Reconstruction of functional brain networks from MEG sensor data

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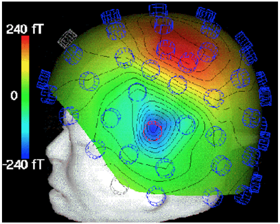
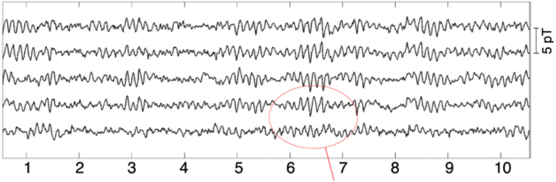
Overview

- Background on MEG
- Inverse modeling of MEG signals
- Assessing brain connectivity with MEG
- Bias reduction

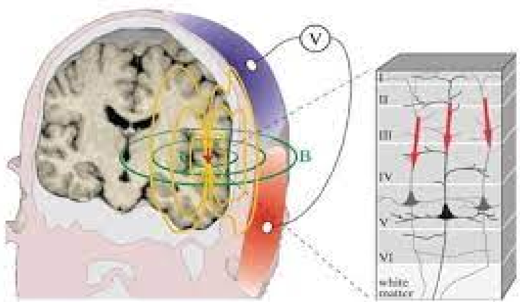
Magnetoencephalography (MEG)





Generation of MEG signals

MEG signals are generated by synchronized currents ($>\approx 1\text{cm}^2$) in the apical dendrites of cortical pyramidal neurons.



The relation between the cortical current density and the generated magnetic fields is described by Maxwell's equations (Biot-Savard).

Forward and inverse modeling of MEG signals

The mapping from the cortical current density to the generated magnetic fields is obtained by numerically solving Maxwell's equations for a volume-conductor model of the head $\rightarrow$ **leadfield matrix** $L \in \mathbb{R}^{n \times p}$ $(n < p)$.

MEG forward model (time-frequency domain): $Y(f) = LX(f) + E(f)$
$X(f) \in \mathbb{C}^{p \times k}$ cortical current density ("brain activity")
$Y(f) \in \mathbb{C}^{n \times k}$ sensor data
$E(f) \in \mathbb{C}^{n \times k}$ sensor noise

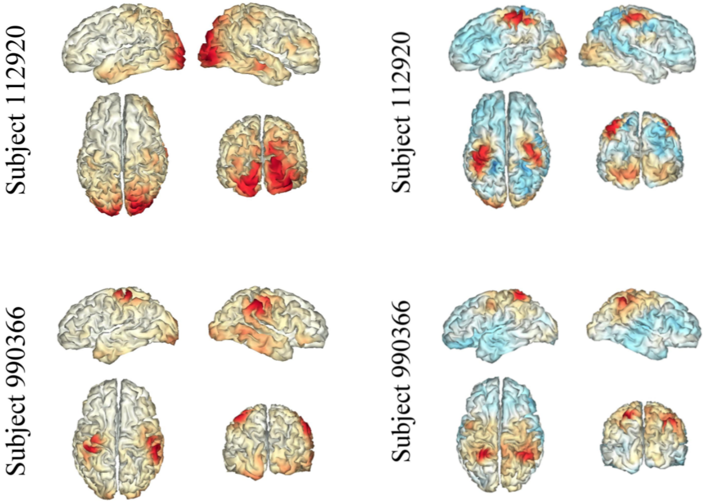
Two popular linear reconstruction methods:

Ridge regression: $X^* = \underset{X \in \mathbb{C}^{p \times k}}{\operatorname{argmin}} ||Y - LX||_F^2 + \lambda ||AX||_F^2$

Adaptive filter: $X_j^* = w_j^\dagger Y$ with $w_j = \underset{w \in \mathbb{C}^n}{\operatorname{argmin}} w^\dagger \Sigma_Y w$ subject to $w^\dagger L_j = 1$

$\Sigma_Y = YY^\dagger$ sensor covariance matrix
$w^\dagger \Sigma_Y w = \mathbb{V}(X_j^*)$ output power
$w^\dagger L_j = w^\dagger (Le_j) = 1$ unit-gain constraint

Example: Spontaneous alpha (~ 10 Hz) oscillations[1]



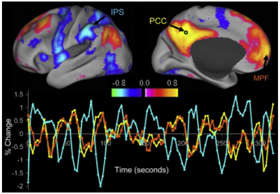
[1] Hindriks et al. 2017

Functional brain connectivity

Refers to any statistical relationship between signals x and y recorded from different cortical locations.

One of the many measures is **coherence:** $\rho = \dfrac{\langle xy^* \rangle}{\sqrt{\langle |x|^2 \rangle \langle |y|^2 \rangle}} = \dfrac{\langle a_x a_y e^{i(\phi_x - \phi_y)} \rangle}{\sqrt{\langle |a_x|^2 \rangle \langle |a_y|^2 \rangle}}$.

Spontaneous hemodynamic activity[2] is organized into distributed correlated patterns (**resting-state networks**).



Do resting-state networks have an electrophysiological origin? This question can (potentially) be answered with MEG.

[2] As measured with functional MRI.

Assessing functional connectivity with MEG

Reconstructed signals X^* are low-dimensional projections of the true signals X:

$Y = LX \to X^* = L^\sharp Y = L^\sharp LX = RX$
$L^\sharp \in \mathbb{R}^{p\times n}$ inverse operator
$R = L^\sharp L \in \mathbb{R}^{p\times p}$ resolution operator with rank $\leq n < p$.

$\to$ spurious interactions: $\Sigma^* = R\Sigma R^T$, in particular: $\Sigma = I \to \Sigma^* = RR^T$.

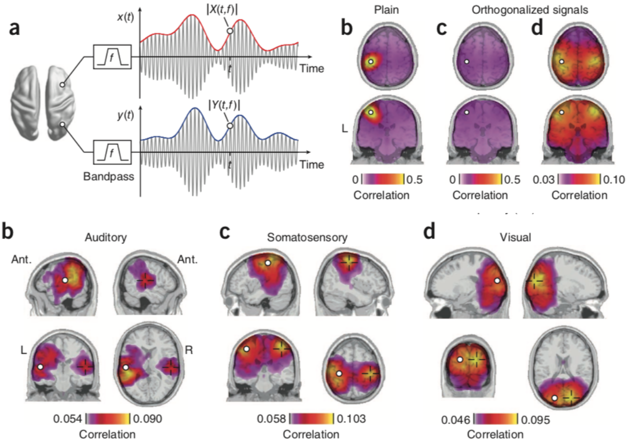
However: R is real-valued, hence if Σ is real-valued, then so is Σ^*. This can be exploited by only considering $\mathrm{Im}(\Sigma^*)$.

Most currently used connectivity measures are variations on this.

One of these is the **imaginary phase-locking factor** $\left\langle \frac{\mathrm{Im}(xy^*)}{|x||y|} \right\rangle = \langle \sin\left(\phi_x - \phi_y\right)\rangle$.

Detecting resting-state networks with MEG

Using one of these measures (orthogonalized amplitude correlation) resting-state networks were observed using MEG[3].



[3] Hipp et al. 2012.

Bias reduction

Main drawback of existing measures is that instantaneous interactions (Σ real-valued) cannot be detected.

To mitigate this, exploit the relation $Vec(\Sigma^*) = (R \otimes R)Vec(\Sigma)$.[4] Write $\Sigma = \Sigma^+ + \Sigma^-$.

$$Vec(\Sigma^*) = (R \otimes R)Vec(\Sigma^+) + (R \otimes R)Vec(\text{Re}(\Sigma^-)) + (R \otimes R)Vec(\text{Im}(\Sigma^-))i.$$

The three vectors are contained in:

$S_0 = \text{span}\{r_i \otimes r_i\}$ (leakage subspace)

$S_1 = \text{span}\{r_i \otimes r_j + r_j \otimes r_i\}$ (instantaneous interaction subspace)

$S_2 = \text{span}\{r_i \otimes r_j - r_j \otimes r_i\}$ (lagged interaction subspace)

Approach: Construct a series of operators π_j on $\mathbb{C}^{p^2}$ that project $Vec(\Sigma^*)$ onto the orthogonal complement of the first j left singular vectors of $[r_i \otimes r_i]$.

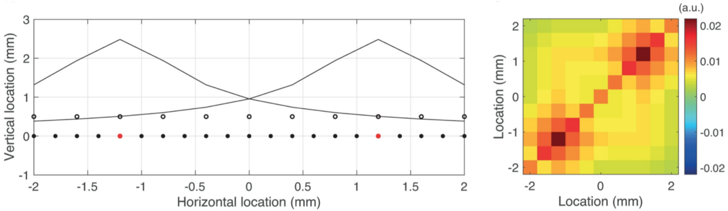
[4] Ossadtchi et al. (2018) and Hindriks (2020).

Toy model (cortical segment)

Intra-cortical potentials generated by a current source (Poisson's equation):

$$V = \frac{1}{\sqrt{h^2+(x-y)^2}}.$$

Two oscillatory monopoles with covariance matrix $\Sigma = \begin{bmatrix} 1 & \gamma e^{i\phi} \\ e^{-i\phi} & 1 \end{bmatrix}$



Effect of bias reduction