The ML Group at CWI + Monte Carlo Tree Search

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CWI Scientific Meeting, Friday 16th June, 2017
The Machine Learning Group
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Misspecification
Hypothesis Testing
Statistical Learning Theory

Spiking Neural Networks
Deep Reinforcement Learning

Online Learning and Optimisation
Monte Carlo Tree Search
Overview

Questions:
- Which data points to collect?
- How many data points do I need?
- How to draw provable conclusions?

Today:
- Best Arm Identification
- Monte Carlo Tree Search
Clinical Trials Example
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Clinical Trials Example

[Image]
Clinical Trials Example
Clinical Trials Example
Clinical Trials Example
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Clinical Trials Example
Clinical Trials Example
Probably Approximately Correct (PAC) Learning

World: drug success rates $\mu = (\mu_1, \ldots, \mu_K)$

Strategy:
- Adaptive sampling rule $l_t$
- Stopping rule $\tau$
- Recommendation rule $\hat{i}$
Probably Approximately Correct (PAC) Learning

World: drug success rates $\mu = (\mu_1, \ldots, \mu_K)$

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- Adaptive sampling rule $I_t$
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Definition

A strategy is $\delta$-PAC if for all $\mu$

$$\mathbb{P}_\mu \left( \hat{i} \neq \arg \max_i \mu_i \right) \leq \delta.$$
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Want: $\delta$-PAC strategy with low sample complexity $\mathbb{E}[\tau]$. 
Thompson Sampling

- Assume prior distributions on success rate $\mu_i$ of each drug $i$.
- Each round $t$
  - Draw a world $\tilde{\mu}_t$ from posteriors
  - Try the best drug for it $l_t = \arg \max_i \tilde{\mu}_{t,i}$
  - Update the posterior

Video.
Information Theoretic Lower Bounds

How many samples are really necessary in a world $\mu$?
Information Theoretic Lower Bounds

How many samples are **really necessary** in a world $\mu$?

**Change of measure idea**

Consider a world $\mu'$ that is close to $\mu$ but has a different best drug

$$\arg\max_i \mu_i' \neq \arg\max_i \mu_i$$

**Few** observations might as well have come from $\mu'$ . . .

. . . but then your answer is wrong.

Need **many** observations.
Information Theoretic Lower Bounds

How many samples are really necessary in a world \( \mu \)?

**Change of measure idea**

Consider a world \( \mu' \) that is close to \( \mu \) but has a different best drug

\[
\arg \max_i \mu'_i \neq \arg \max_i \mu_i
\]

Few observations might as well have come from \( \mu' \) . . .

. . . but then your answer is wrong.

Need many observations.

**Theorem (KCG’15)**

Any \( \delta \)-PAC algorithm needs

\[
\mathbb{E}[\tau] \geq T^*(\mu) \ln \frac{1}{\delta} \quad \text{where} \quad \frac{1}{T^*(\mu)} = \max_{\mu' \in \Delta} \min_{\mu \in \Delta} \sum_i w_i \text{KL}(\mu_i || \mu'_i)
\]
Outlook

Optimal algorithm now available [GK’16].

- Matching lower bound
- Characterise proportion of draws of each arm

"Top Two" Thompson Sampling gets very close [R’16].

How to answer more challenging questions?
Robust Clinical Trials Example
Robust Clinical Trials Example
Robust Clinical Trials Example
My Project

Goal: develop complete theory of tree search

- Lower bounds
  - Optimal weights are often sparse
  - hints at pruning
  - computational challenges

- Well-developed understanding of depth 2 [GKK’15]

- Upgrading efficient algorithms [THT’14, GKK’15, KK’17]
My Project

Goal: develop complete theory of tree search

- Lower bounds
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Applications beyond robust statistics: