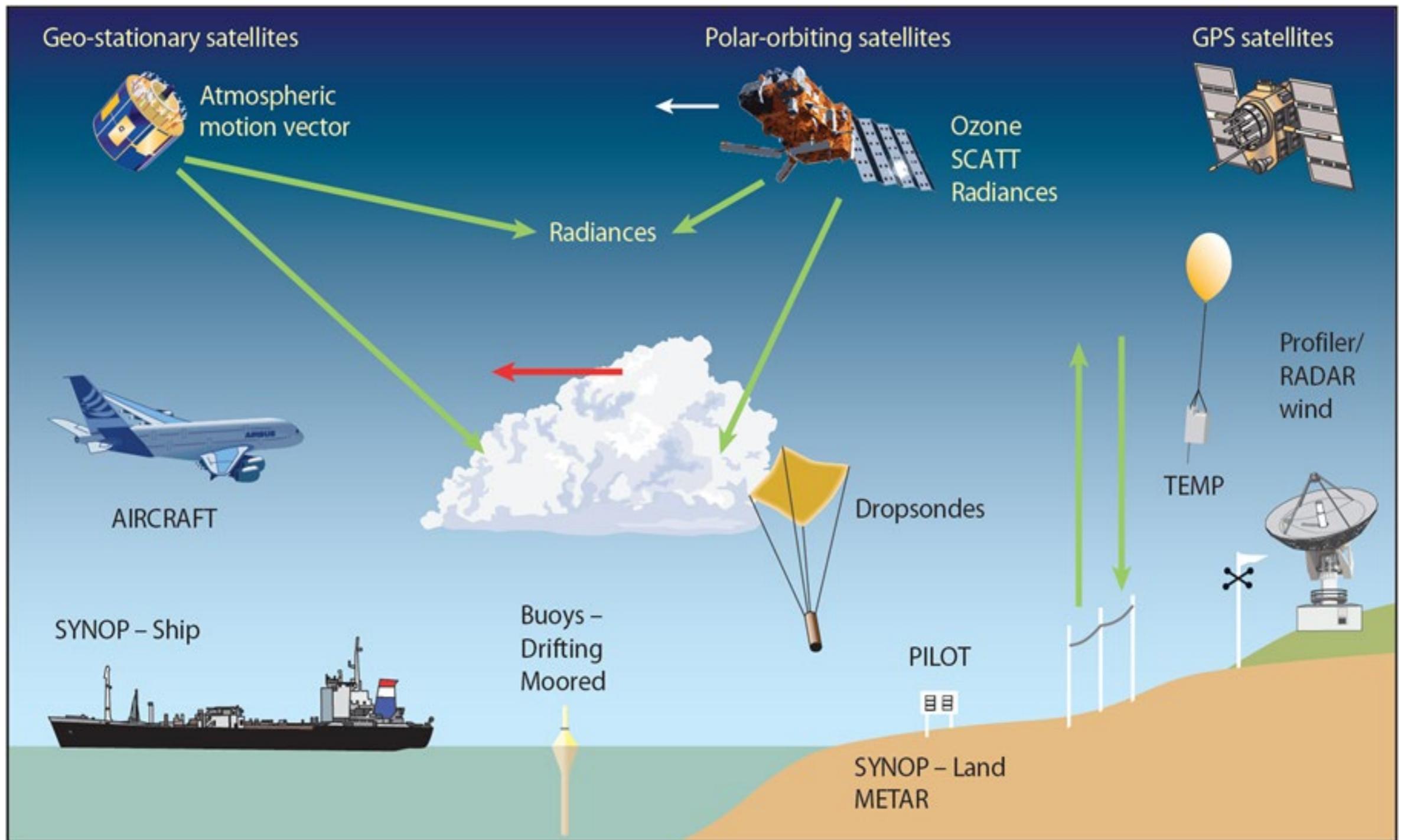
# Hazard of volcanic ash: An accurate forecast is important!



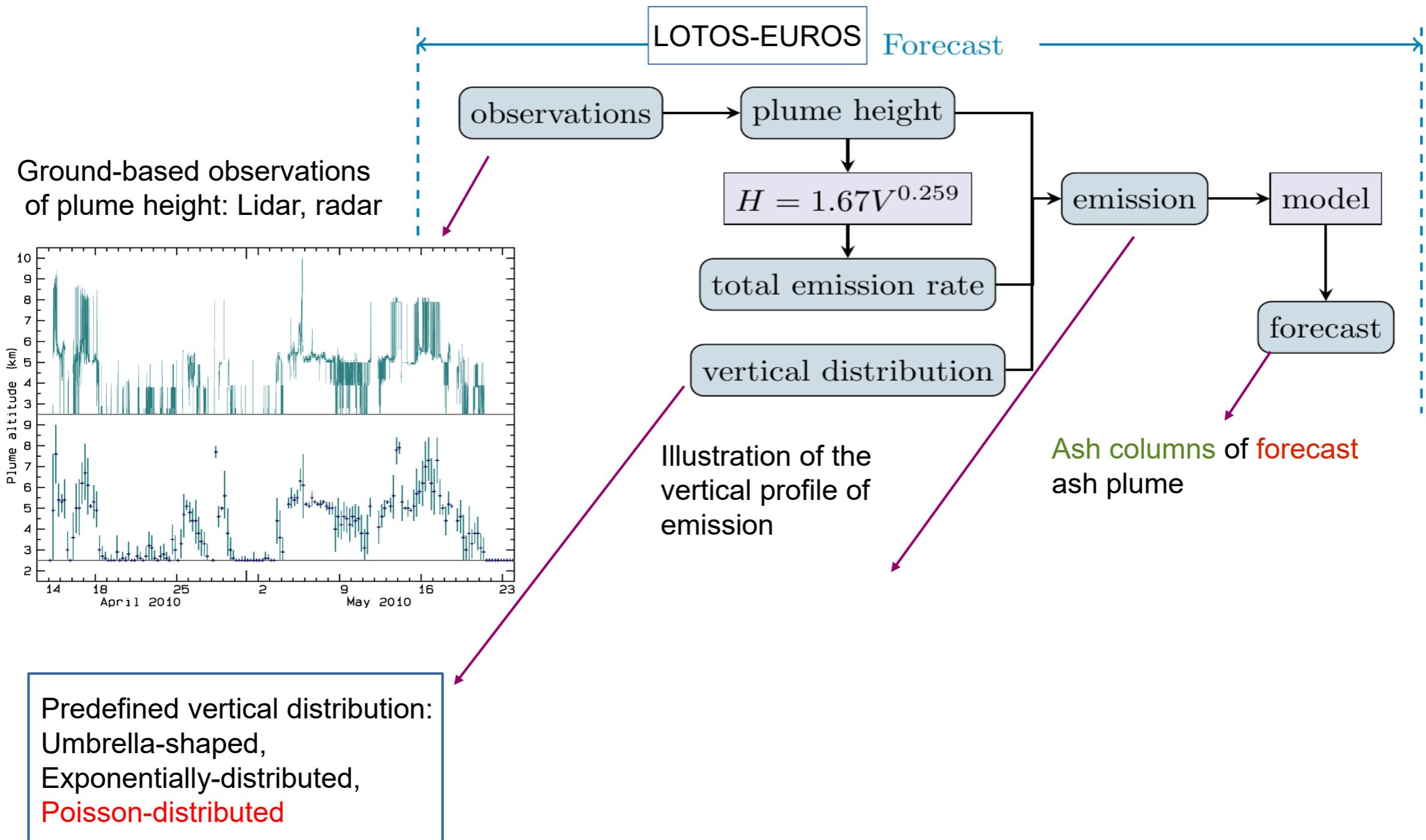




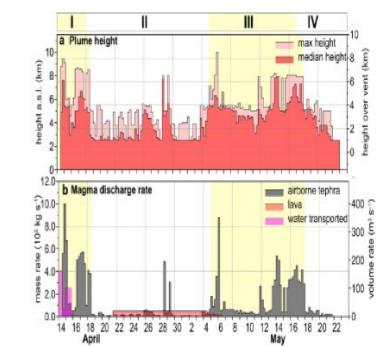
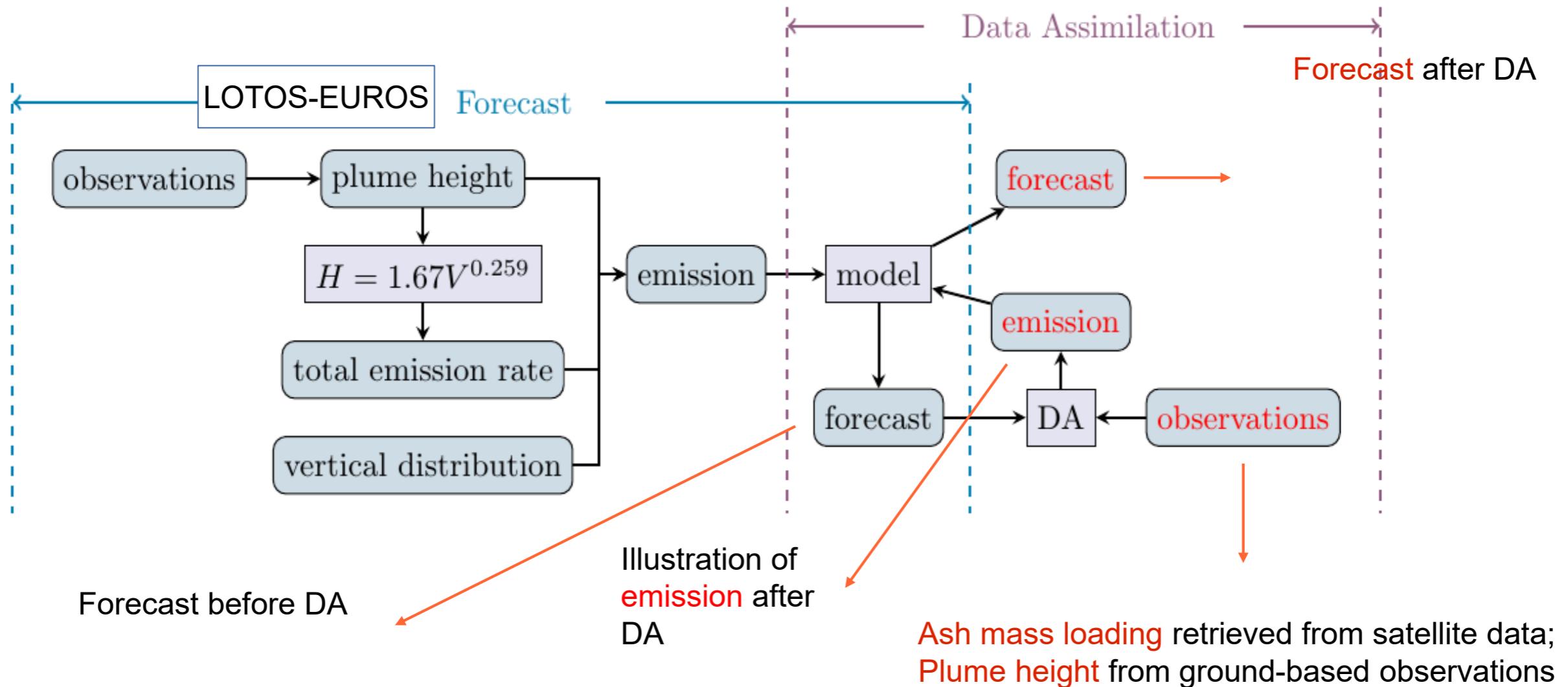
# Observations of volcanic ash activity



# Emission observation: plume height



# Forecast of volcanic ash cloud



# Data Assimilation

Data assimilation combines information of observations and models and their errors to get a best estimate of atmospheric state (or other parameters)

$$X_k^f = M_k(X_{k-1}^a, u) + w_{k-1}$$

$\xrightarrow{\quad N(0, B_k) \quad}$

$$y_k = H_k(X_k) + v_k$$

$\xrightarrow{\quad N(0, R_k) \quad}$

The prior and likelihood are

$$f(X) \propto \exp\left(-\frac{1}{2}(X - X^f)^T B_k^{-1}(X - X^f)\right), \text{ and}$$

$$f(y | X) \propto \exp\left(-\frac{1}{2}(y - X)^T R_k^{-1}(y - X)\right)$$

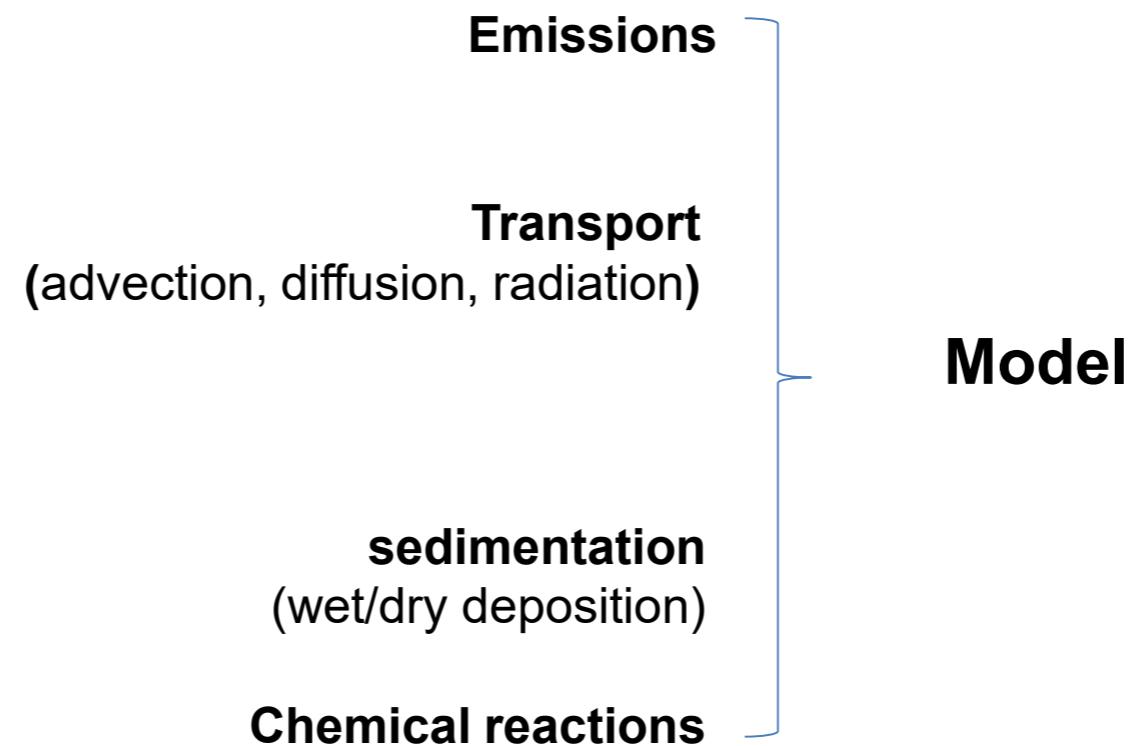
Posterior:  $f(X | y) \propto f(X)f(y | X) \propto \exp\left(-\frac{1}{2}J(X)\right)$

Minimize the cost function  $J$ :

$$J(X^a) = (X^a - X^f)^T B_k^{-1}(X^a - X^f) + (X^a - y)^T R_k^{-1}(X^a - y)$$

# Chemical Transport Model – LOTUS-EUROS

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial y} + W \frac{\partial C}{\partial z} = \frac{\partial}{\partial x} \left( K_h \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_h \frac{\partial C}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial C}{\partial z} \right) + E + R + Q - D - H$$



# Using 4D-Var to estimate the emissions

Model representation:

$$\mathbf{x}_k = M_k(\mathbf{x}_{k-1}, \mathbf{u}_k + \mathbf{w}_k)$$

Model uncertainty lies in the emission 'u'

$$\mathbf{y}_k = H_k(\mathbf{x}_k) + \mathbf{v}_k.$$

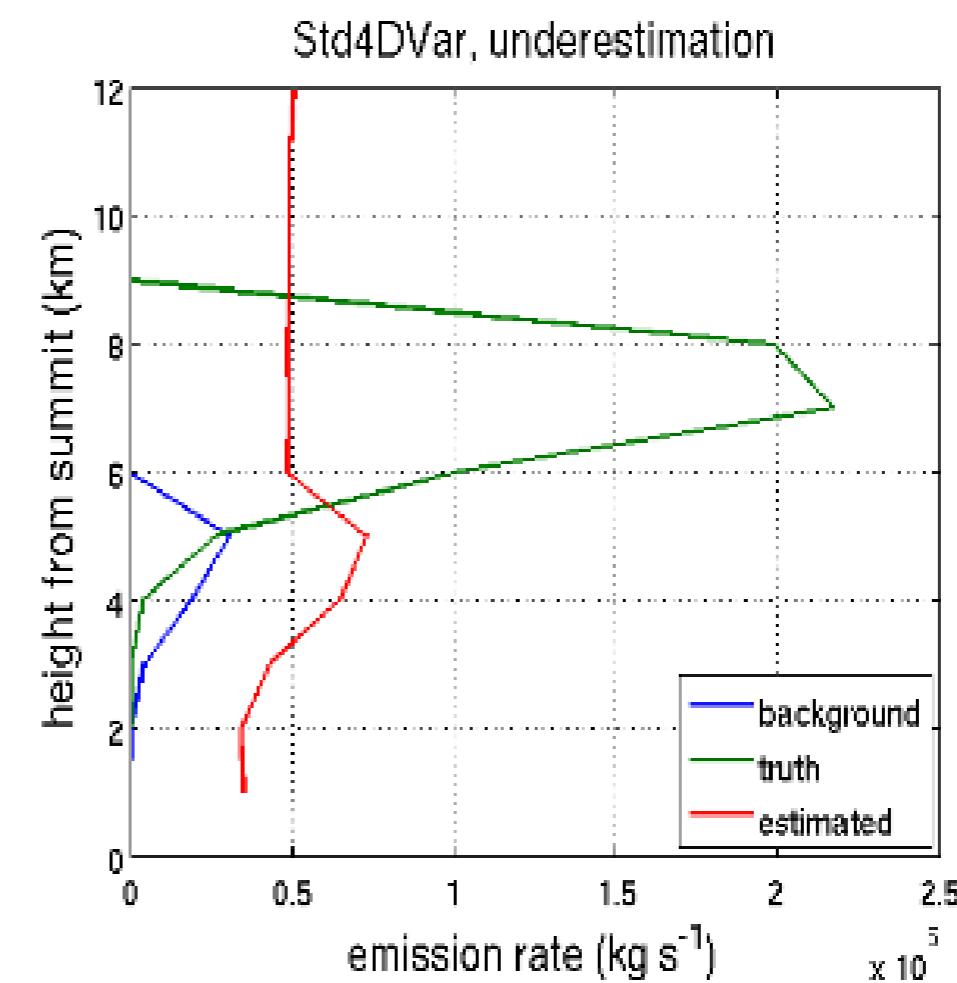
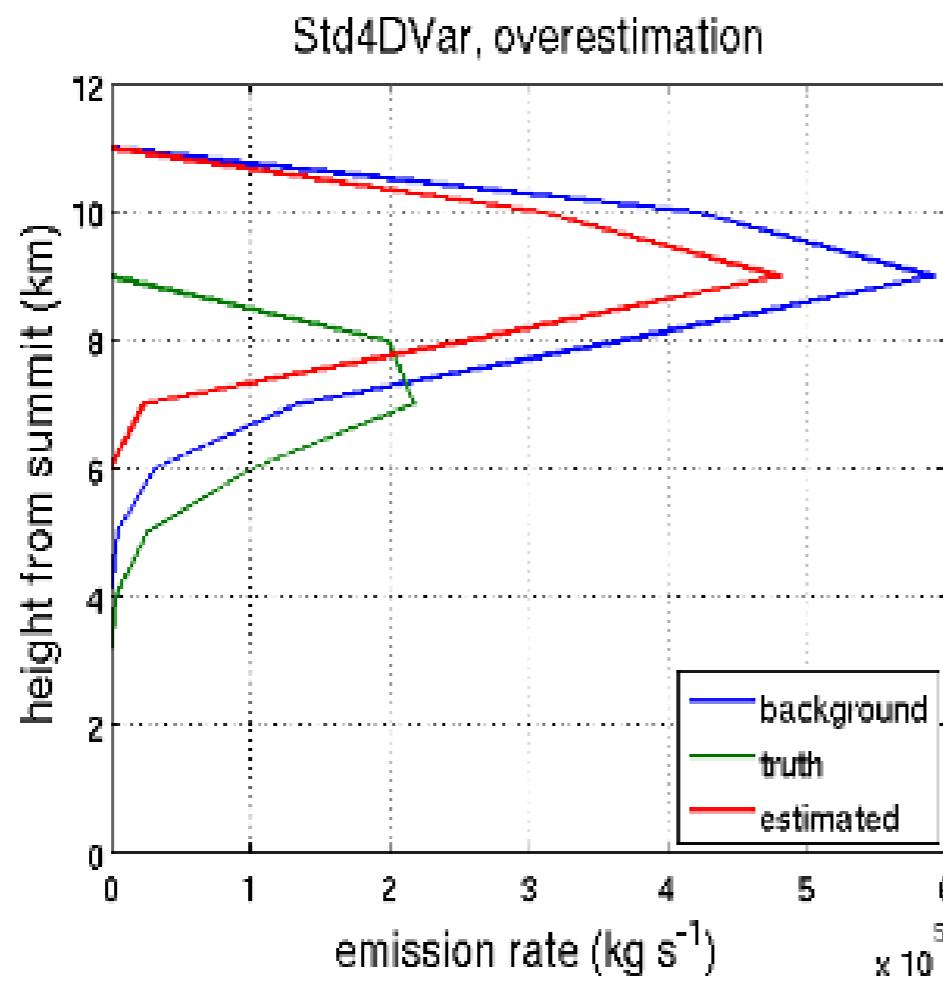
Measurement uncertainty (error)  $\mathbf{v}_k$

Typical cost function of standard 4DVar:

$$J(\mathbf{u}_k) = \frac{1}{2} \sum_{k=0}^N \left( \mathbf{u}_k - \mathbf{u}_k^b \right)^T \mathbf{B}_k^{-1} \left( \mathbf{u}_k - \mathbf{u}_k^b \right) + \frac{1}{2} \sum_{k=0}^N (\tilde{\mathbf{y}}_k - \mathbf{y}_k)^T \mathbf{R}_k^{-1} (\tilde{\mathbf{y}}_k - \mathbf{y}_k)$$

$$= J^b + J^o,$$

Ill-conditioned problem due to ‘spurious relationship’



# Trajectory-based 4D-Var

The emission is assumed to be a linear combination of the perturbation sets:

$$\mathbf{u} = \mathbf{u}^b + \sum_{i=1}^p \beta^i \Delta \mathbf{u}^i,$$

Trajectories

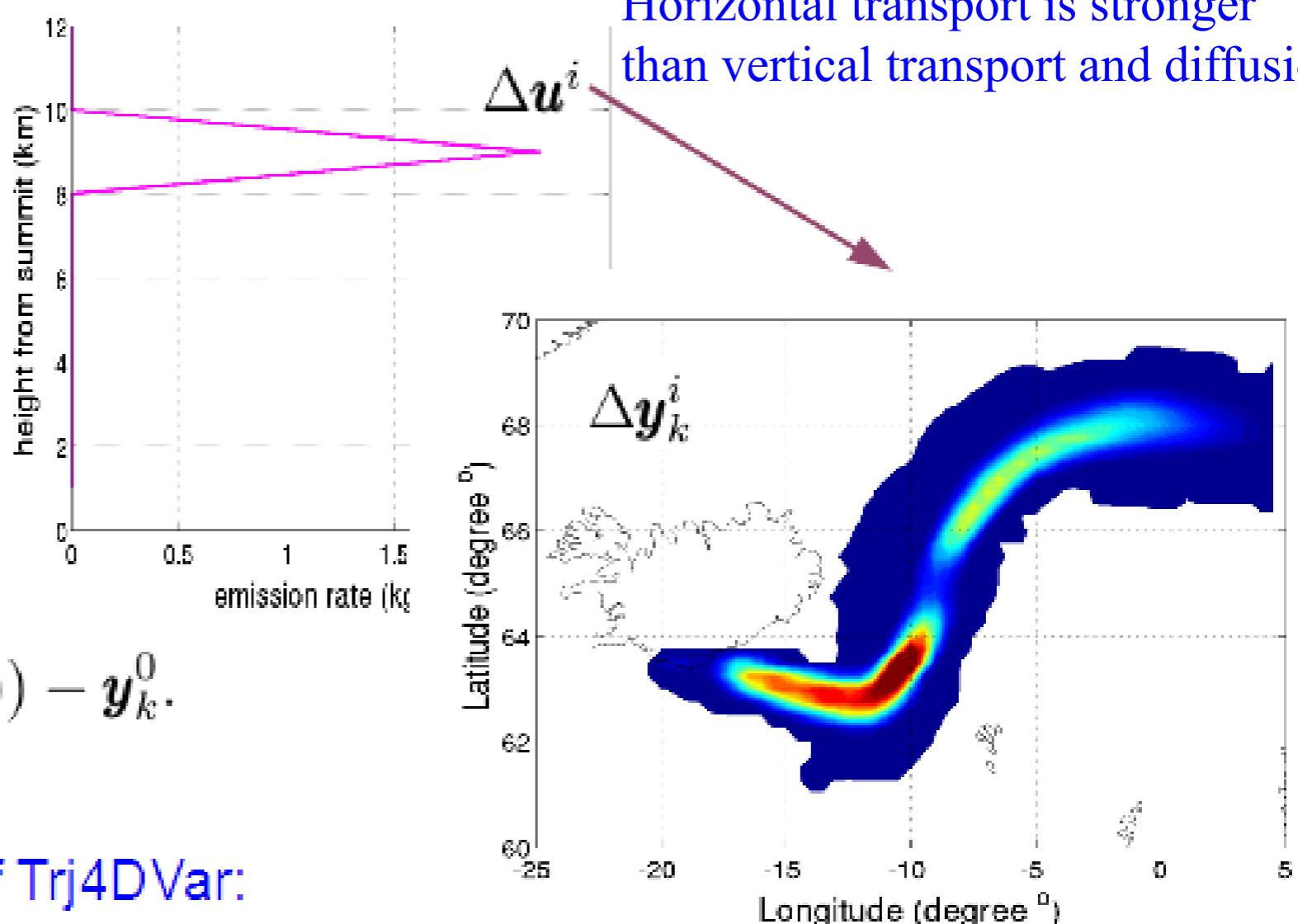
$$\mathbf{y}_k^0 = \mathcal{H}_k(\mathcal{M}_k(\mathbf{x}_{k-1}, \mathbf{u}^b)).$$

$$\Delta \mathbf{y}_k^i = \mathcal{H}_k(\mathcal{M}_k(\mathbf{x}_{k-1}, \mathbf{u}^b + \Delta \mathbf{u}^i)) - \mathbf{y}_k^0.$$

The reformulated cost function of Trj4DVar:

$$\begin{aligned} J(\boldsymbol{\beta}) &= \frac{1}{2} \sum_{k=1}^{Nt} \left( \sum_{i=1}^p \beta^i \Delta \mathbf{y}_k^i + \mathbf{y}_k^0 - \mathbf{y}_k \right)^T [\mathbf{R}_k]^{-1} \left( \sum_{i=1}^p \beta^i \Delta \mathbf{y}_k^i + \mathbf{y}_k^0 - \mathbf{y}_k \right) \\ &\quad + \frac{1}{2} (\mathbf{u} - \mathbf{u}^b)^T [\mathbf{B}_k]^{-1} (\mathbf{u} - \mathbf{u}^b) \\ &= J^o + J^b, \end{aligned}$$

Assumption/observation:  
Horizontal transport is stronger than vertical transport and diffusion.



Given:

$$\mathbf{x}_k = M_k(\mathbf{x}_{k-1}, \mathbf{u}_k + \mathbf{w}_k), \quad (1)$$

$$\mathbf{y}_k = H_k(\mathbf{x}_k) + \mathbf{v}_k. \quad (2)$$

with  $\mathbf{u} = \mathbf{u}^b + \sum_{i=1}^p \beta^i \Delta \mathbf{u}^i$  Eq.(2) can be rewritten as

$$\begin{aligned} \mathbf{y}_k &\approx H_k[M_k(\mathbf{x}_{k-1}, \mathbf{u}^b)] + \sum_{i=1}^p \beta^i \mathbf{H}_k \mathbf{M}_k(\mathbf{x}_{k-1}, \mathbf{u}^b) \Delta \mathbf{u}^i + \mathbf{v}_k \\ &\approx \mathbf{y}_k^0 + \sum_{i=1}^p \beta^i \{H_k[M_k(\mathbf{x}_{k-1}, \mathbf{u}^b + \Delta \mathbf{u}^i)] - \mathbf{y}_k^0\} + \mathbf{v}_k \\ &= \mathbf{y}_k^0 + \sum_{i=1}^p \beta^i \Delta \mathbf{y}_k^i + \mathbf{v}_k, \end{aligned} \quad (5)$$

leading to the trajectory-based 4D-Var formulation:

$$\begin{aligned} J(\beta) &= \frac{1}{2} \sum_{k=1}^{Nt} \left( \sum_{i=1}^p \beta^i \Delta \mathbf{y}_k^i + \mathbf{y}_k^0 - \mathbf{y}_k \right)^T [\mathbf{R}_k]^{-1} \left( \sum_{i=1}^p \beta^i \Delta \mathbf{y}_k^i + \mathbf{y}_k^0 - \mathbf{y}_k \right) \\ &\quad + \frac{1}{2} \sum_{k=1}^{Nt} (\mathbf{u} - \mathbf{u}^b)^T [\mathbf{B}_k]^{-1} (\mathbf{u} - \mathbf{u}^b) + \mu \|\nabla \mathbf{u}\|^2 \\ &= J^o + J^b + J^r, \end{aligned}$$

# Estimates of emissions in twin experiments

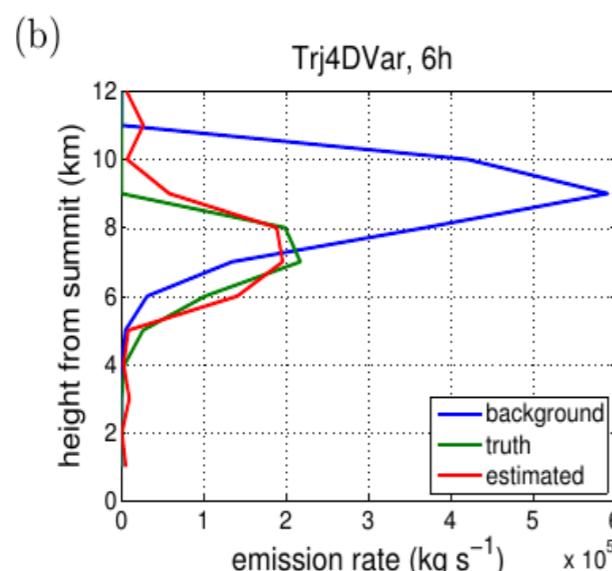
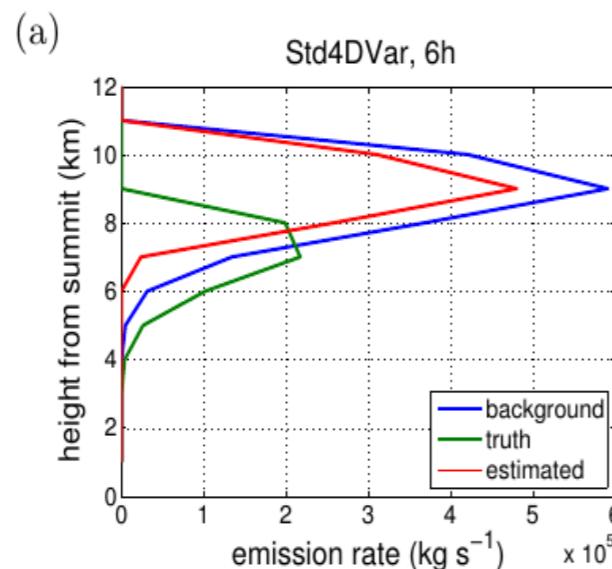
E15 2010 eruption events as a case study: LOTOS-EUROS model,  
meteorological situation, synthetic observations

Testing the  
methodology:

- Standard 4D-Var vs.  
▫ Trajectory-based 4D-Var
- Deterministic model

▫ Observations:  
synthetic ash column data

▫ 6-hour  
assimilation window

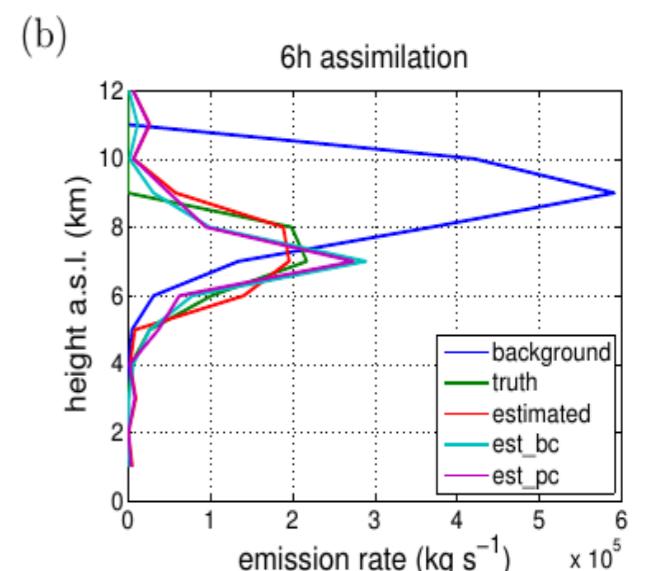
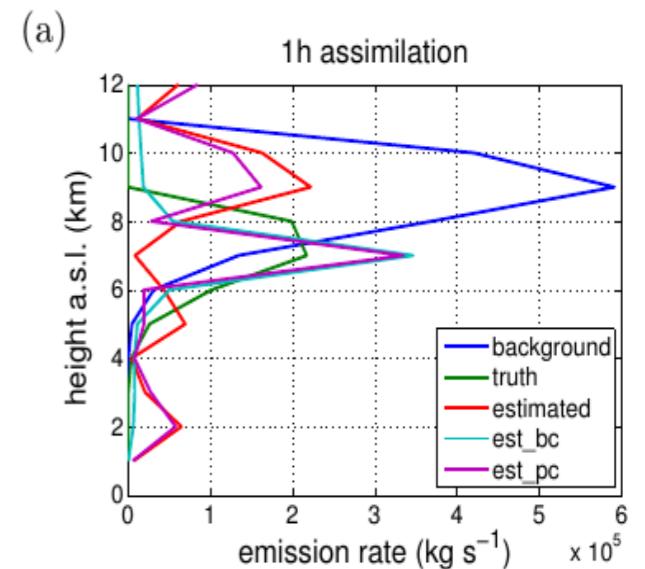


Testing some of the choices:

- Trajectory-based 4D-Var
- Stochastic model
- Observations:  
synthetic ash columns with 50% uncertainty  
& plume height and mass eruption rate

Settings and choices:

- Length of assimilation windows
- Uncertainties of Observations
- Schemes to integrate multi-observations



## Problem 2

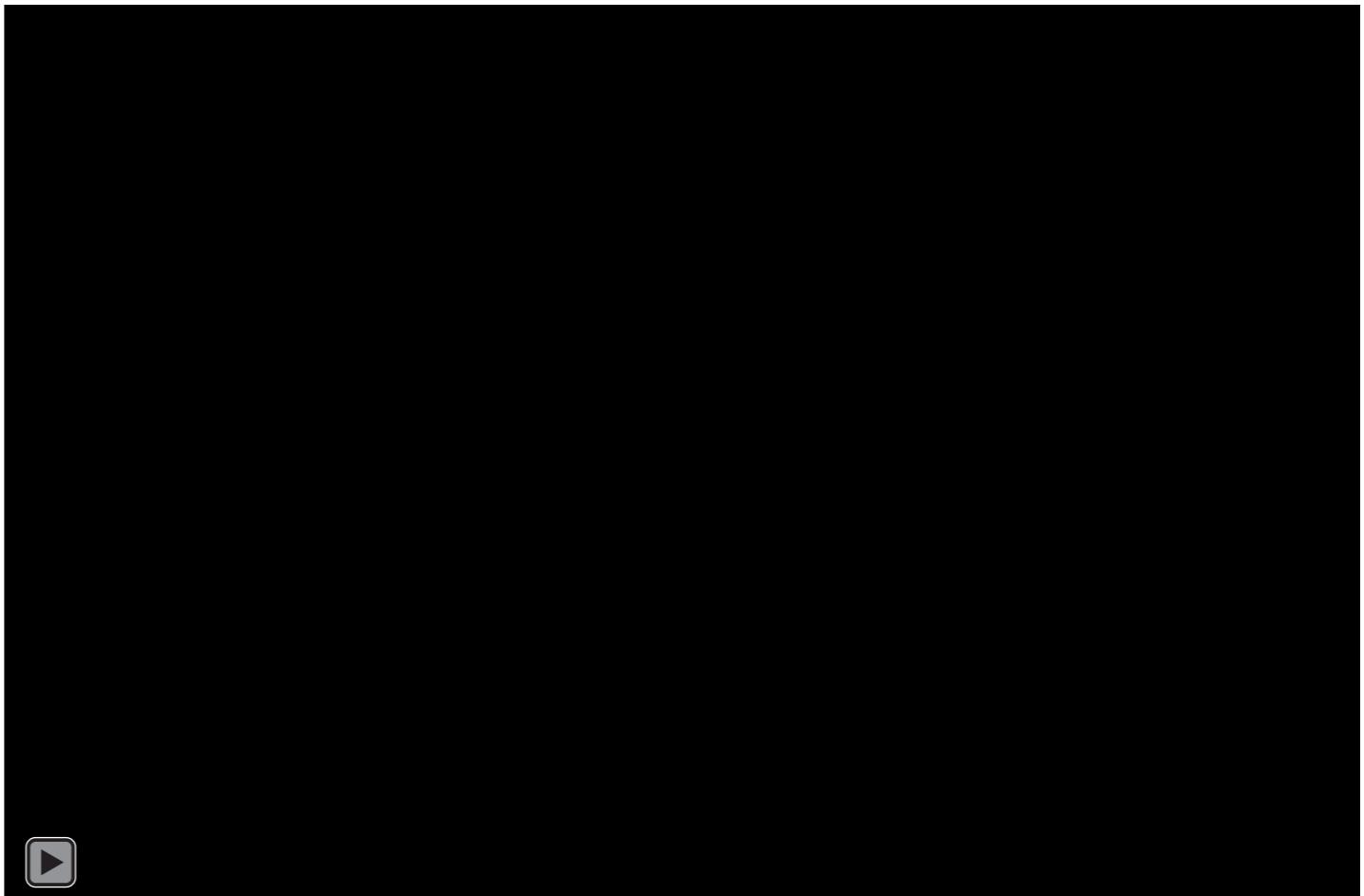
# Dust storm emission inversion using multiple data sources

## Dust storm models (chemical transport model)

- **Emissions;**
- **Transport;**  
advection, diffusion, radiation
- **Sedimentations.**  
wet, dry deposition

$$\begin{aligned} \frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial y} + W \frac{\partial C}{\partial z} \\ = \frac{\partial}{\partial x} \left( K_h \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_h \frac{\partial C}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial C}{\partial z} \right) \\ + E + R + Q - D - H \end{aligned}$$

**Example:** Lotos-Euros/Dust over East Asia





## 2.1. Data assimilation with Lotos-Euros: algorithm

**Data assimilation:** to find a solution that fits both the observation and priori.

**Traditional 4 dimensional variational (4DVar) data assimilation:**

$$J(\delta \mathbf{f}) = \frac{1}{2} \delta \mathbf{f} \mathbf{B}^{-1} \delta \mathbf{f} + \frac{1}{2} \sum_{i=1}^k (\mathbf{H}_i \mathbf{M}_i \delta \mathbf{f} + \mathbf{d}_i)^T \mathbf{O}_i^{-1} (\mathbf{H}_i \mathbf{M}_i \delta \mathbf{f} + \mathbf{d}_i)$$

$$\mathbf{d}_i = \mathcal{H}_i(\mathcal{M}_i(\mathbf{f})) - \mathbf{y}_i$$

$\mathbf{M}_i$  full tangent linear model  
Order of  $O(10^5)$

**Reduced-tangent-linearization 4DVar**

$$\mathbf{B} = \mathbf{U} \mathbf{U}^T \approx \tilde{\mathbf{U}} \tilde{\mathbf{U}}^T$$

$$\delta \mathbf{f} \approx \tilde{\mathbf{U}} \delta \mathbf{w}$$

$$J(\delta \mathbf{w}) = \frac{1}{2} \delta \mathbf{w}^T \delta \mathbf{w} + \frac{1}{2} \sum_{i=1}^k (\mathbf{H}_i \tilde{\mathbf{M}}_i \tilde{\mathbf{U}} \delta \mathbf{w} + \mathbf{d}_i)^T \mathbf{O}_i^{-1} (\mathbf{H}_i \tilde{\mathbf{M}}_i \tilde{\mathbf{U}} \delta \mathbf{w} + \mathbf{d}_i)$$

$\tilde{\mathbf{M}}_i$  reduced tangent linear model  
Order of  $O(10^2)$

**Sensitivity-based parameter filters:** To reduce the size of  $\delta \mathbf{f}$  and improve the computation efficiency

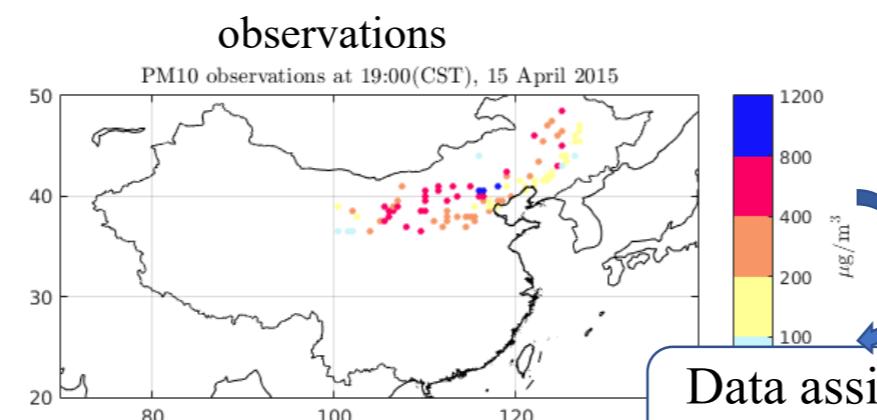
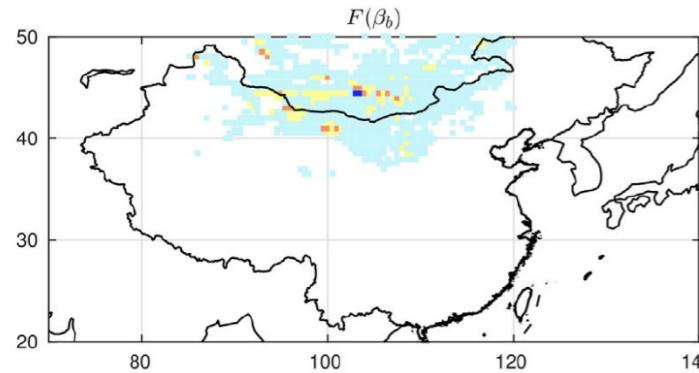


## 2.1 Data assimilation with Lotos-Euros: assimilating PM10

China MEP monitoring network:

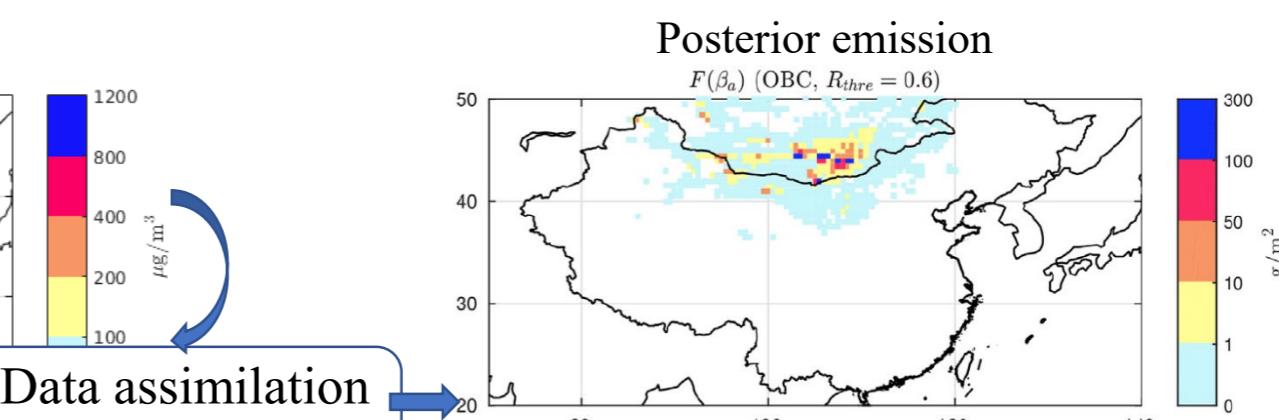
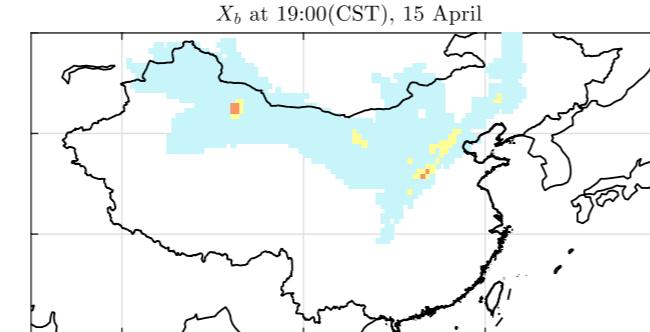
- $\text{PM}_{10}$ ,  $\text{PM}_{2.5}$ ,  $\text{SO}_2$ ,  $\text{NO}_x$ ,  $\text{O}_3$
- wide coverage
- high accuracy
- hourly measured
- Over 1,500 sites

Prior emission

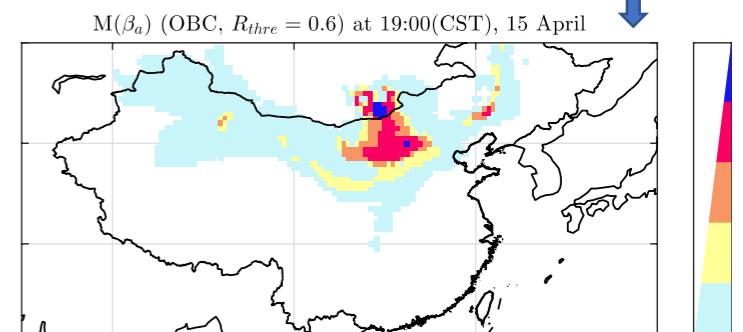


Data assimilation  
(emission inversion)

Prior dust simulation

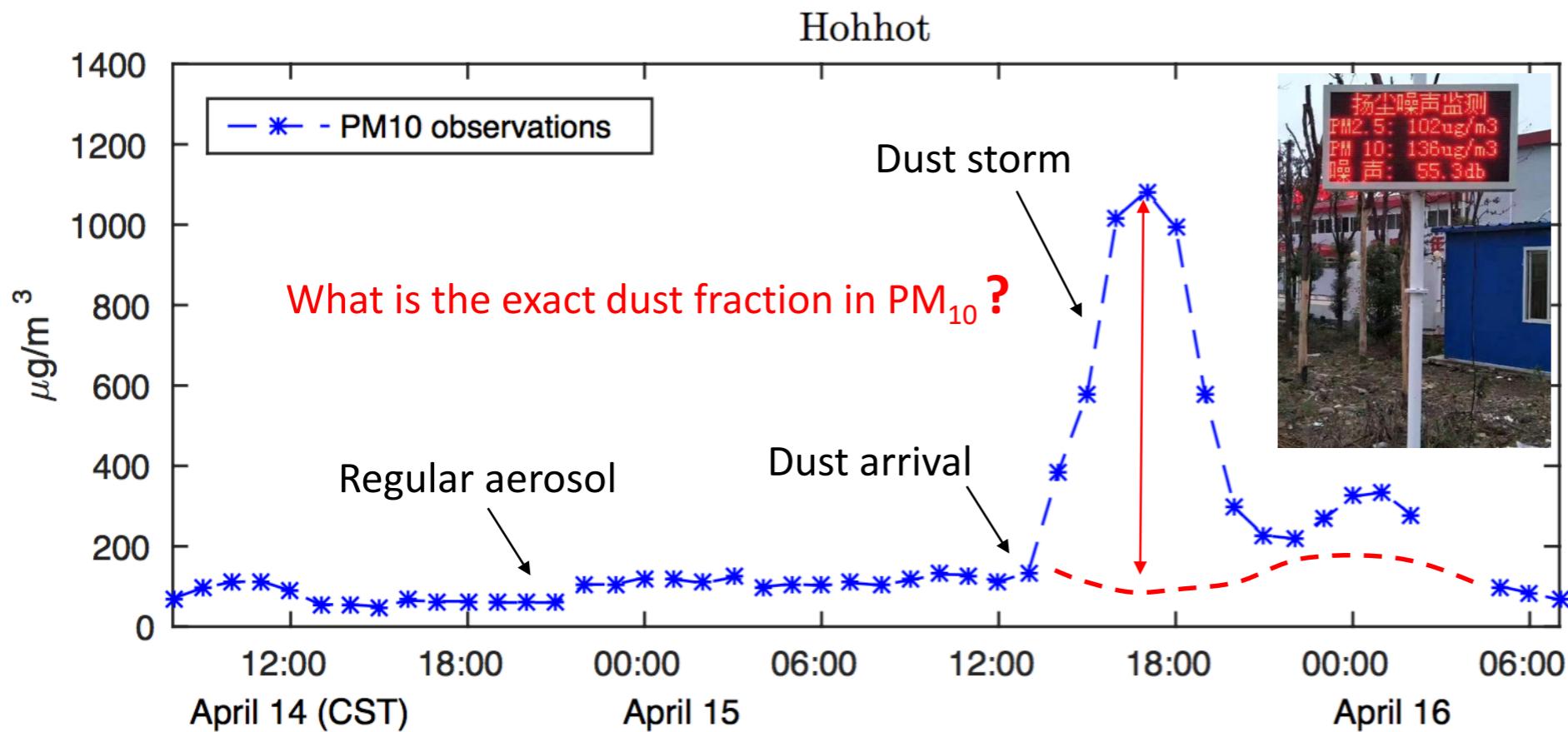


Posterior dust simulation



## 2.2 Machine learning based observation bias correction: bias/baseline

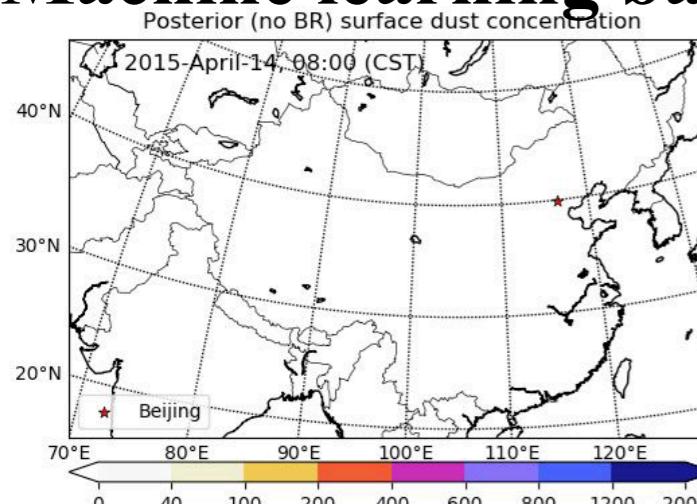
**Existence of bias in PM10 concentration for its use in data assimilation**



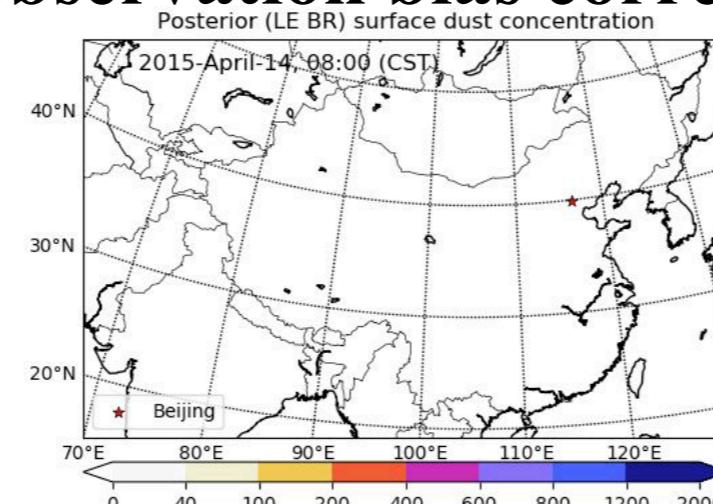
- **PM10 observation is a sum of non-dust and dust aerosols, thus includes a bias when representing the dust concentration.**
- **Issue:** the data assimilation algorithm cannot calculate whether the error is caused by the model deficiency or observation bias.
- **Challenge:** bias with strong spatial and temporal variability
- **Why not full aerosol model???**



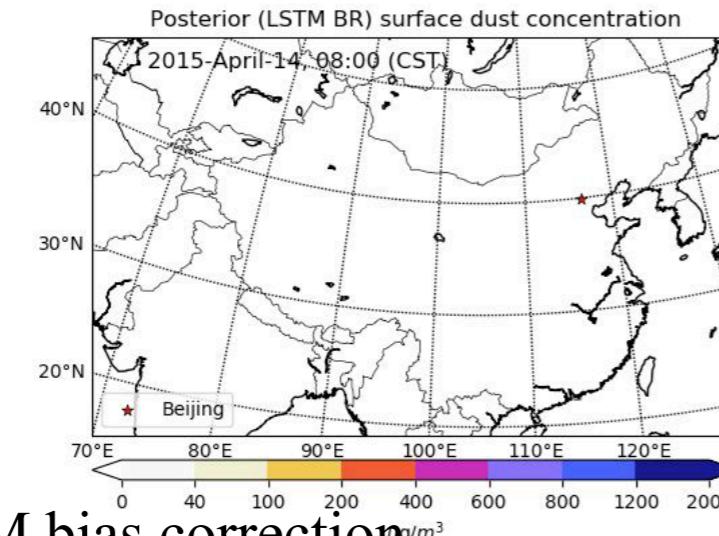
## 2.2 Machine learning based observation bias correction: assimilation evaluation



Posteriors no bias correction



Posteriors CTM bias correction



Posteriors LSTM bias correction

- Assimilation of machine learning bias corrected data gives the most accurate posterior;
- Direct assimilation of PM<sub>10</sub> causes overestimation of dust simulations.

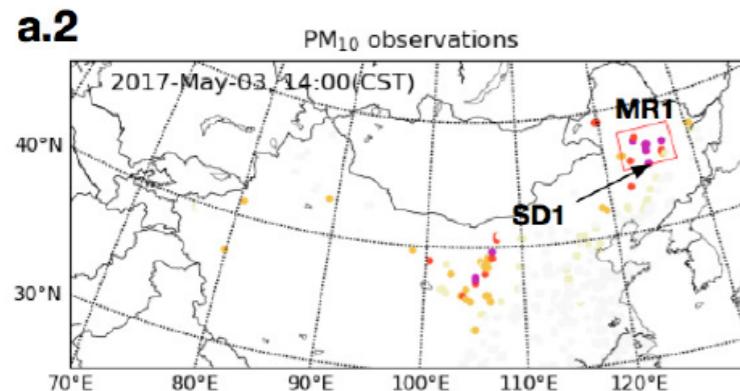
**J.Jin et al,** *Machine learning for observation bias correction with application to dust storm data assimilation.* (ACP Discussion)

**H.X.Lin, J.Jin et al.** *air quality forecast through integrated data assimilation and machine learning.* ICCART, Prague, 2019.

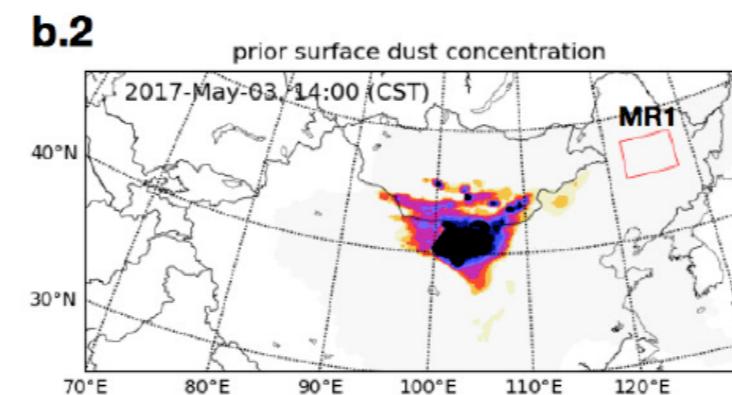


## 2.3 Emission detection using adjoint: no dust simulated in northeast China

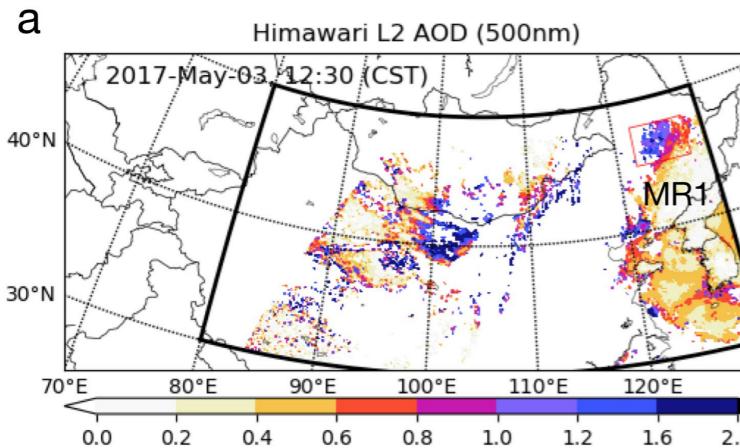
PM10 observations (independent)



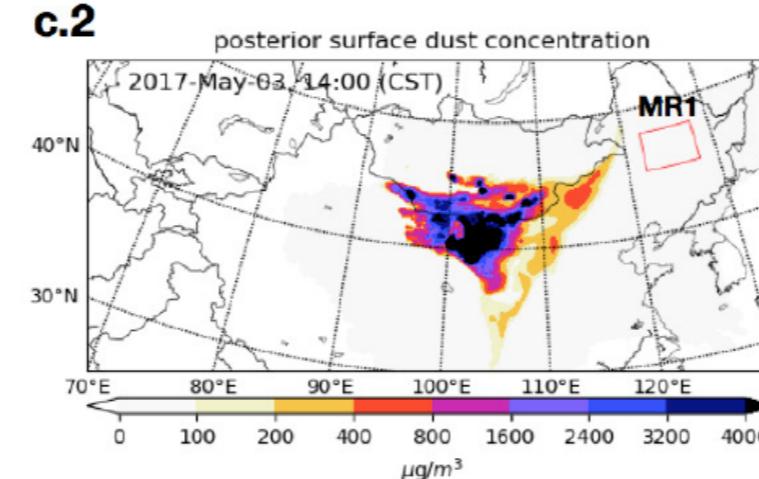
Prior



Assimilated AODs



Posterior (by assimilating AODs)



- No dusts are simulated in prior or posterior model;
- Other two dust outbreaks are also not reproduced.
- **Solution:** to detect the (missing) sensitive emissions for the dust outbreak



## 2.3 Emission detection using adjoint: theory

- Semi-analytical calculation  $\mathcal{M}^i(x^i, f^i)$

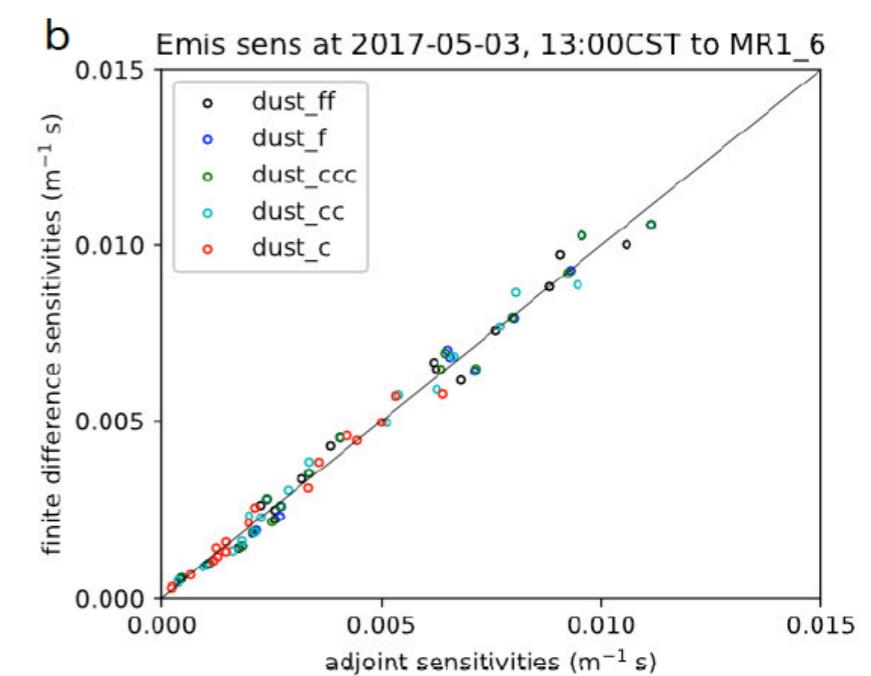
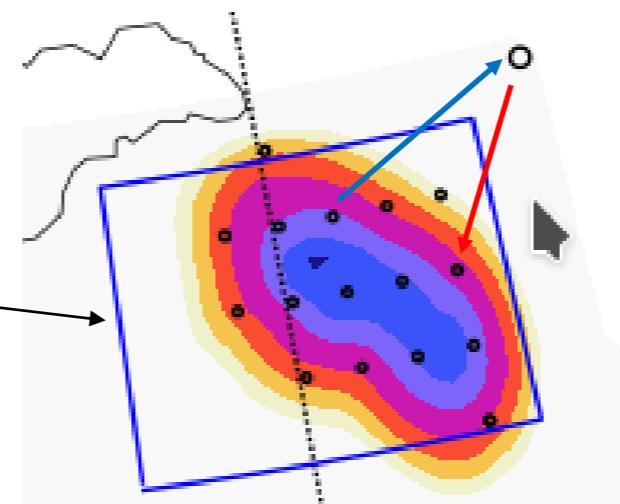
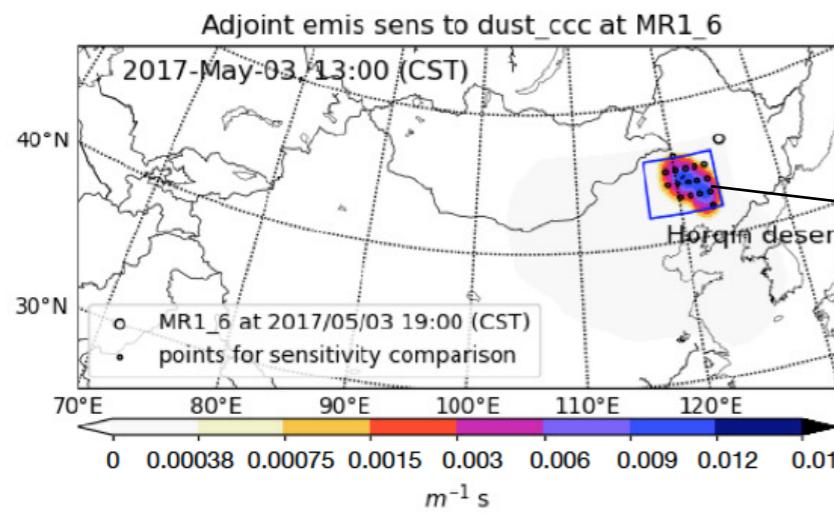
- Linearized model operator intensive finite difference accurate but  $\frac{\partial x^{i+1}}{\partial f^i} = \frac{\partial \mathcal{M}^i}{\partial f^i} = M_f^i$  and  $\frac{\partial x^{i+1}}{\partial x^i} = \frac{\partial \mathcal{M}^i}{\partial x^i} = M_x^i$

$(x^i, f^i)$  gradient of model response J to parameters  $x^{i+1}$ :

$$\nabla_{f^j} \mathcal{J}(x^i) = (M_f^j)^T \cdot (M_x^{j+1})^T \cdots (M_x^{i-1})^T \cdot \left\{ \frac{\partial \mathcal{J}(x^i)}{\partial x^i} \right\}^T$$

adjoint method: efficient but .....

(adjoint vs. finite difference):

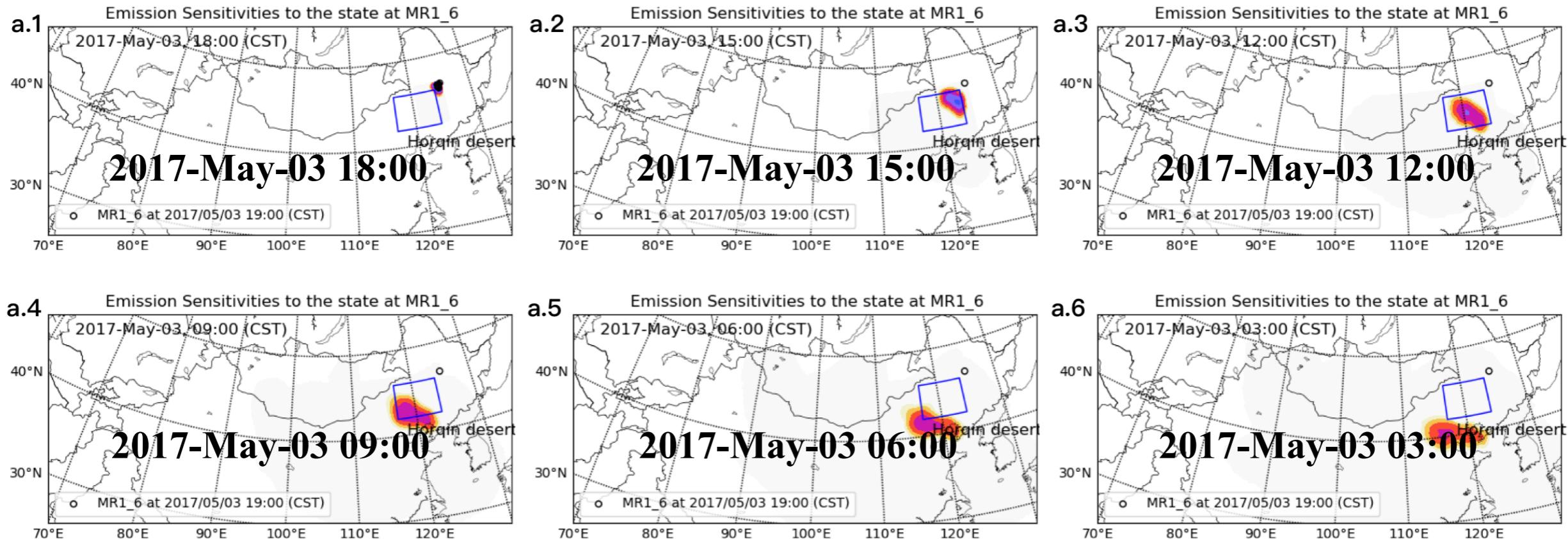




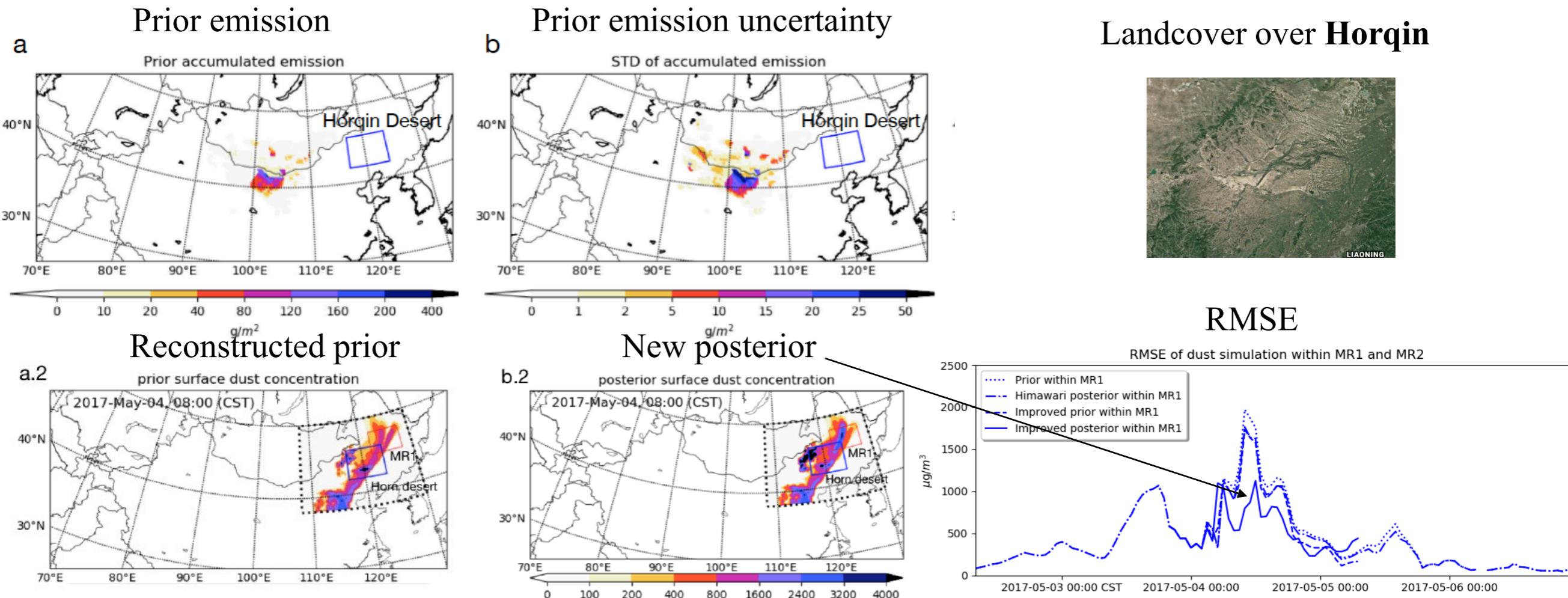
## 2.3 Emission detection using adjoint: emission backtracking

Time series of emission sensitivities to a state X at **2017-May-03 19:00**

Sensitivities to dust simulation at MR1\_6 2017-05-03 19:00 CST

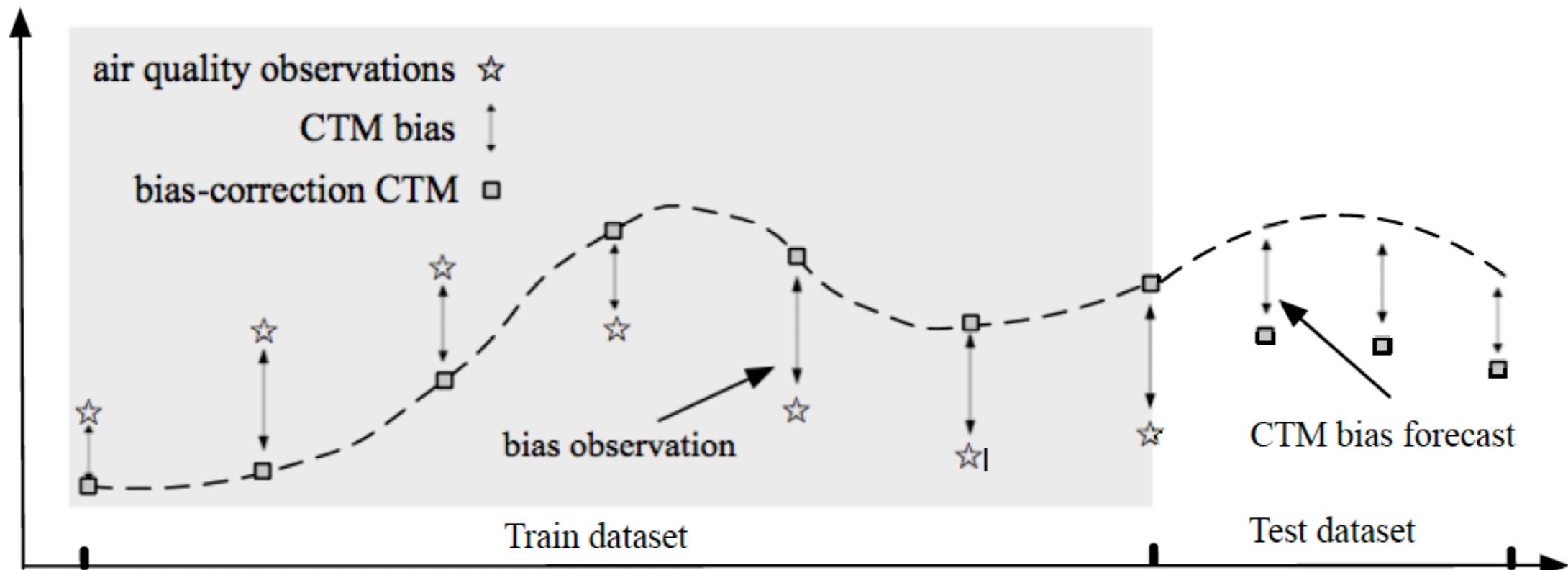


## 2.3 Emission detection using adjoint: guided emission reconstruction



J.Jin et.al., Source backtracking for dust storm emission inversion using an adjoint method.  
*Atmospheric Chemistry and Physics*, 2021

## 2.4 Machine Learning: error bias correction



# Improved forecast after ML bias correction

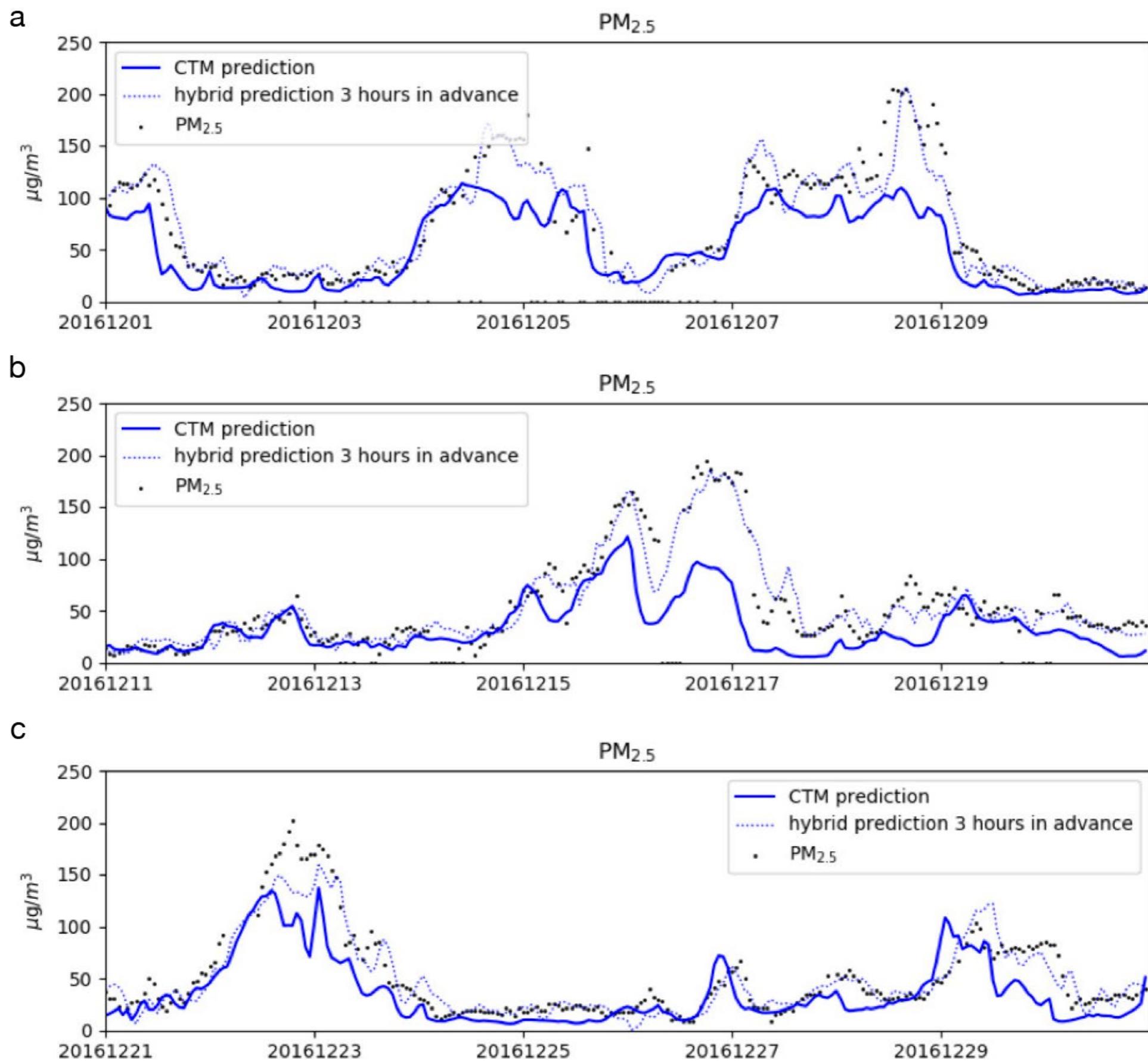


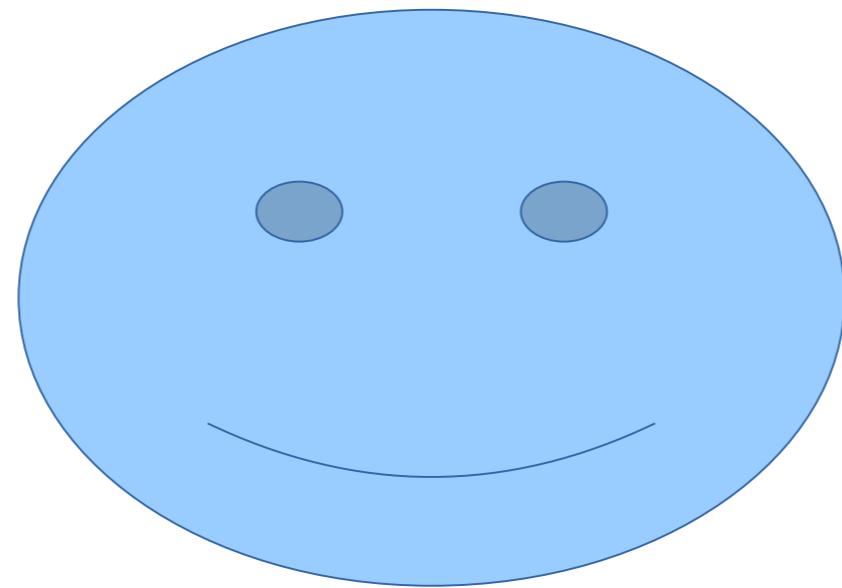
Figure 4: Time series of the  $\text{PM}_{2.5}$  observation, CTM prediction and hybrid forecasts with 3 hours in advance. (a): Dec 01-10; (b): Dec 11-20; (c): Dec 21-30.

## Data assimilation and machine learning for air quality forecasts

- Trajectory-based 4DVar is effective in estimating volcano eruption plume shapes using satellite data
- The integration of machine learning and data assimilation results in more accurate air quality forecast during dust storms.
- Inclusion of physical model knowledge further enhances the learning process in (atmospheric) modeling.
- Integration ML and DA opens many new possibilities, such as filling unmodelled processes in CTM with ML, using an ML surrogate to replace computation intensive (sub)models, ...

# Challenges

- Explainable ML model
- Can DA sometimes converge to ‘truth’ (certainty)? Accurate Uncertainty quantification is the key.
- Computational cost
- ...



**Thank you!**

# References

1. J. Jin, A.J. Segers, H.X. Lin, B. Henzing, X. Wang, A.W. Heemink, H. Liao (2021). [Position correction in dust storm forecast using LOTUS-EUROS v2.1: grid distorted data assimilation v1.0](#), Geoscientific Model Development.
2. M. Xu, J. Jin, G. Wang, A Segers, T. Deng, H.X. Lin (2021), [Machine learning based bias correction for numerical chemical transport models](#), Atmospheric Environment, 118022
3. J. Jin, A. Segers, H. Liao, A.W. Heemink, R. Kranenburg, H.X. Lin (2020), [Source backtracking for dust storm emission inversion using adjoint method: case study of northeast China](#), Atmospheric Chemistry and Physics.
4. C. Xiao, O. Leeuwenburgh, H.X. Lin, A.W. Heemink (2021), [Conditioning of Deep-Learning Surrogate Models to Image Data with Application to Reservoir Characterization](#), Knowledge Based Systems.
5. C. Xiao, H.X. Lin, O. Leeuwenburgh, A.W. Heemink (2022), [Surrogate-assisted inversion for large-scale history matching: comparative study between projection-based reduced-order modelling and deep neural network](#), Journal of Petroleum Science and Engineering.
6. H.X. Lin, J. Jin, H.J. van den Herik (2019), [Air Quality Forecast through Integrated Data Assimilation and Machine Learning](#), in Proc. 11th International Conference on Agents and Artificial Intelligence (ICAART 2019).
7. J. Jin, H.X. Lin, A. Segers, Y. Xie, A.W. Heemink (2019), [Machine learning for observation bias correction with application to dust storm data assimilation](#), Atmospheric Chemistry and Physics, Vol.19, pp. 10009-10026.
8. J. Jin, A.W. Heemink, A. Segers, M. Yoshida, W. Han, H.X. Lin (2019), [Dust Emission Inversion Using Himawari-8 AODs Over East Asia: an Extreme Dust Event in May 2017](#), Journal of Advances of Modeling Earth Systems, pp. 446-467.
9. J. Jin, H.X. Lin, A.W. Heemink, A. Segers (2018), [Spatially varying parameter estimation for dust emissions using reduced-tangent-linearization 4DVar](#), Atmospheric Environment, Vol.187, pp. 358-373.
10. S. Lu, A. Heemink, H.X. Lin, G. Fu, A Segers (2017), [Evaluation criteria on the design for assimilating remote sensing data using variational approaches](#), *Monthly Weather Review* 145(6), pp.2165-2175
11. G. Fu, H.X. Lin, A.W. Heemink, S. Lu, A. Segers, N. van Velzen, T.C. Lu, and S.M. Xu (2017), [Accelerating volcanic ash data assimilation using a mask-state algorithm based on an ensemble Kalman filter: a case study with the LOTOS-EUROS model \(version 1.10\)](#), *Geoscientific Model Development*, 10, pp.1751–1766
12. G. Fu, F. Prata, H.X. Lin, A.W. Heemink, S. Lu, A.J. Segers (2017) [Data assimilation for volcanic ash plumes using a Satellite Observational Operator: a case study on the 2010 Eyjafjallajokull volcanic eruption](#), *Atmospheric Chemistry and Physics*, Vol17(2), pp. 1187—1205, DOI: 10.5194/acp-17-1187-2017
13. S. Lu, H.X. Lin, A. Heemink, A Segers, G. Fu (2016) [Estimation of volcanic ash emissions through assimilating satellite data and ground-based observations](#), *Journal of Geophysical Research: Atmospheres*, Vol. 121 (18), pp. 10971–10994.
14. G. Fu, A.W. Heemink, S. Lu, A.J. Segers, K. Weber, H.X. Lin (2016) [Model-based aviation advice on distal volcanic ash clouds by assimilating aircraft in-situ measurements](#), *Atmospheric Chemical Physics*, doi:10.5194/acp-2016-166,
15. S. Lu, H.X. Lin, A.W. Heemink, G. Fu, A. Segers (2016), [Estimation of volcanic ash emissions using trajectory-based 4D-Var data Assimilation](#), *Monthly Weather Review*, Vol. 143, pp.575-589.
16. G. Fu, H.X. Lin, A.W. Heemink, A.J. Segers, S. Lu, T. Palsson (2015) [Assimilating aircraft-based measurements to improve Forecast Accuracy of Volcanic Ash Transport](#), *Atmospheric Environment*, Vol.115, pp. 170-184.