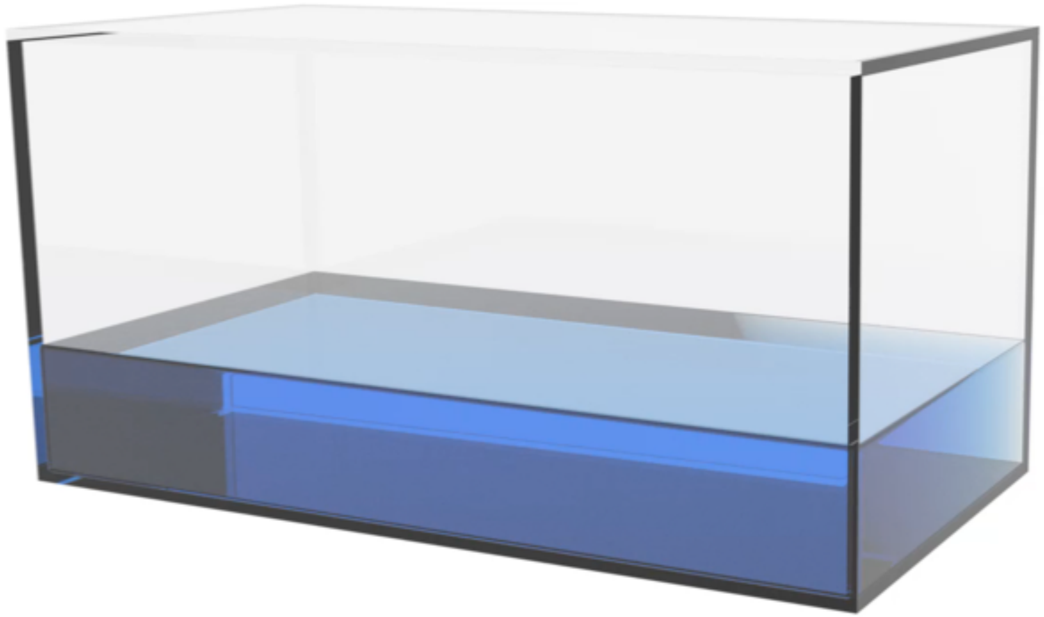


Uncertainty quantification for fluid dynamics applications

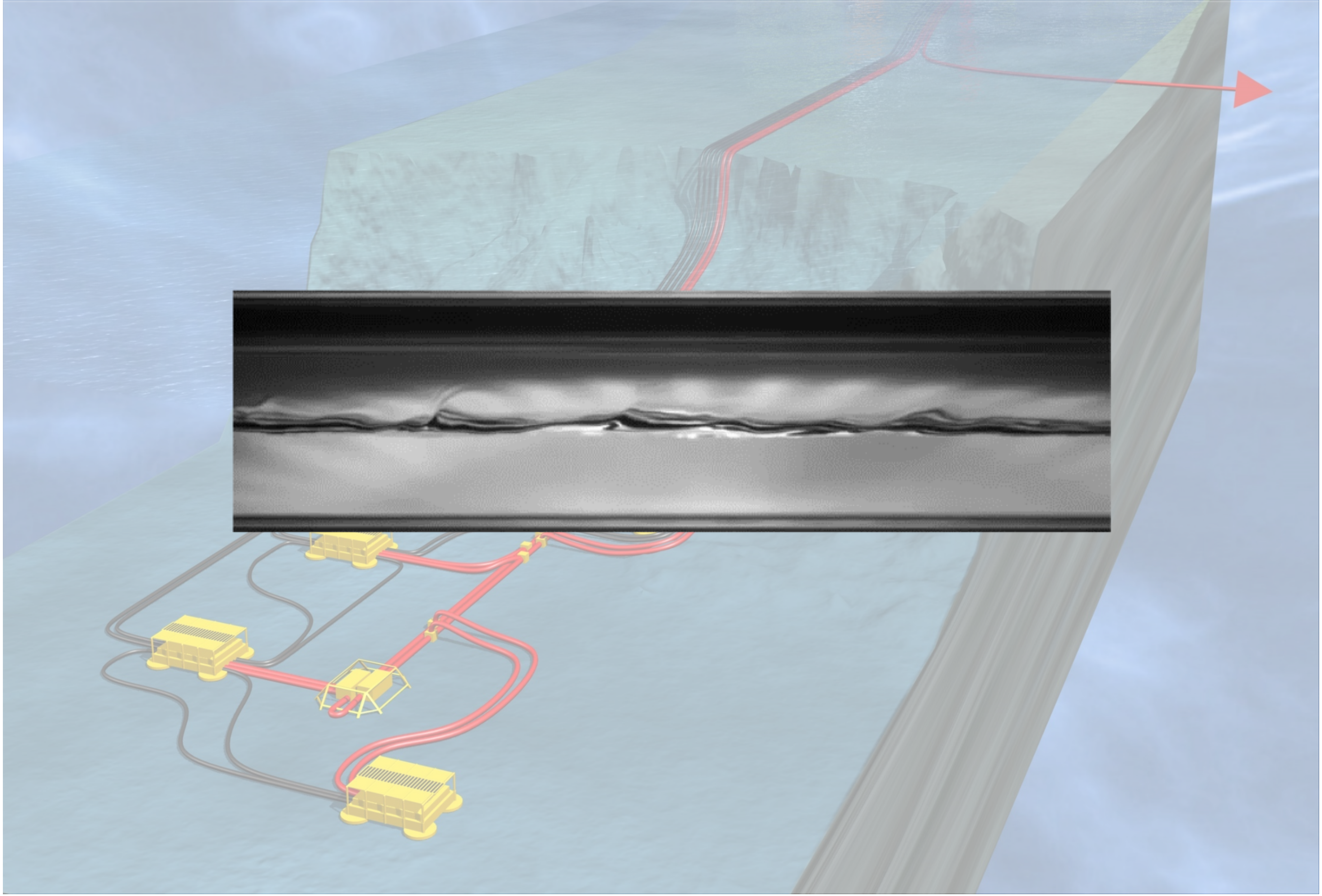
Benjamin Sanderse, Laurent van den Bos, Yous van Halder,
Sirshendu Misra, Julia Klinkert

CWI

Variability in sloshing impacts

Variability in sloshing impacts

Improved prediction of multiphase flow in pipelines



Uncertainty reduction in offshore wind

EUROS

Bel Air Aviation - Helicopter Services

UQ and fluid dynamics

Fluid dynamics

Partial differential equations (Navier-Stokes)

Conservation laws

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$$

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} \otimes \boldsymbol{u}) = -\nabla p + \nabla \cdot (\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T)$$

Uncertainties

Random variables

Probability density functions

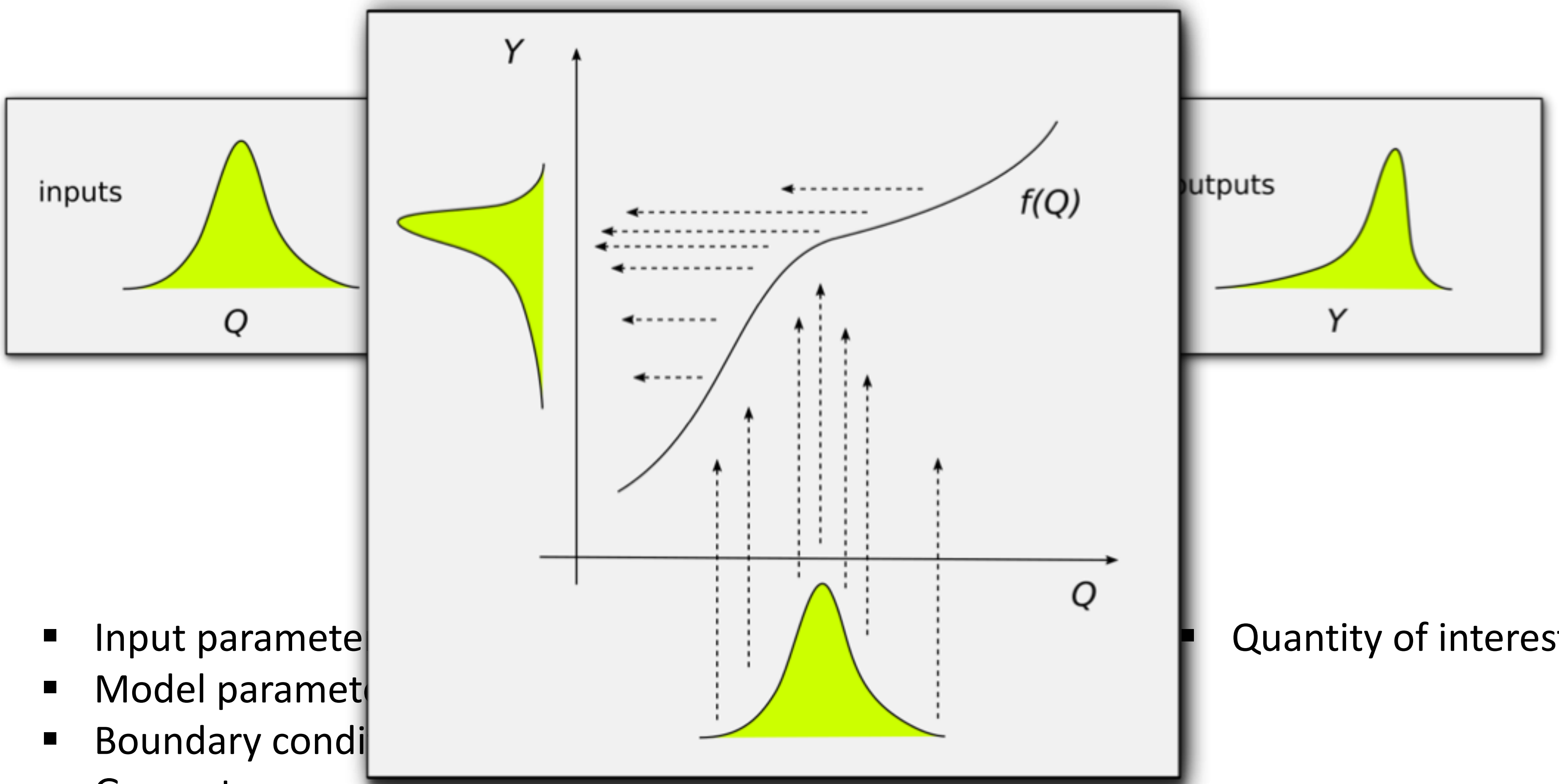
Stochastic processes

$$q \sim \mathcal{N}(\mu, \sigma^2)$$

- PDEs with random coefficients
- Stochastic PDEs

Uncertainty Quantification

- **Propagation** – a forward problem

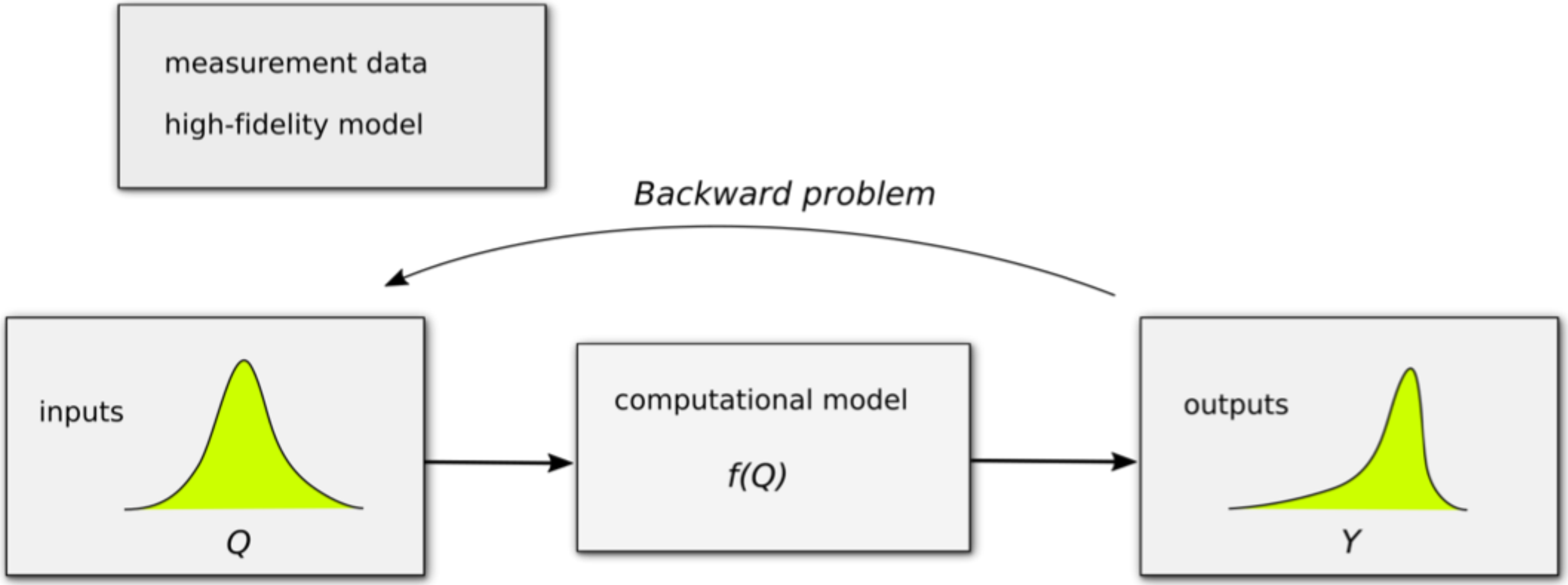


- Input paramete
- Model paramet
- Boundary condi
- Geometry

- Quantity of interest

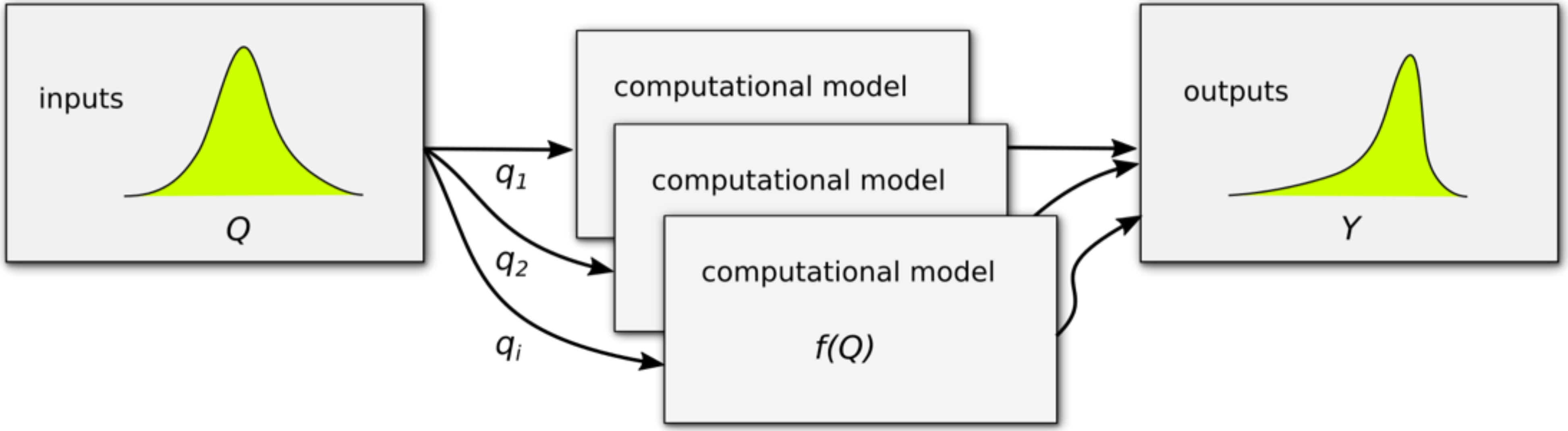
Uncertainty Quantification

- **Calibration** – a backward problem



Monte Carlo

random samples q_i with *equal* weights



$$Q = (Q_1, ..., Q_d)$$

$$\rho_Q(q) = \prod_{i=1}^{d} \rho_{Q_i}(q_i)$$

$$\mu_Y = \mathbb{E}[Y] \approx \frac{1}{N} \sum_{i=1}^{N} f(q_i)$$

Monte Carlo?

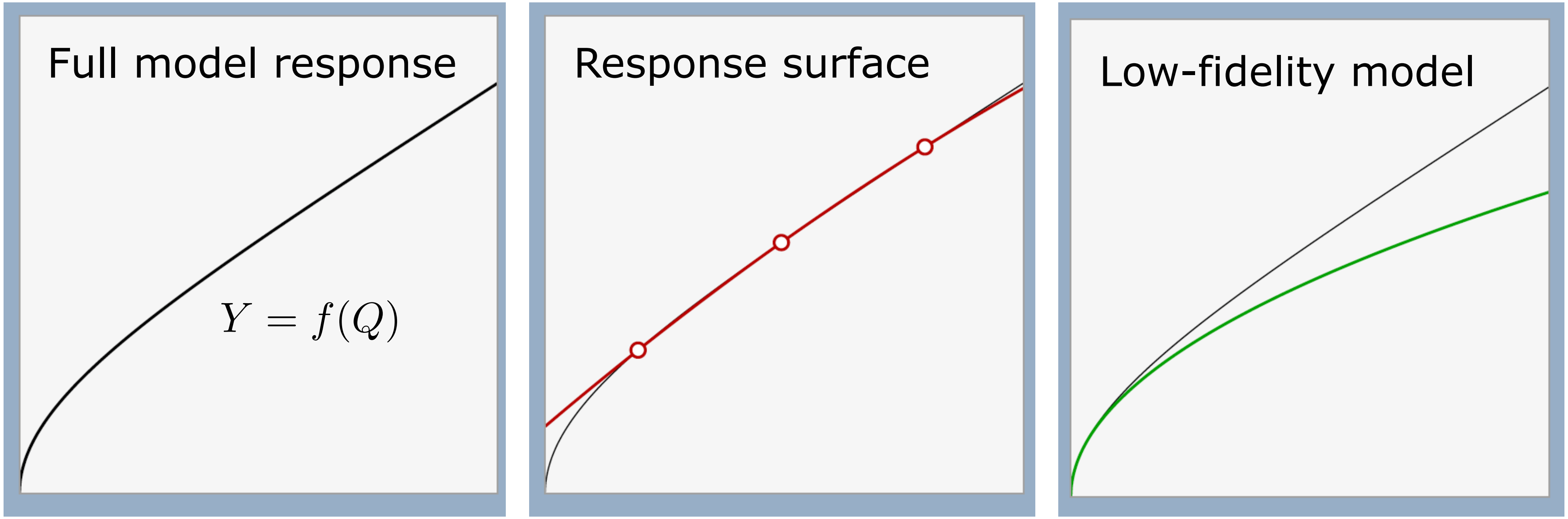
Challenge:

Fluid dynamic models feature very **expensive models** *$f(Q)$*

Our approach:

Approximate full model with **surrogate** models

Surrogate models



'mathematics-based':
- **Polynomials**
- Radial basis functions
- Gaussian processes
- Neural networks

'physics-based':
- **Averaging**
- Asymptotics
- Modal expansions

M. Sinsbeck,
PhD thesis 2016

Polynomial response surface

- Polynomial Chaos Expansion
- Fourier expansion in random space:

$$Y = f(q) \approx f_{\mathrm{PCE}}(q) = \sum_{i=0}^{N} \hat{f}_i \phi_i(q)$$

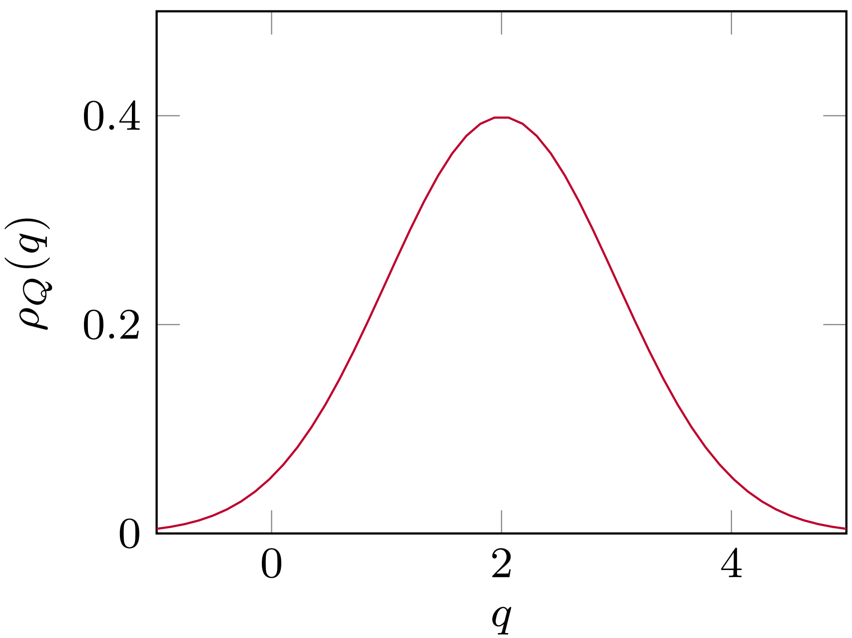
- Use polynomials $\phi(q)$ *orthogonal* with respect to PDF $\rho(q)$:

$$\int \phi_i(q)\phi_j(q)\rho_Q(q)\mathrm{d}q = \begin{cases} \gamma_i & i = j, \\ 0 & i \neq j. \end{cases}$$

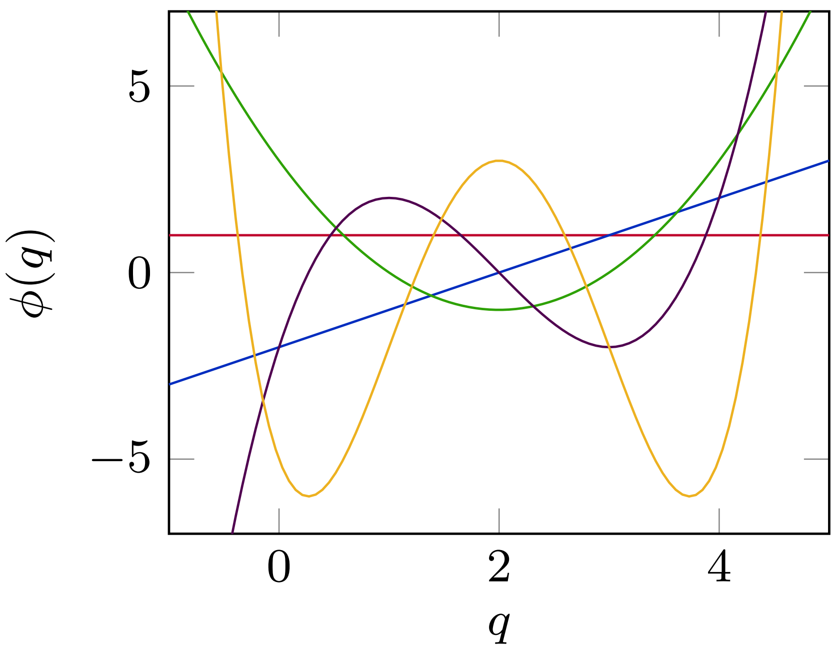
Polynomial Chaos Expansion

- Example orthogonality polynomials and PDF

Gaussian random variable



Hermite polynomials



Polynomial Chaos Expansion

- Fourier expansion in random space:

$$Y = f(q) \approx f_{\mathrm{PCE}}(q) = \sum_{i=0}^{N} \hat{f}_i \phi_i(q)$$

- Coefficients follow from

$$\hat{f}_i = \frac{1}{\gamma_i} \int f(q)\phi_i(q)\rho_Q(q)\mathrm{d}q$$

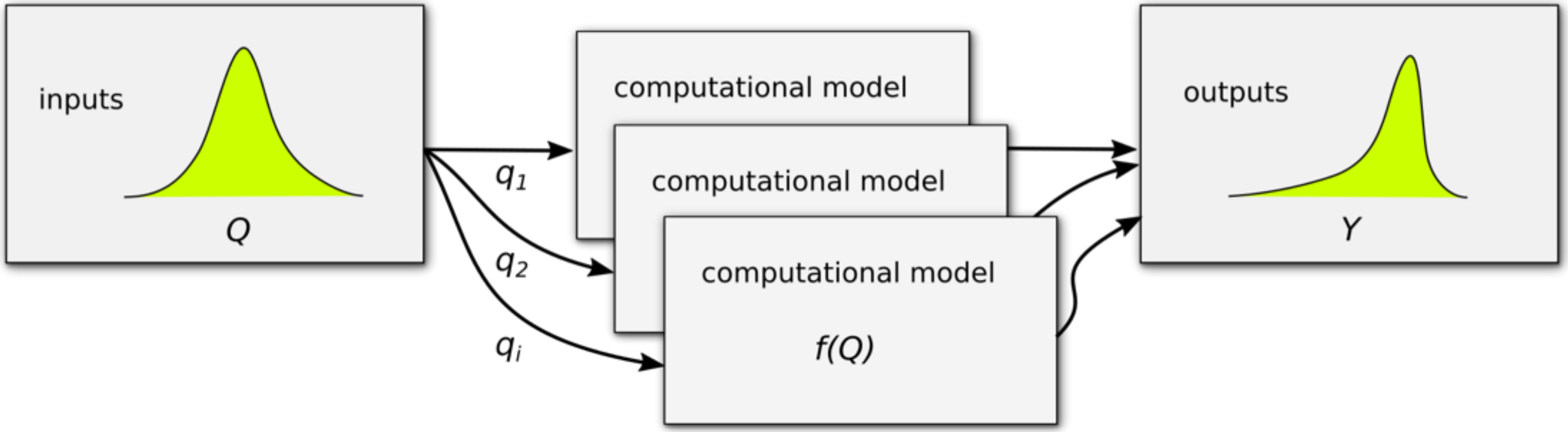
- Approximate with quadrature rules

$$\int f(q)\phi_i(q)\rho_Q(q)\mathrm{d}q \approx \sum_{k=1}^{K} f(q_k)\phi_i(q_k)w_k$$

Model sampling at nodes q_k

Polynomial Chaos Expansion

deterministic samples q_i



analytic output $f_{\mathrm{PCE}}(q)$

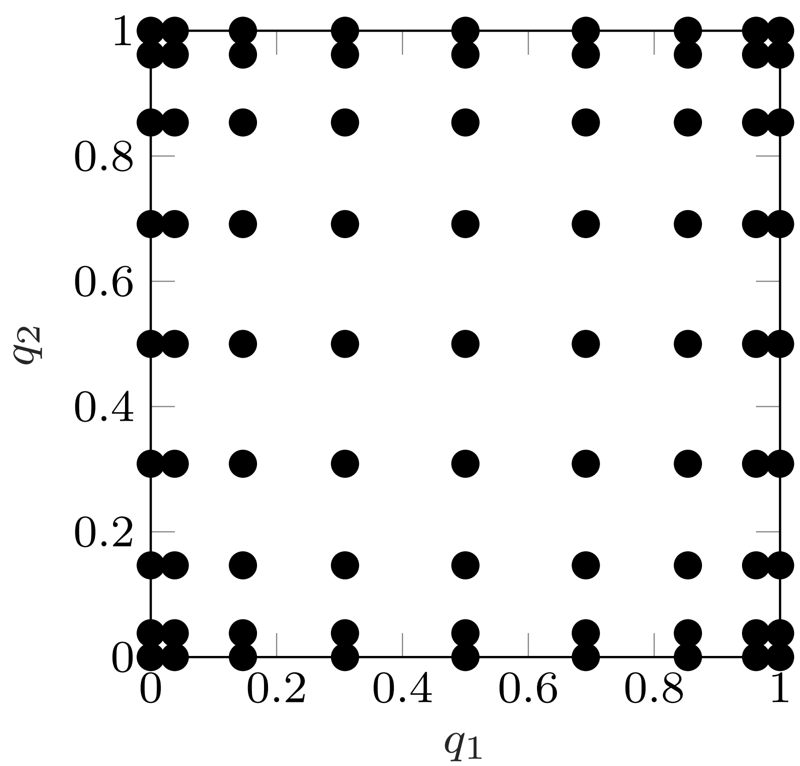
$$\mu_Y = \hat{f}_0$$

$$\sigma_Y = \sum_{i=1} \hat{f}_i^2$$

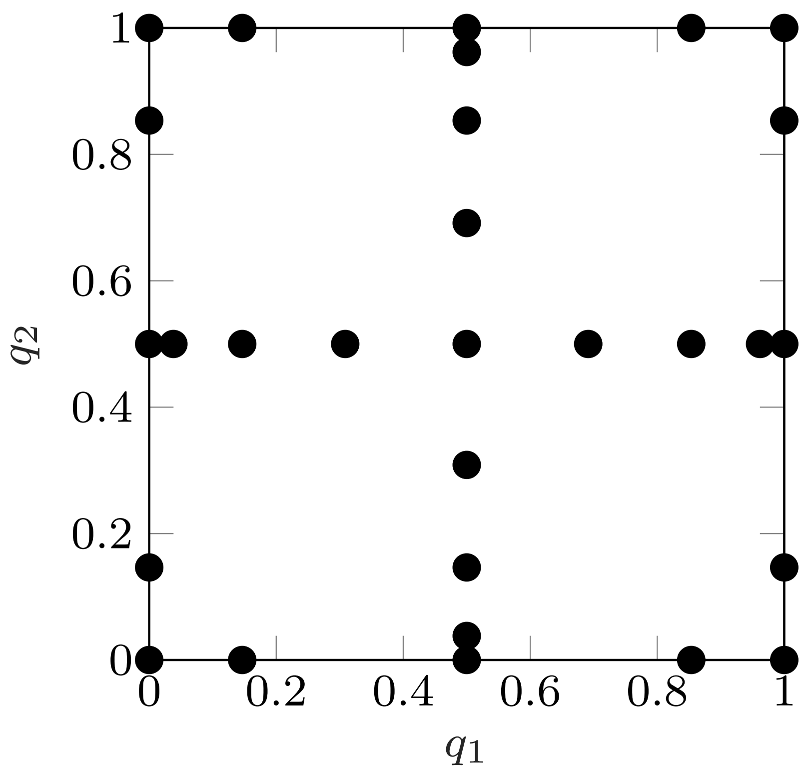
Monte Carlo

Model sampling

Tensor grid

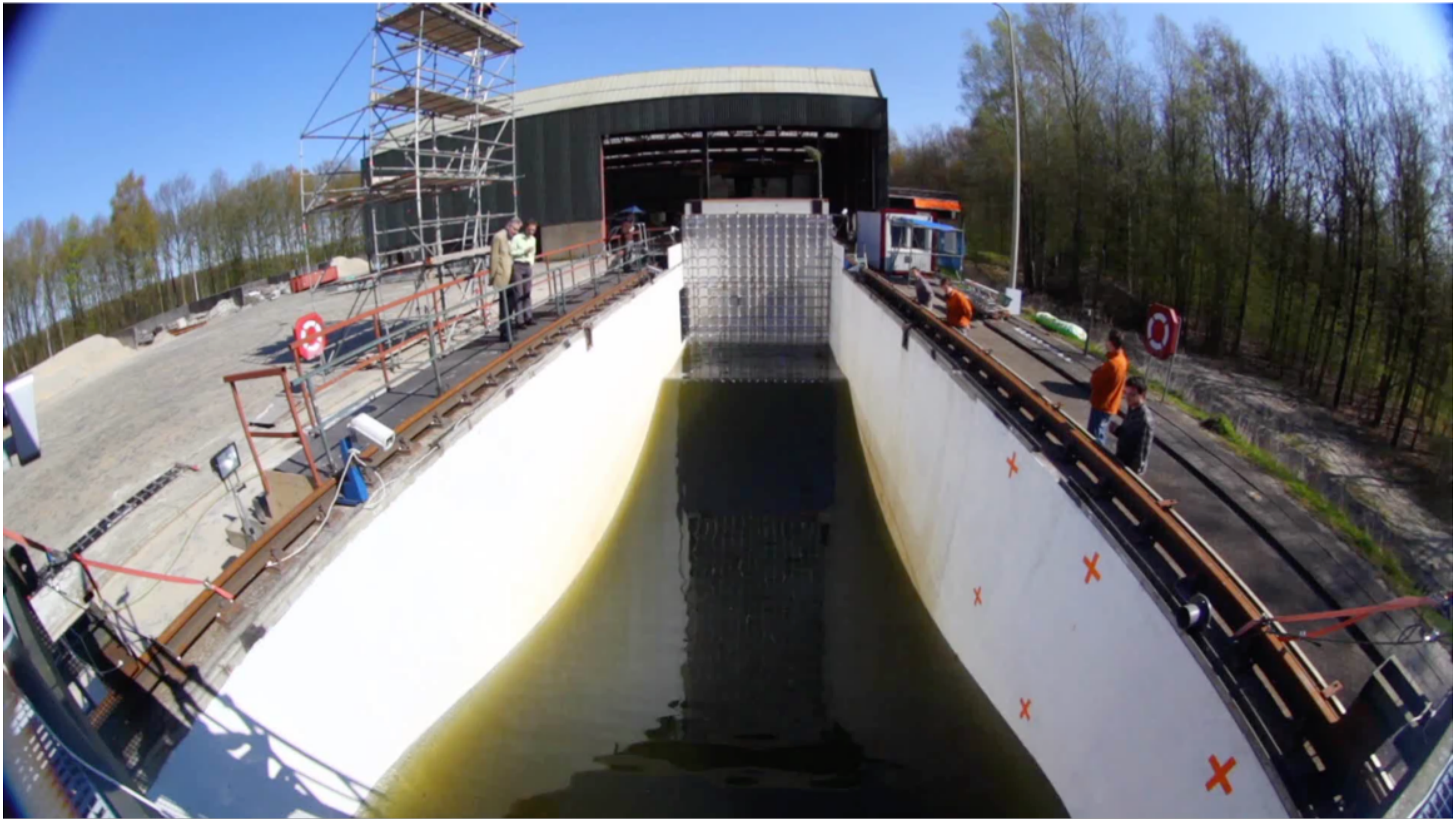


Sparse grid

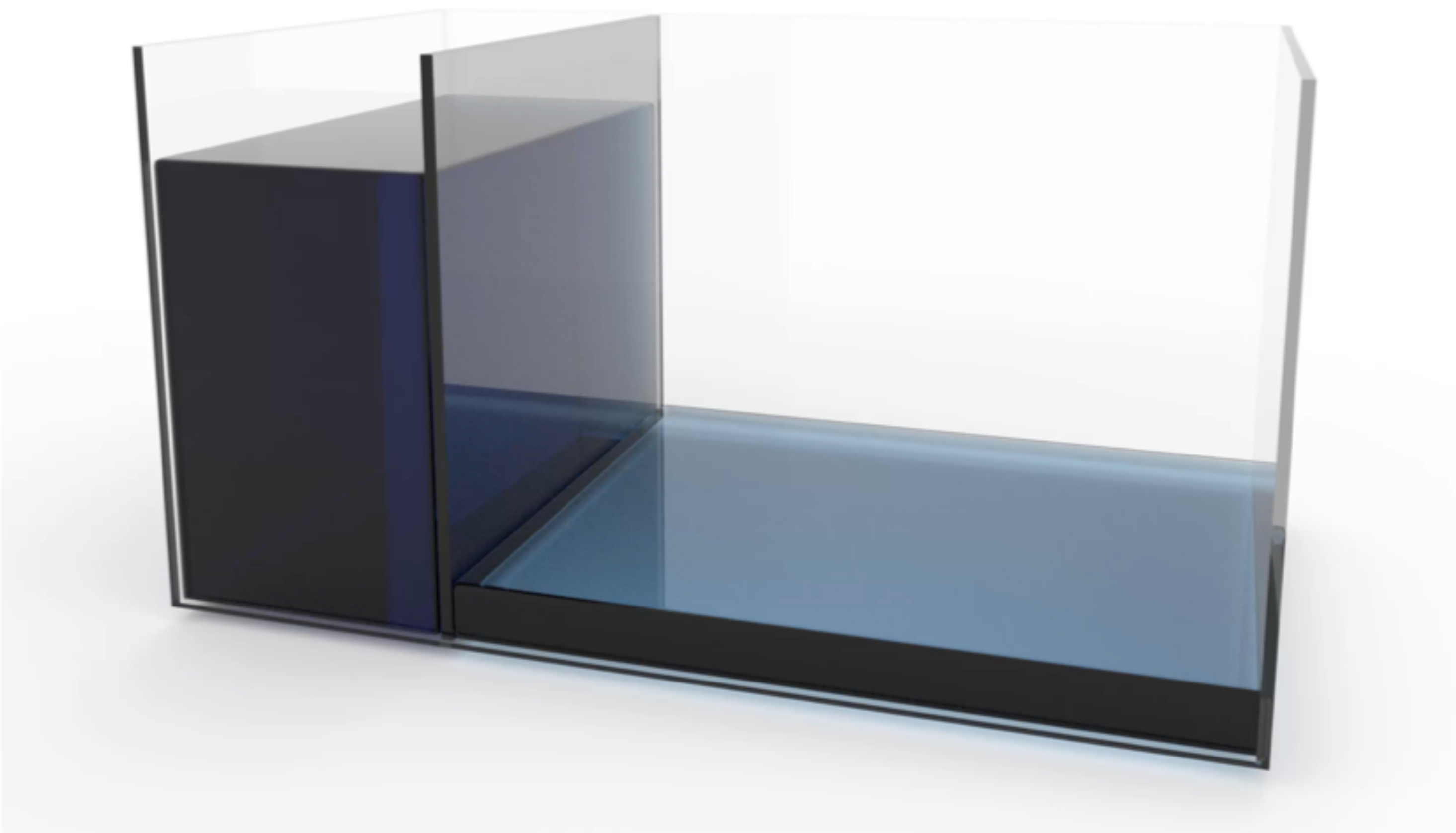


- Random space is **high-dimensional**: curse of dimensionality $N = M^d$
- Where to sample the model?
- Use model behavior to adaptively select nodes

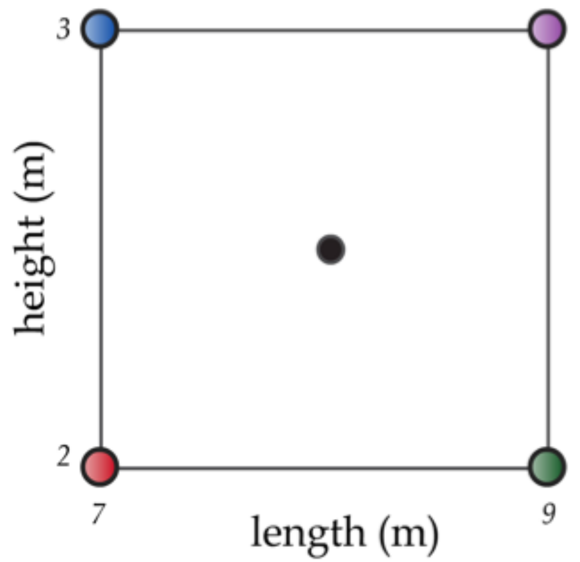
Sloshing

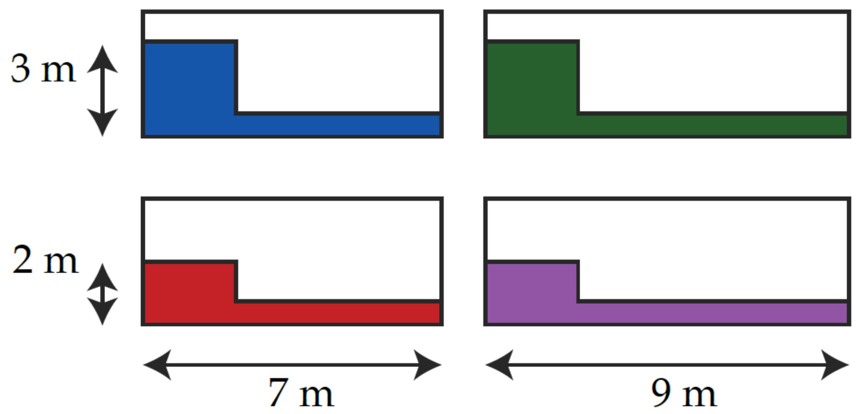


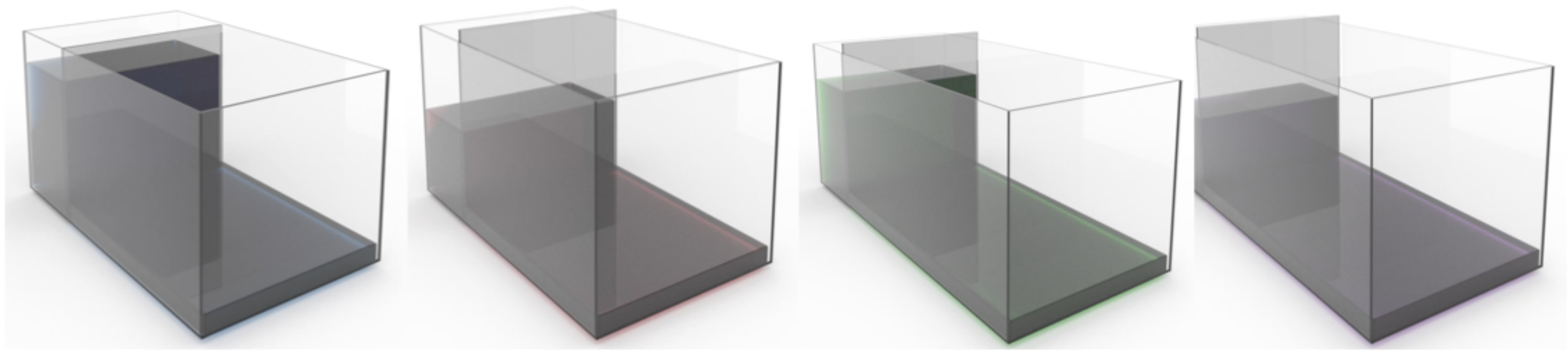
Sloshing



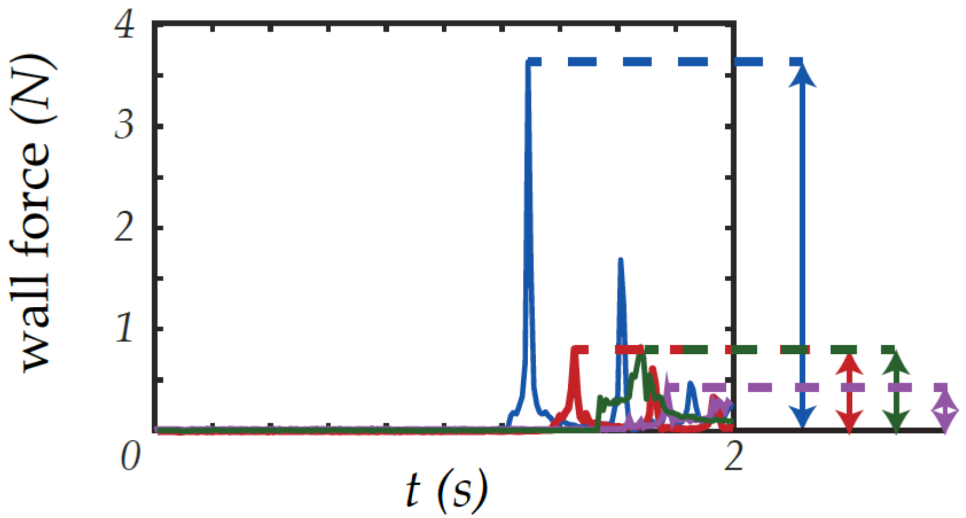
Adaptive sampling for sloshing

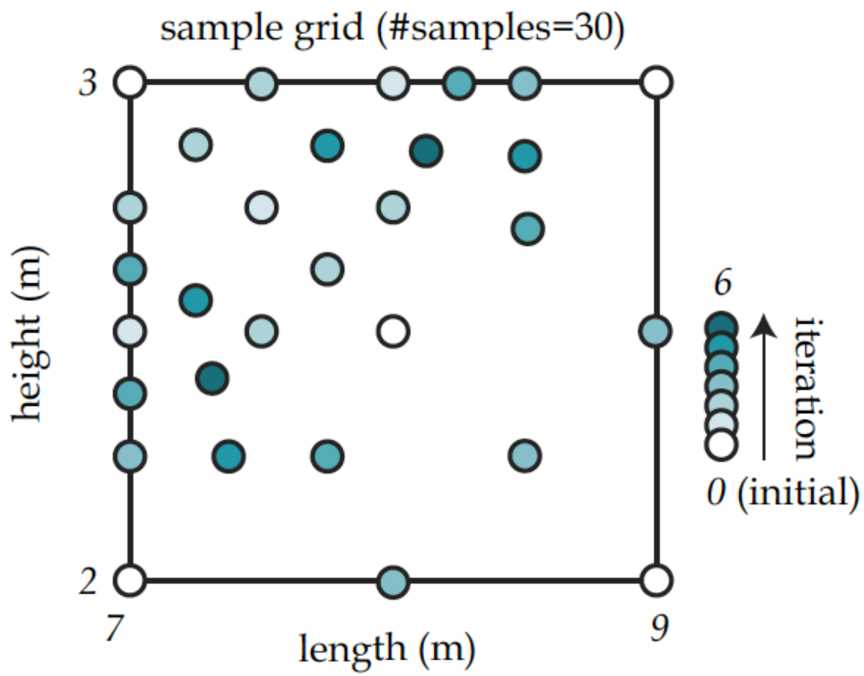


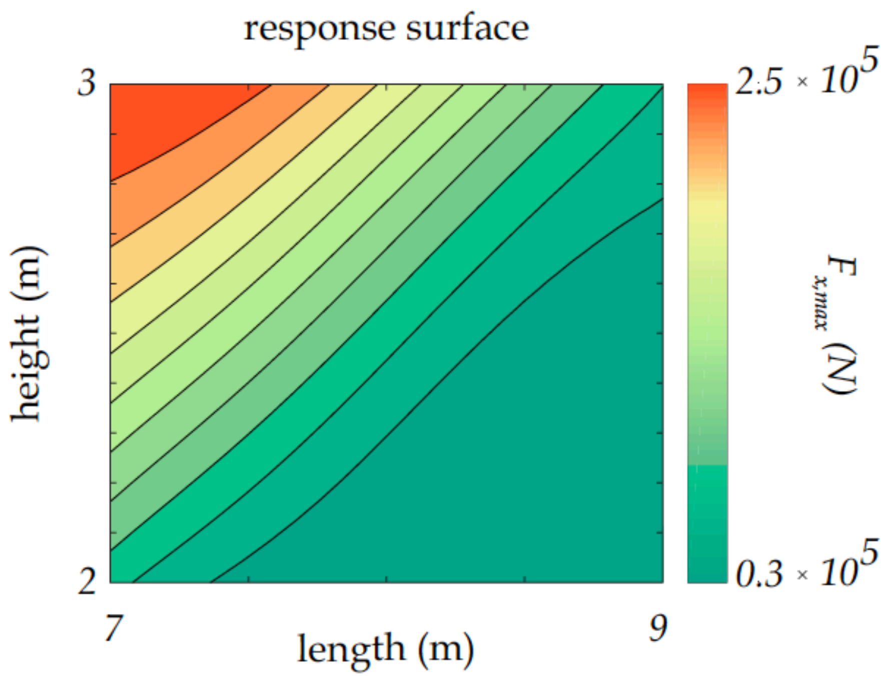




Adaptive sampling for sloshing





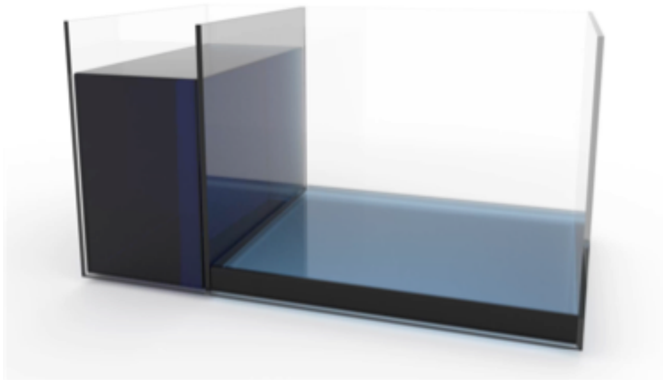


Physics-based surrogate model

Full model

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$$

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} \otimes \boldsymbol{u}) = -\nabla p + \nabla \cdot (\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T)$$

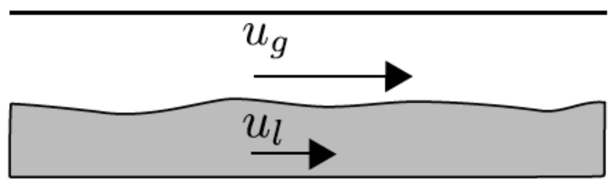


Averaging

Low-fidelity model

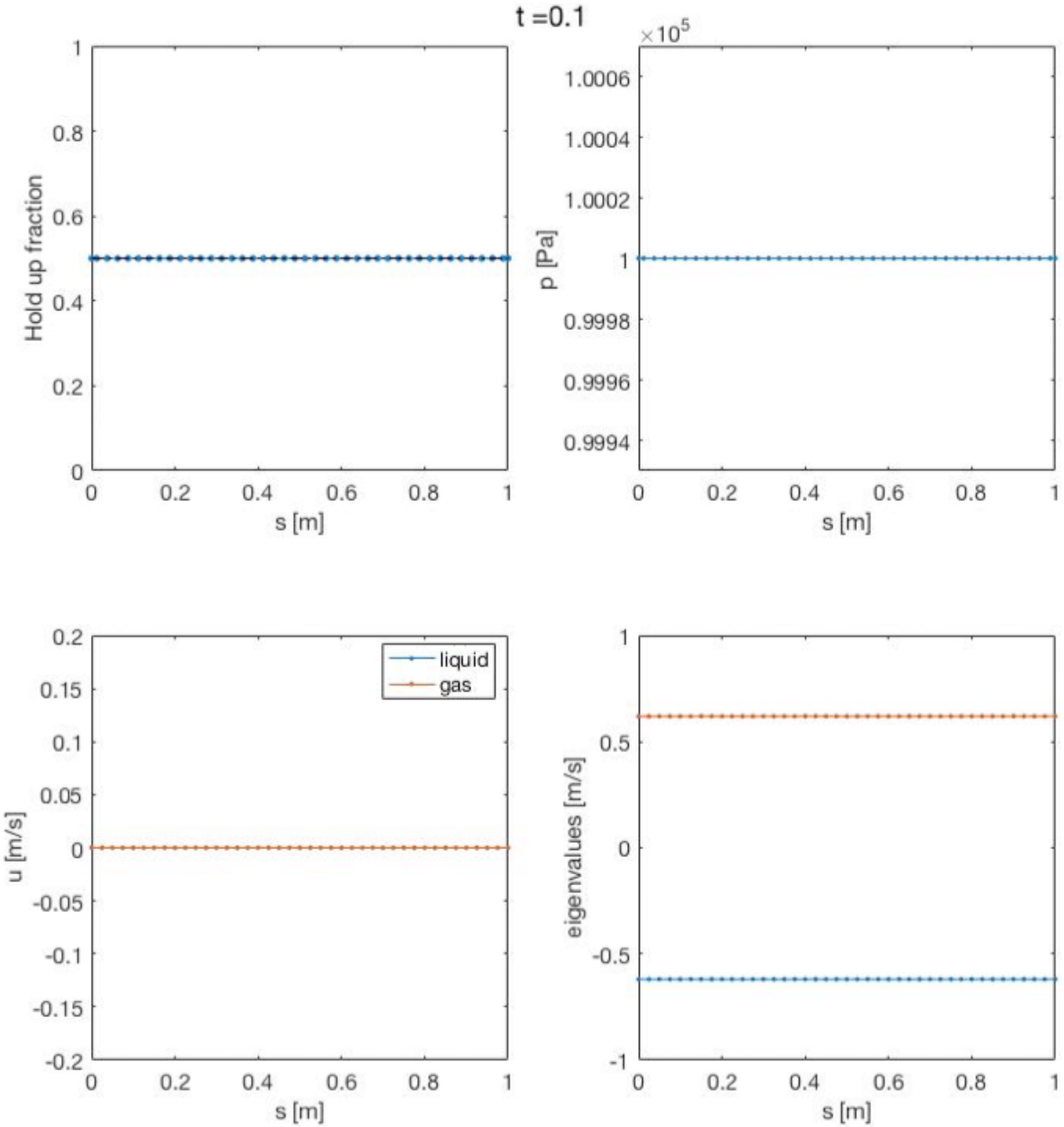
$$\frac{\partial}{\partial t}(\rho_g A_g) + \frac{\partial}{\partial s}(\rho_g u_g A_g) = 0$$

$$\frac{\partial}{\partial t}(\rho_g u_g A_g) + \frac{\partial}{\partial s}(\rho_g u_g^2 A_g) = -\frac{\partial p}{\partial s} A_g + LG_g - F_g$$



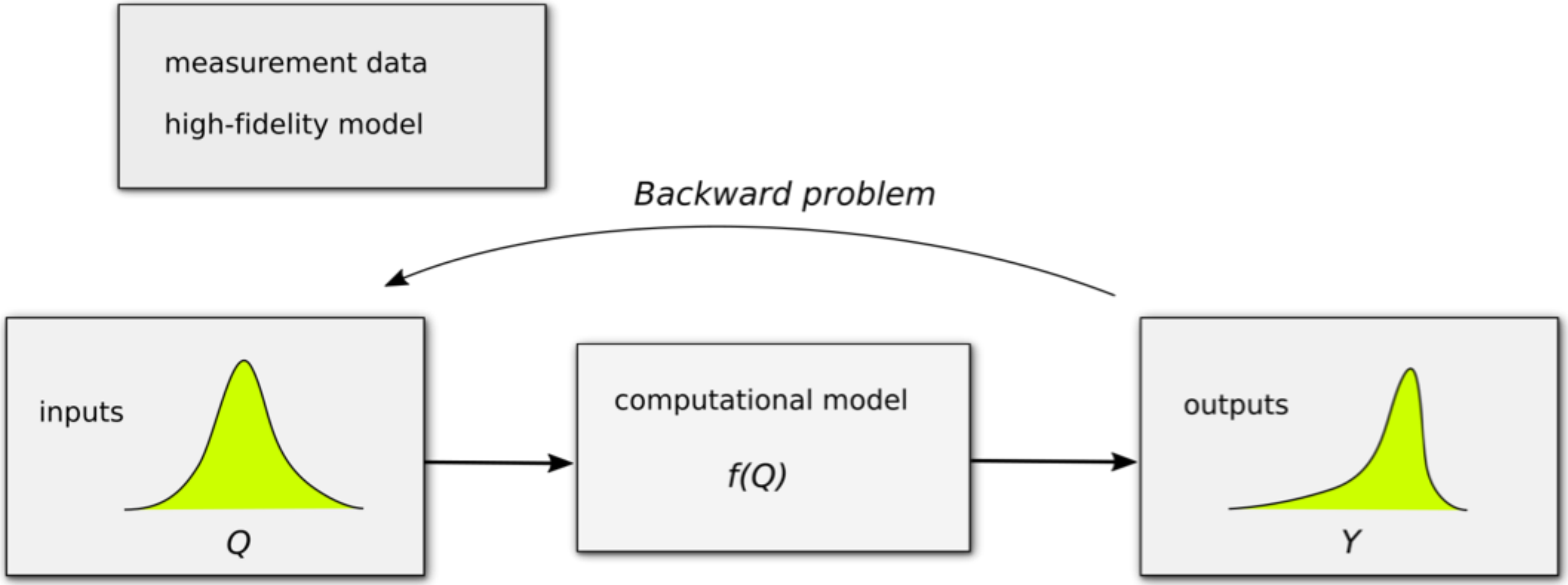
Closure problem!

Physics-based approach



Bayesian model calibration

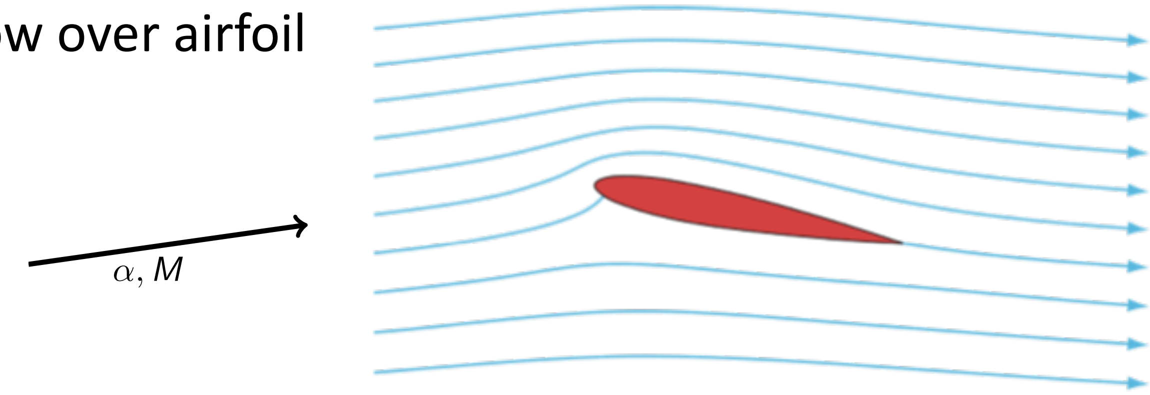
- Bayesian approach: elegant, but expensive
- Surrogate models to reduce computational cost



$$\underbrace{p(\text{parameters}|\text{data})}_{\text{posterior}} \propto \underbrace{p(\text{data}|\text{parameters})}_{\text{likelihood}} \; \underbrace{p(\text{parameters})}_{\text{prior}}$$

Calibration of turbulence model parameters

Flow over airfoil

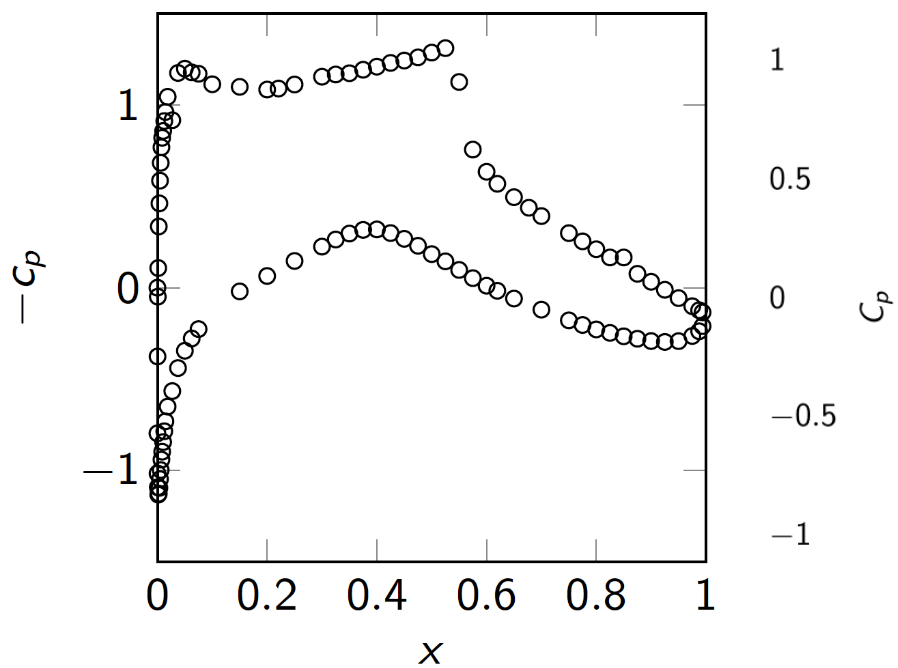


Reynolds-averaged
Navier-Stokes equations
7 closure parameters

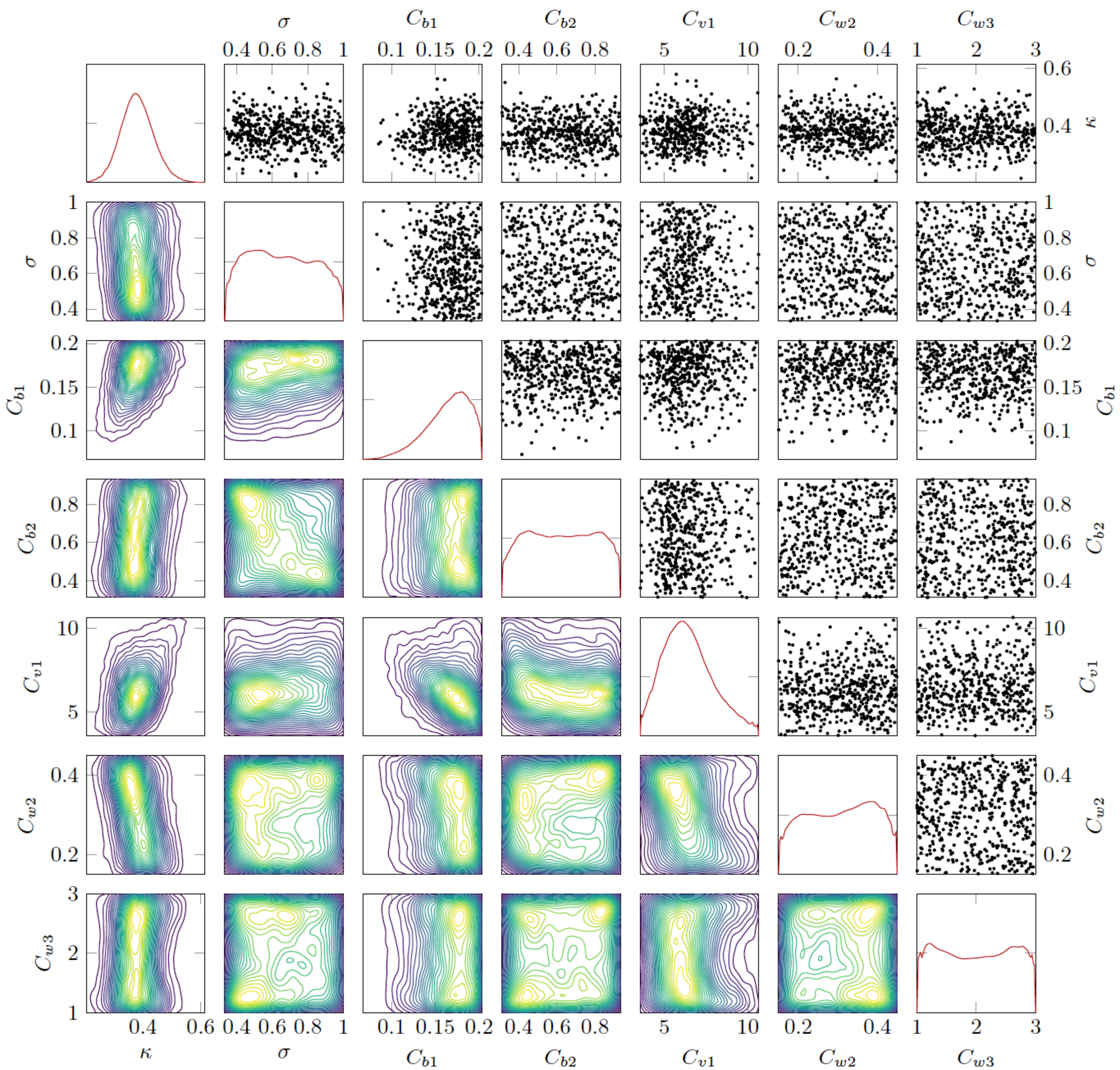
σ	2/3	C_{b1}	0.1355	C_{b2}	0.622
κ	0.41	C_{w2}	0.3	C_{w3}	2.0
C_{t3}	1.2	C_{t4}	0.5	C_{v1}	7.1

Can we calibrate these parameters
given the experimental data?

Experimental data

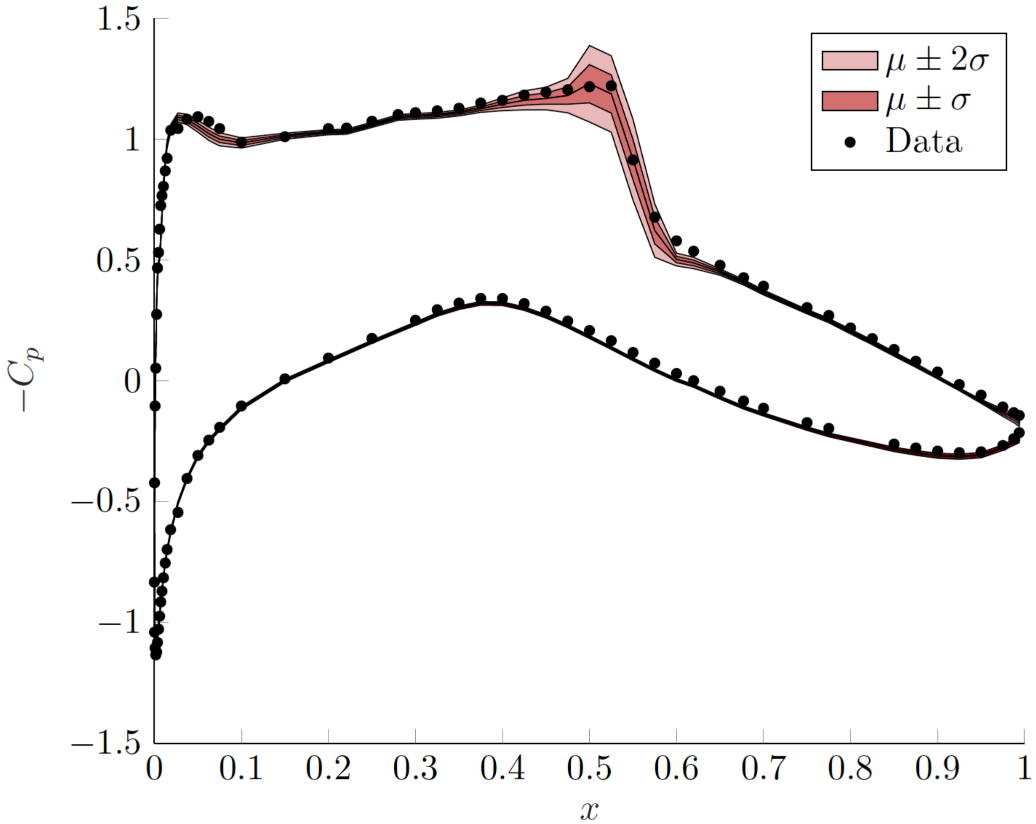


Bayesian model calibration



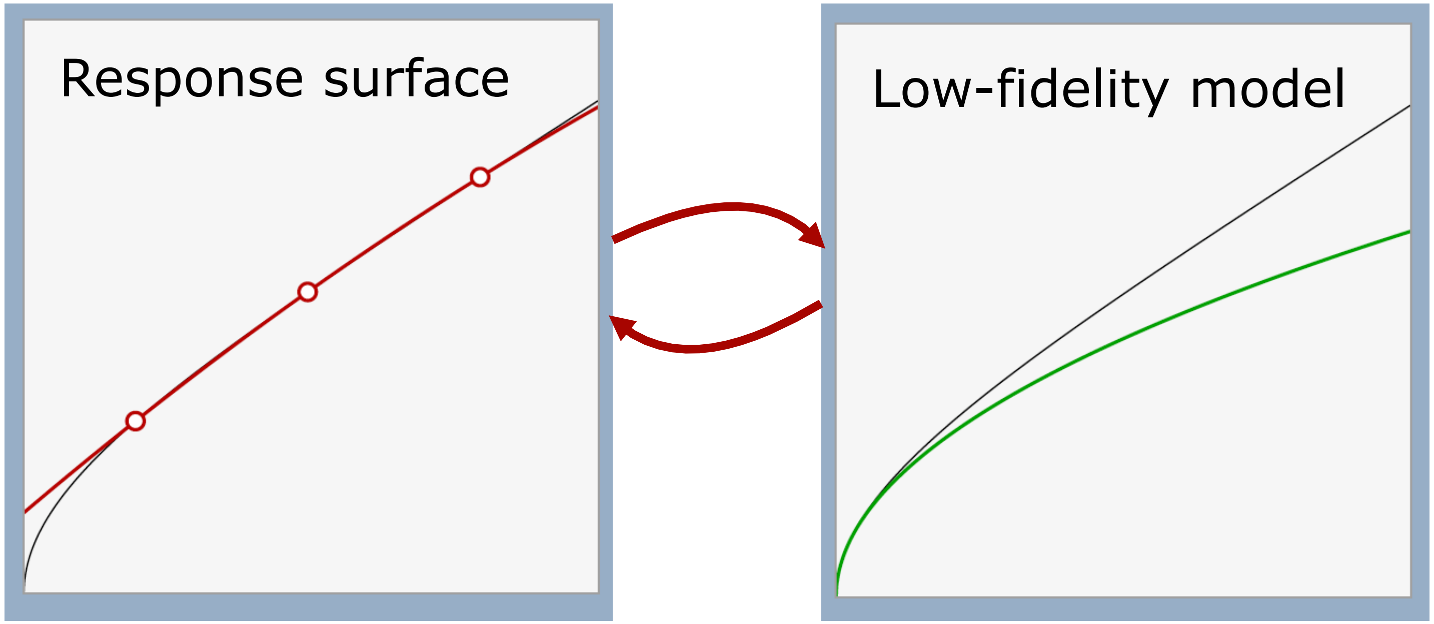
Bayesian model calibration

- *Calibrated* model constants with probability distribution
- Quantified uncertainty for predictive capability



Summary

- Surrogate models make UQ computable for fluid dynamics problems:
  - Propagation
  - Calibration



Interested? Some further reading