How to minimize time-discretization error?

Krzysztof Bisewski[*+]

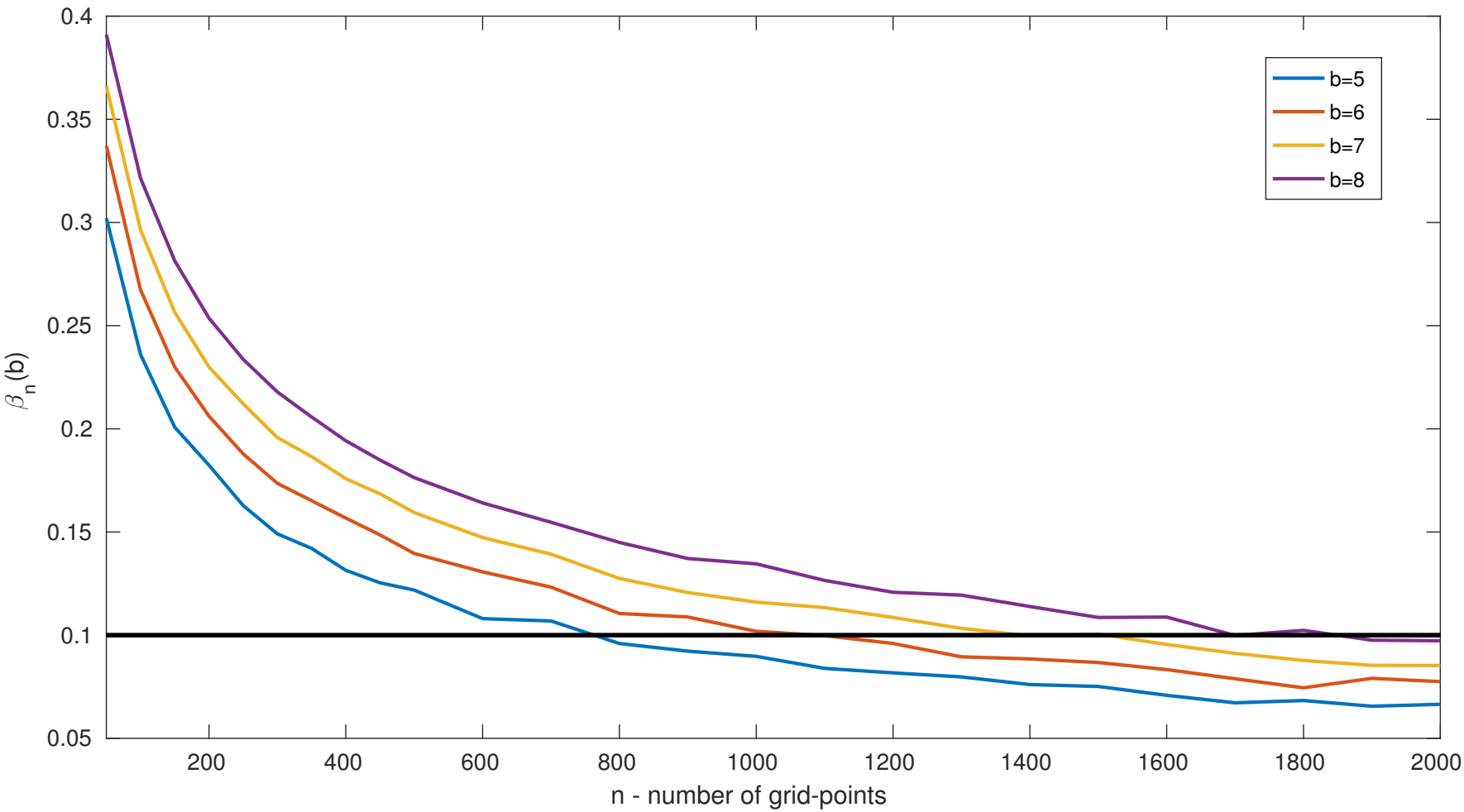
Joint work with:

Daan Crommelin[*+], **Michel Mandjes**[+]

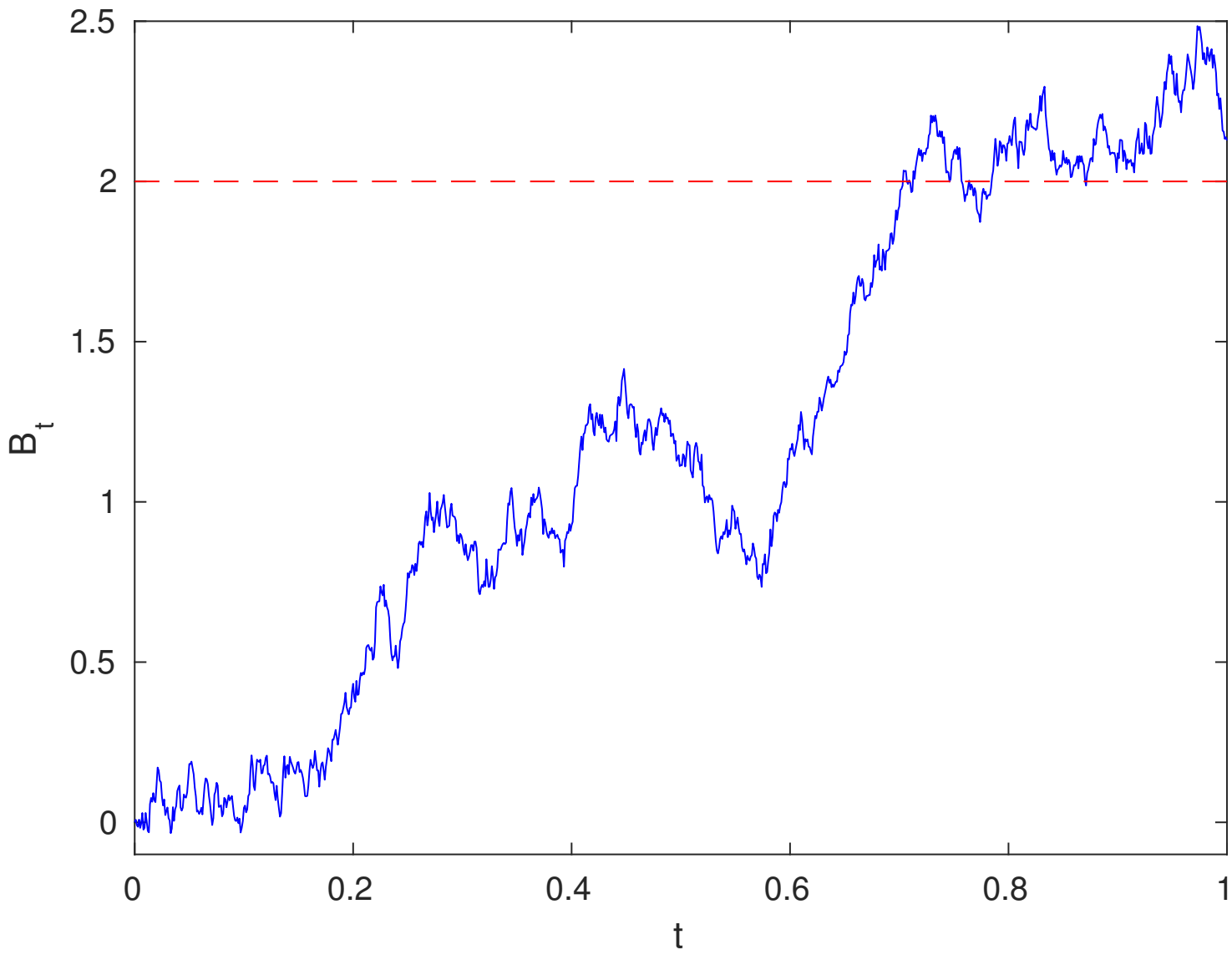
([*]Centrum Wiskunde & Informatica, [+]KdVI University of Amsterdam)

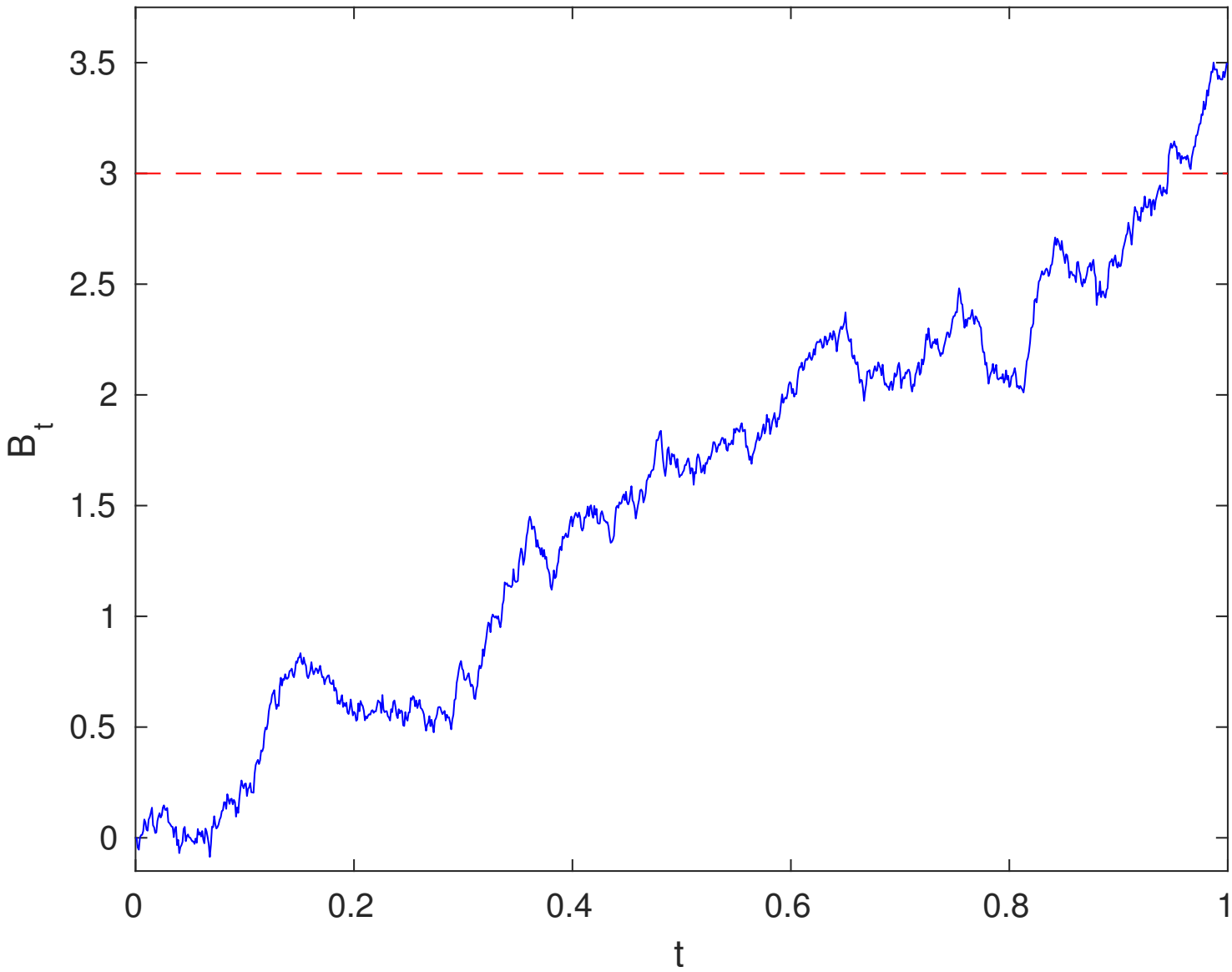
Introduction

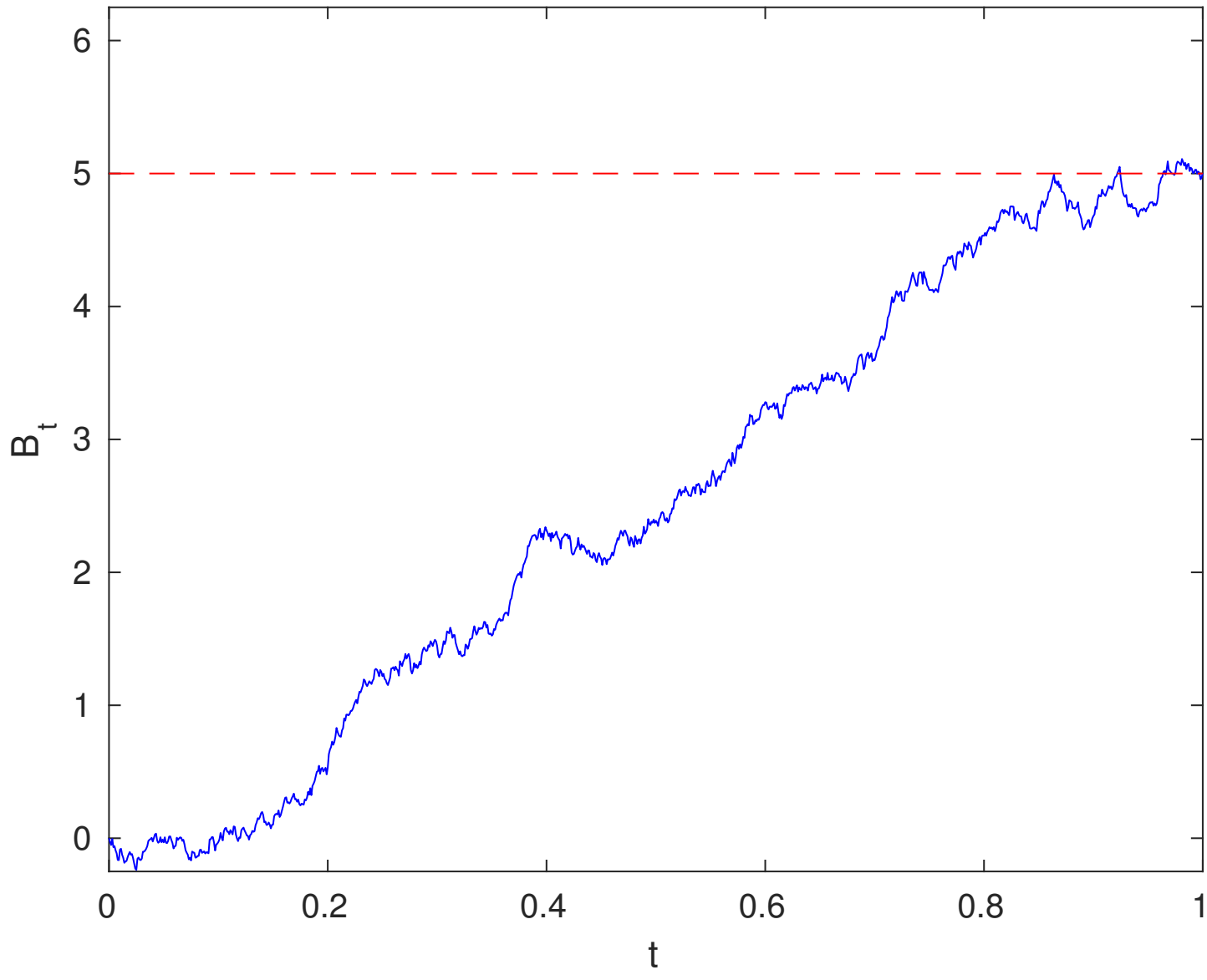
Equidistant grids

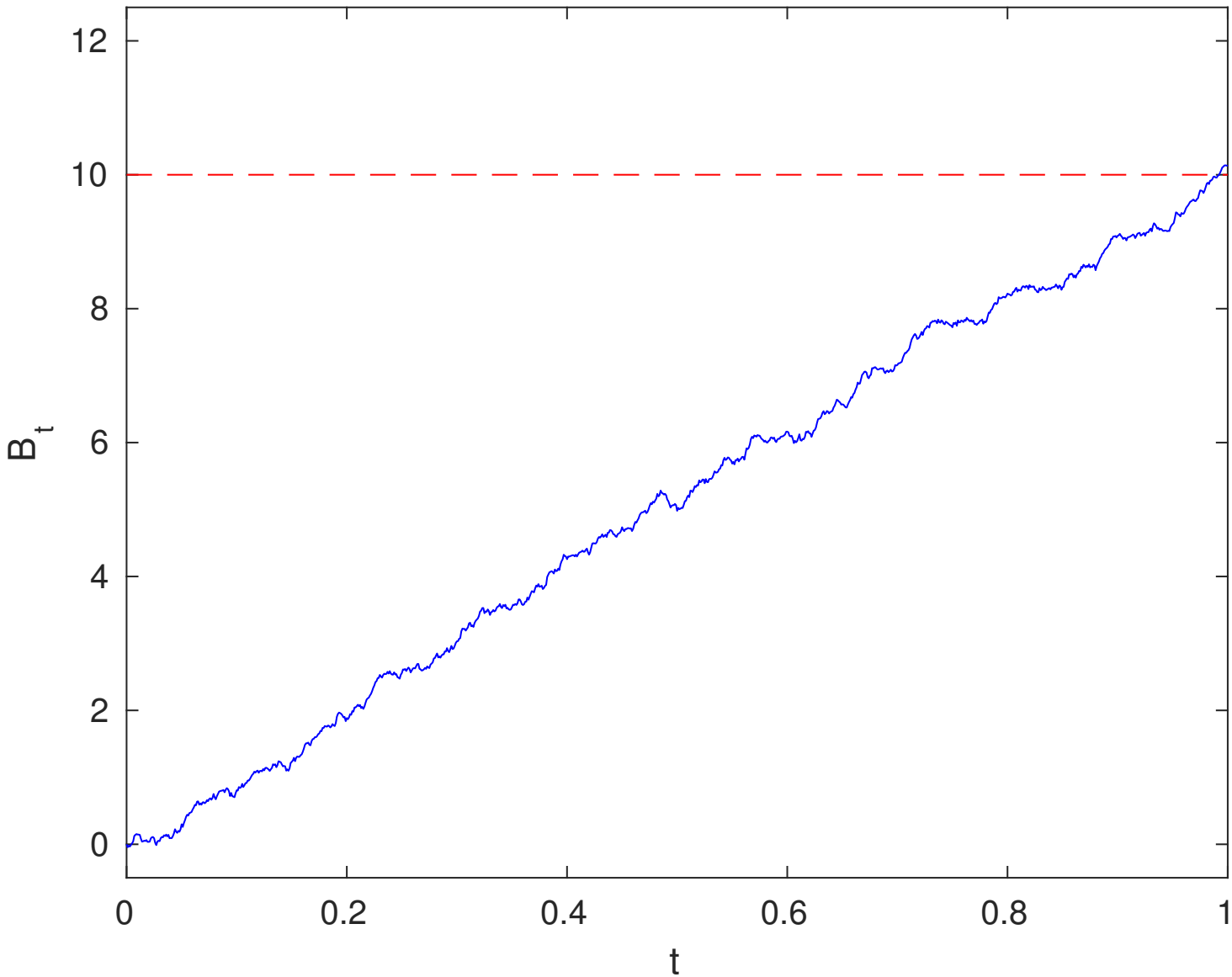


How does a path leading to the threshold look like?


2.5
2
1.5
1
0.5
0
B_t
0
0.2
0.4
0.6
0.8
1
t


3.5
3
2.5
2
1.5
1
0.5
0
B_t
0
0.2
0.4
0.6
0.8
1
t


6
5
4
3
2
1
0
B_t
0
0.2
0.4
0.6
0.8
1
t


12
10
8
6
4
2
0
B_t
0
0.2
0.4
0.6
0.8
1
t

Adaptive grids

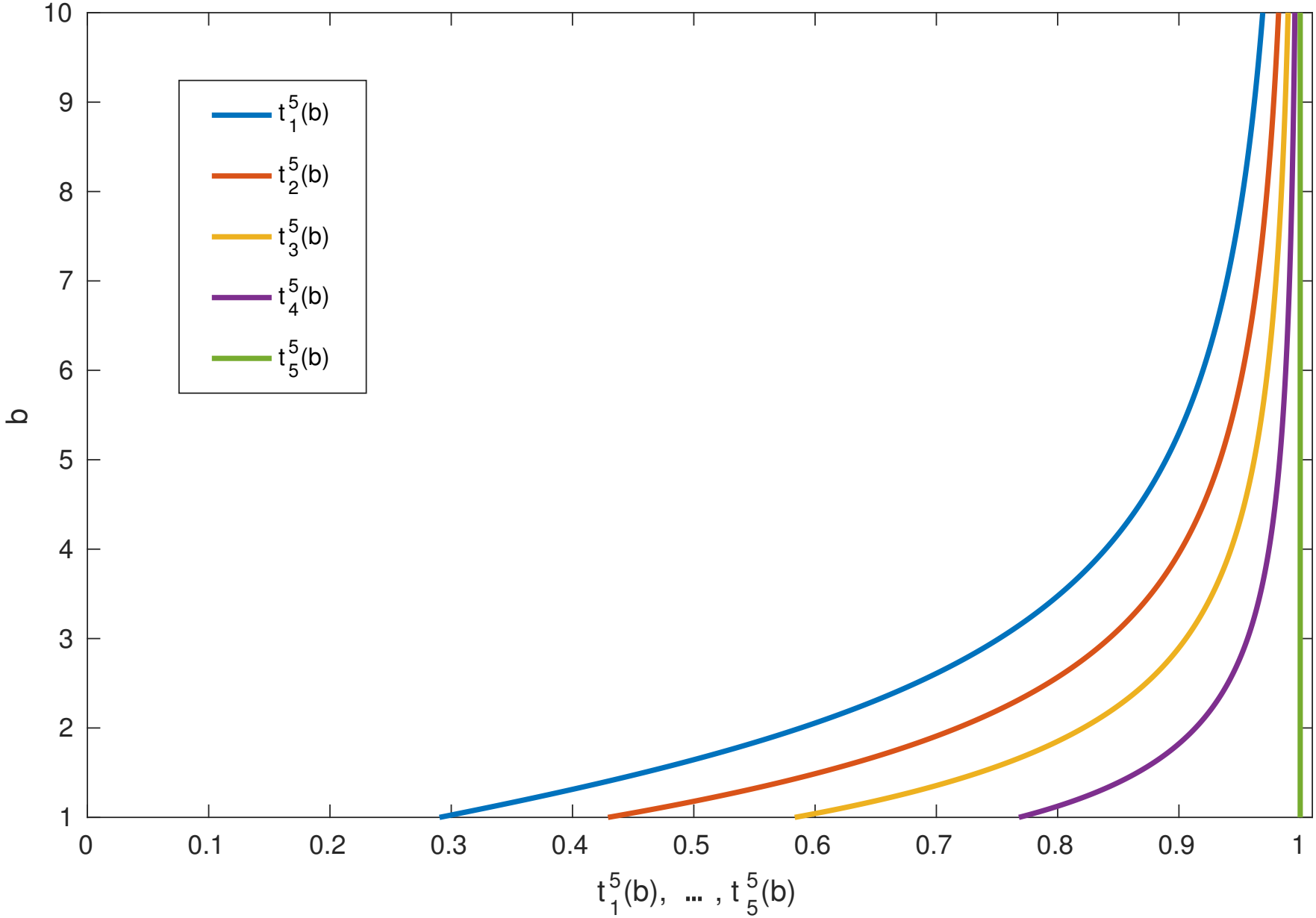
Adaptive grids

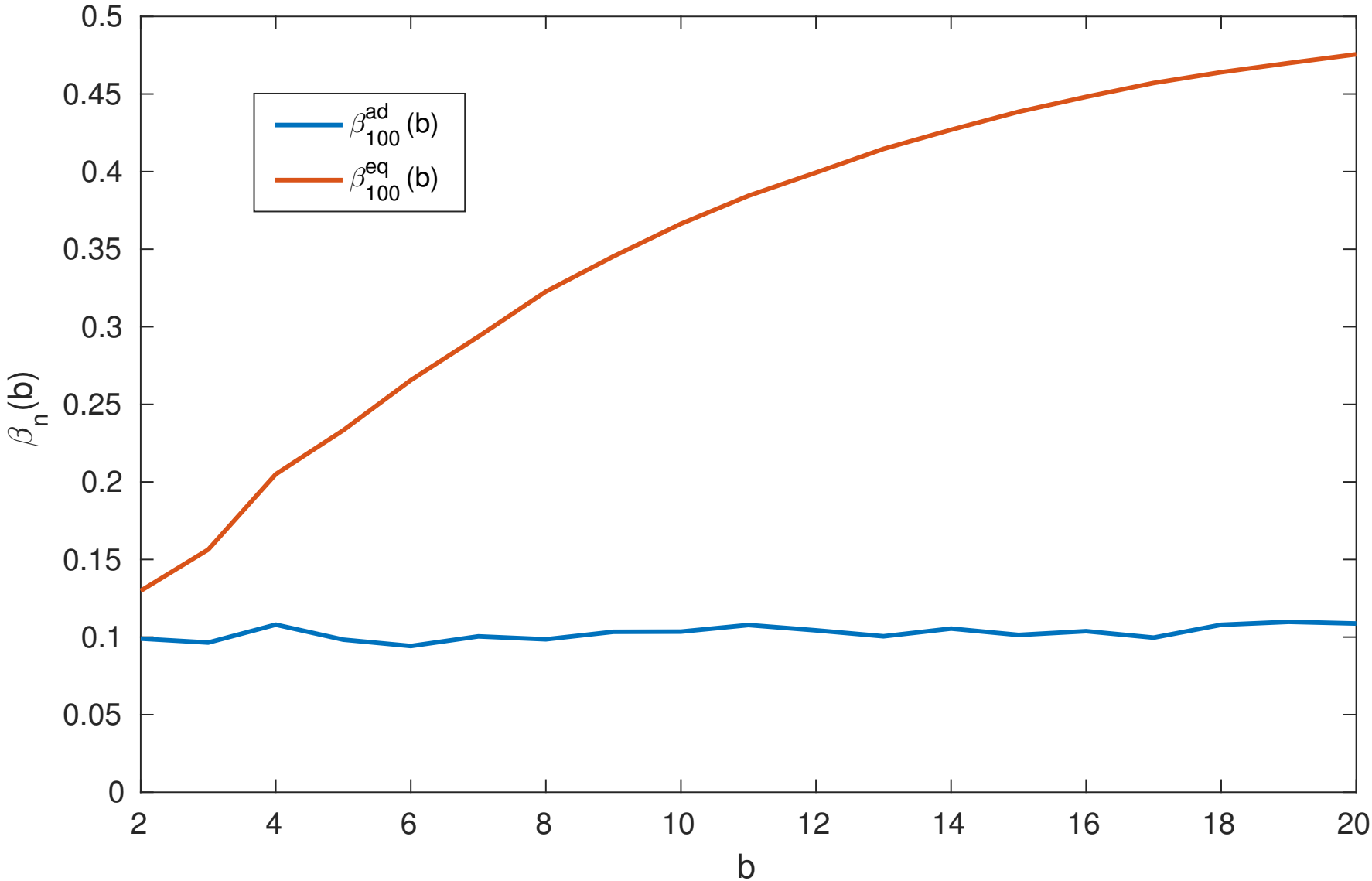
$$t_k^n(b) := \left(\frac{b}{\Phi^{-1}\left(\frac{k}{n}\,\Phi(-b)\right)} \right)^2 \approx 1 - \frac{2\log\left(k/n\right)}{b^2}$$

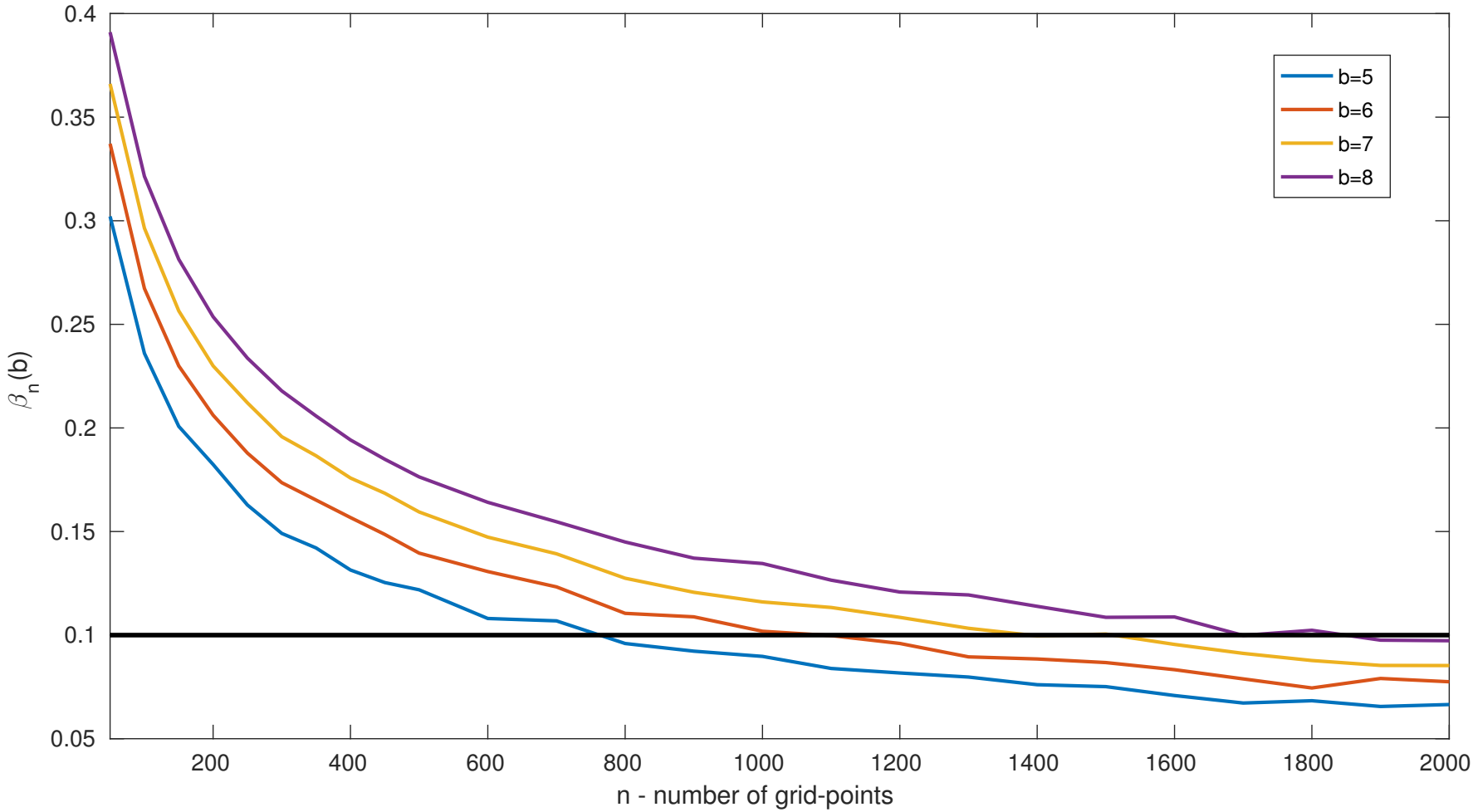
Adaptive grids

$$t_k^n(b) := \left(\frac{b}{\Phi^{-1}\left(\frac{k}{n}\,\Phi(-b)\right)} \right)^2 \approx 1 - \frac{2\log\left(k/n\right)}{b^2}$$

Relative error is bounded independently of b!


10
9
8
7
6
5
4
3
2
1
b
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
$t_1^5(b)$
$t_2^5(b)$
$t_3^5(b)$
$t_4^5(b)$
$t_5^5(b)$
$t_1^5(b), \ldots, t_5^5(b)$


0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
2
4
6
8
10
12
14
16
18
20
b
$\beta_n(b)$
$\beta^{ad}_{100}(b)$
$\beta^{eq}_{100}(b)$


0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
$\beta_n(b)$
200
400
600
800
1000
1200
1400
1600
1800
2000
n - number of grid-points
b=5
b=6
b=7
b=8

Future research

- Identify *optimal* grids.

Future research

- Identify *optimal* grids.
- Generalize the method to a class of Gaussian Processes. What class?

Future research

- Identify *optimal* grids.
- Generalize the method to a class of Gaussian Processes. What class?
- Develop general methods for finding the threshold-crossing probability