

Hash functions in post-quantum cryptography

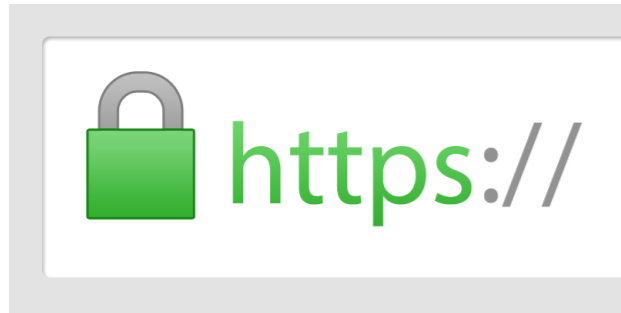
Christian Majenz
CWI



Centrum Wiskunde & Informatica

Cryptography is everywhere

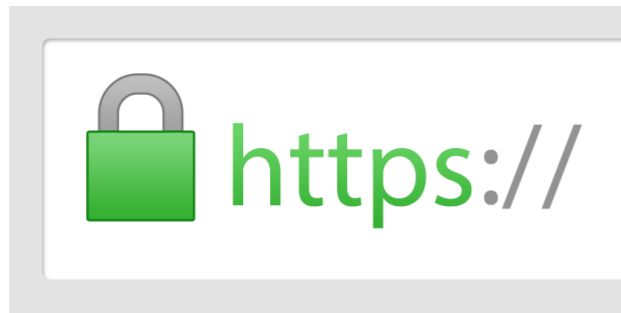
Cryptography is everywhere



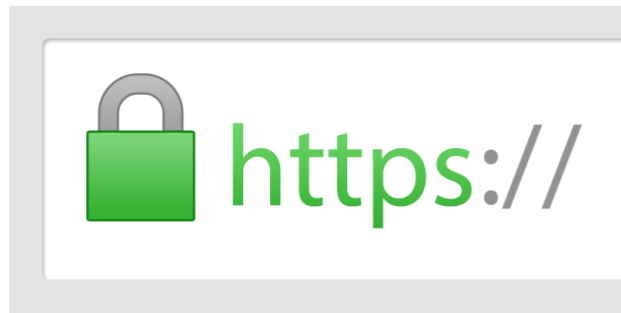
Cryptography is everywhere



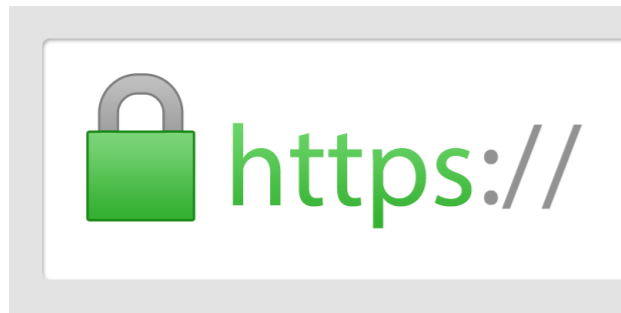
Cryptography is everywhere



Cryptography is everywhere



Cryptography is everywhere



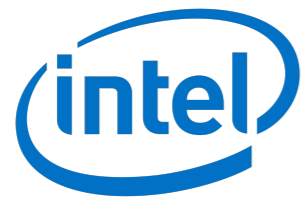
Quantum computers

Quantum computers

- ▶ Accelerating effort to build a quantum computer

Quantum computers

- ▶ Accelerating effort to build a quantum computer
- ▶ Major investments:



Quantum computers

- ▶ Accelerating effort to build a quantum computer
- ▶ Major investments:



We need to prepare cryptography for the arrival of quantum computers!

Quantum computers

- ▶ Accelerating effort to build a quantum computer
- ▶ Major investments:



We need to prepare cryptography for the arrival of quantum computers!

- ▶ Security against quantum attackers



- ▶ Quantum cryptography

Quantum computers

- ▶ Accelerating effort to build a quantum computer
- ▶ Major investments:



We need to prepare cryptography for the arrival of quantum computers!

- ▶ **This talk: Security against quantum attackers (post-quantum cryptography)**

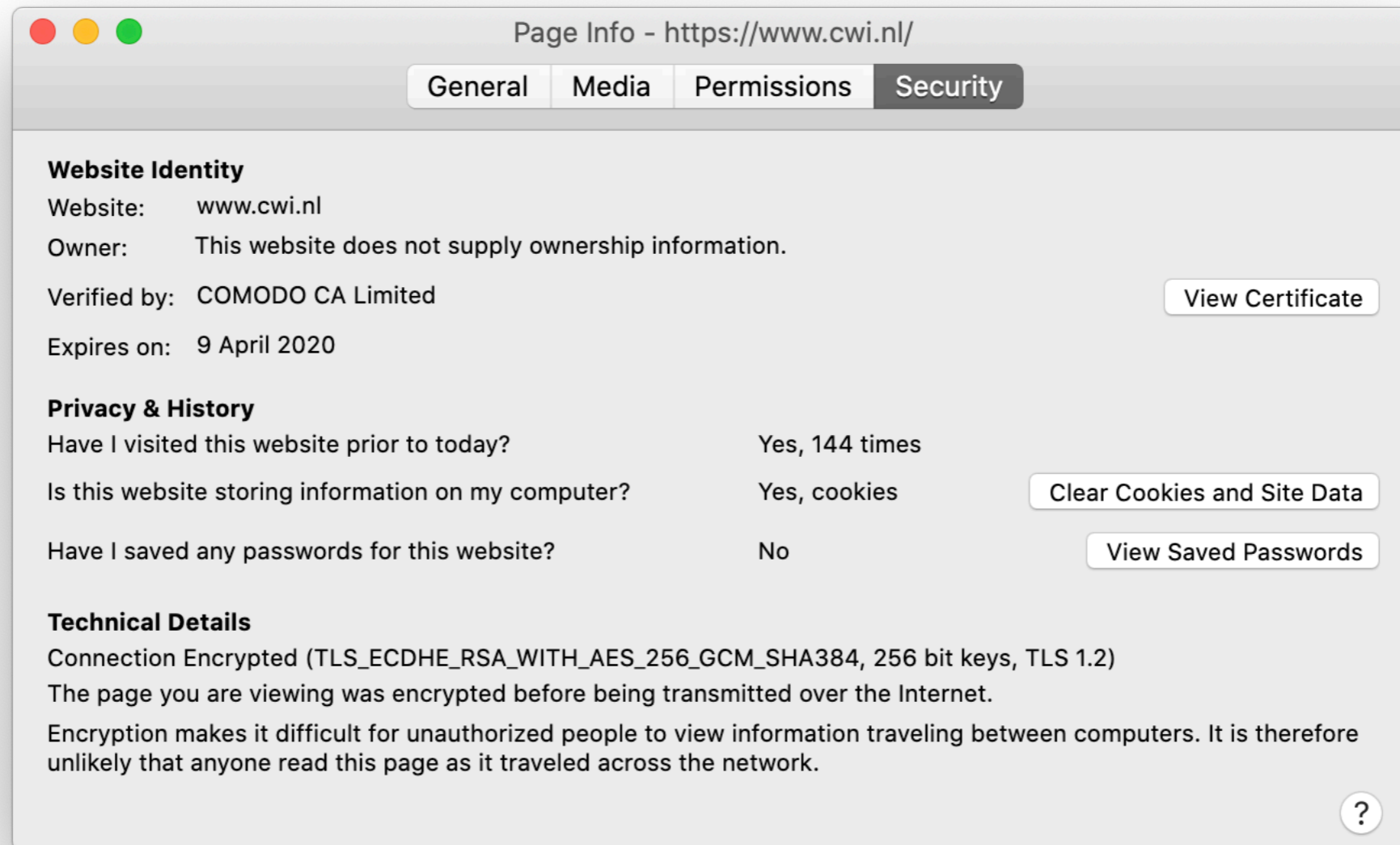
Elements of post-quantum crypto

Elements of post-quantum crypto

- ▶ Quantum Cryptanalysis

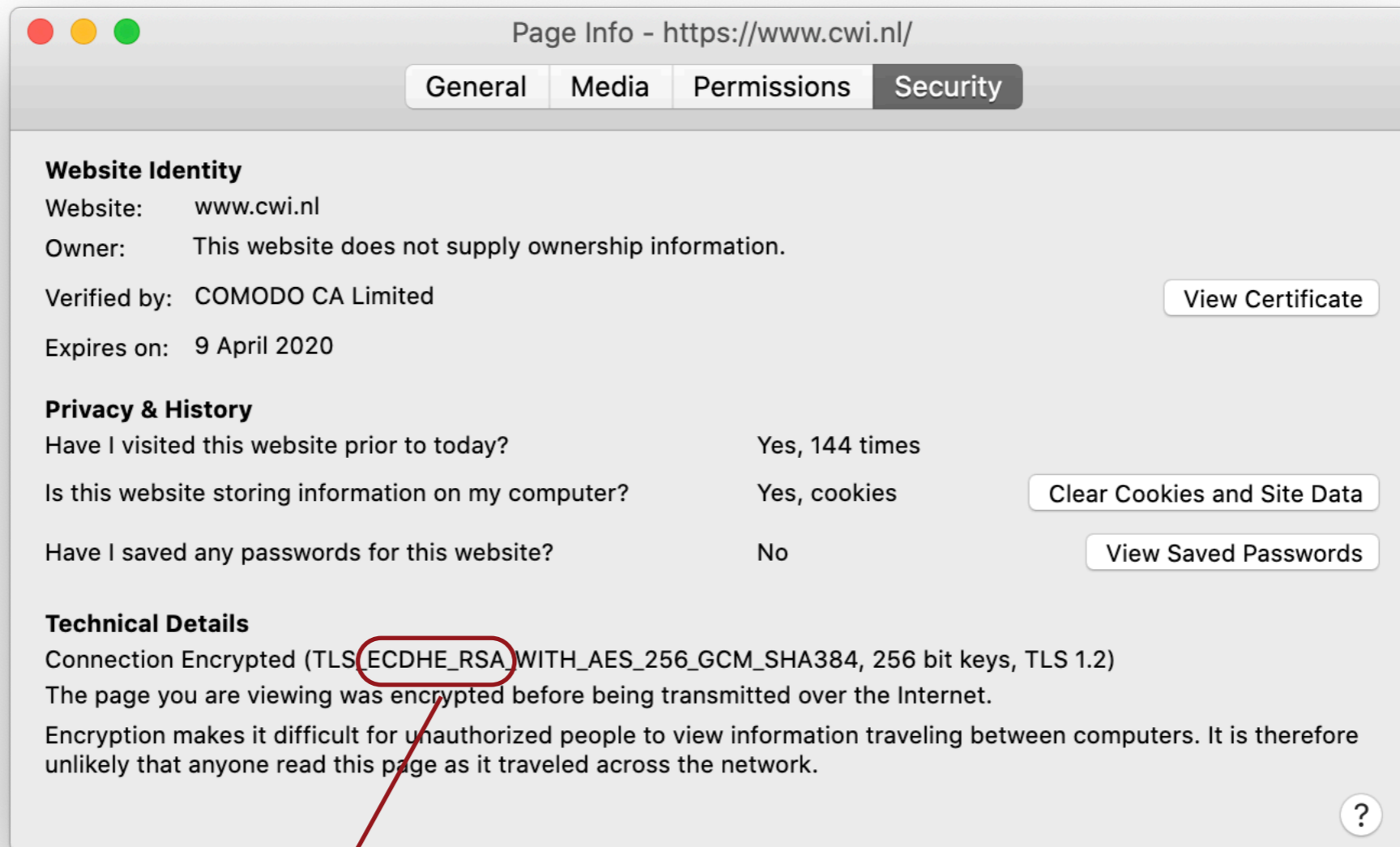
Elements of post-quantum crypto

► Quantum Cryptanalysis



Elements of post-quantum crypto

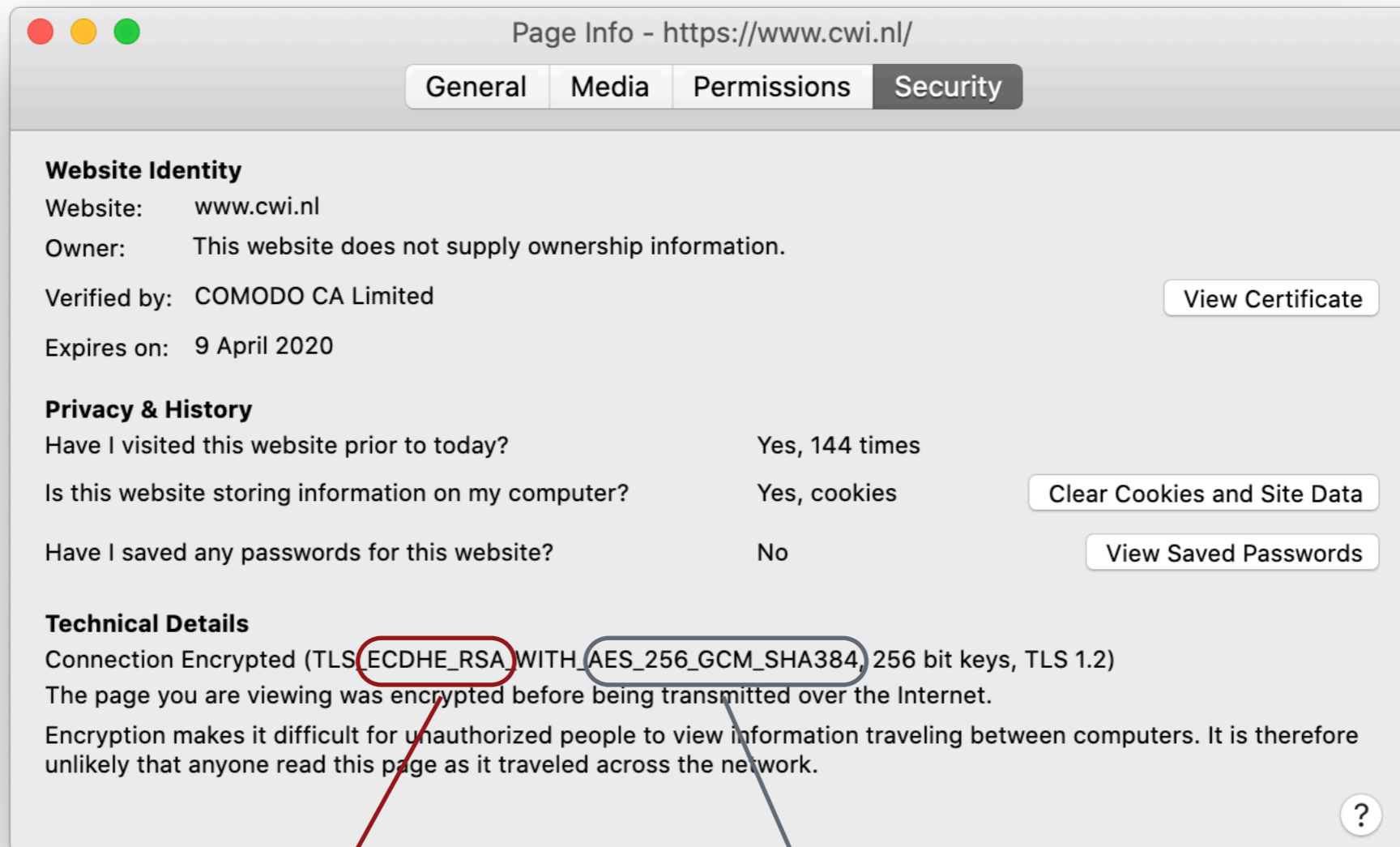
► Quantum Cryptanalysis



Shor's algorithm:
Complete break

Elements of post-quantum crypto

► Quantum Cryptanalysis



Shor's algorithm:
Complete break

Grover's algorithm:
Might necessitate increased key length

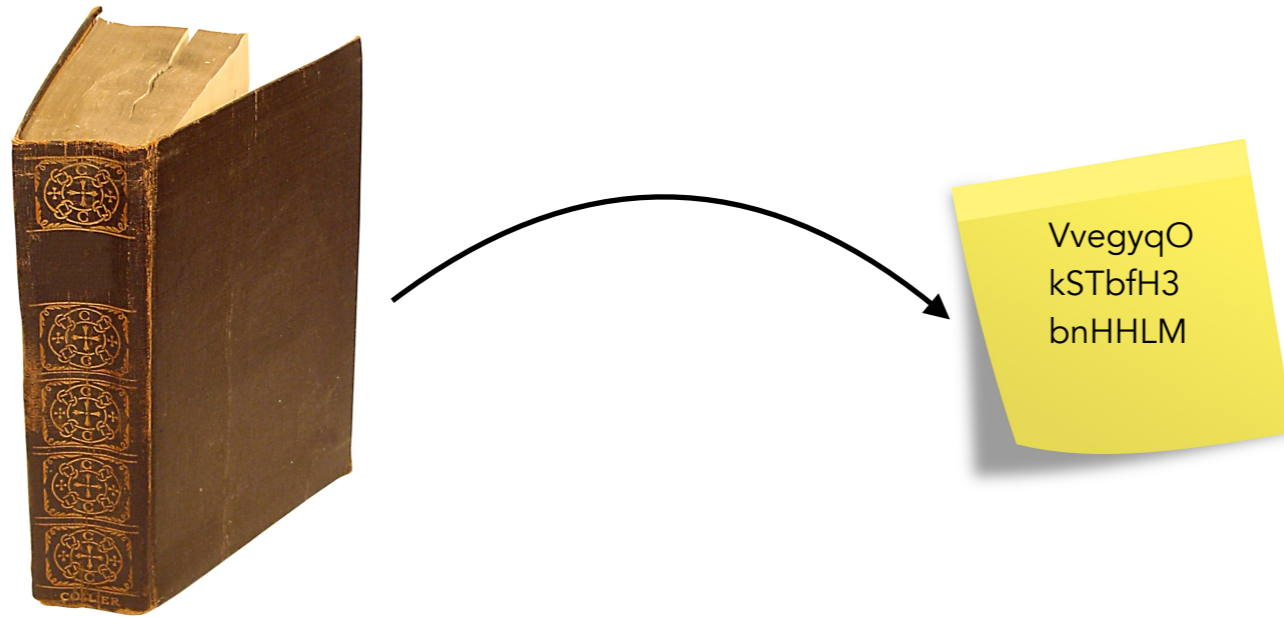
Elements of post-quantum crypto

- ▶ Quantum Cryptanalysis: Shor, Grover
- ▶ Quantum-secure computational assumptions
 - ▶ Lattice problems
 - ▶ Decoding random codes
 - ▶ Inverting multivariate polynomials
 - ▶ Secure hash functions
 - ▶ Supersingular isogeny Diffie-Hellman

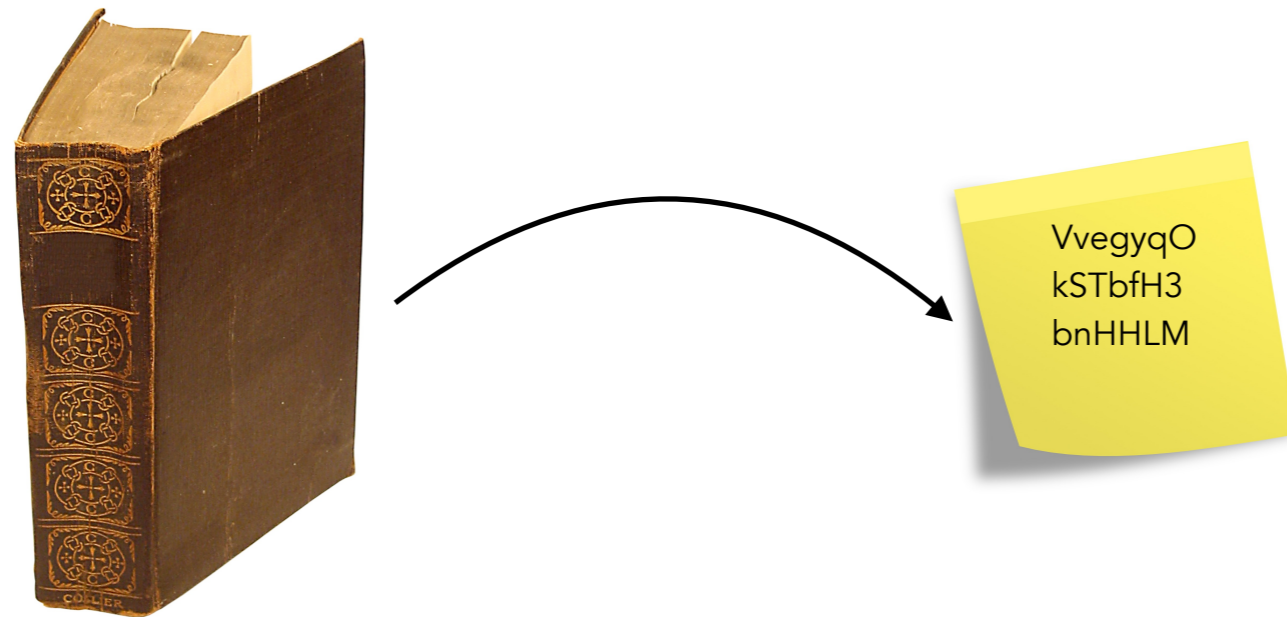
Elements of post-quantum crypto

- ▶ Quantum Cryptanalysis: Shor, Grover
- ▶ Quantum-secure computational assumptions
 - ▶ Lattice problems
 - ▶ Decoding random codes
 - ▶ Inverting multivariate polynomials
 - ▶ Secure hash functions
 - ▶ Supersingular isogeny Diffie-Hellman
- ▶ Models: Quantum Random Oracle Model (QROM)

Hash functions



Hash functions



Ubiquitous in cryptography. Example: digital signatures

The (Q)ROM

The (Q)ROM

Reality

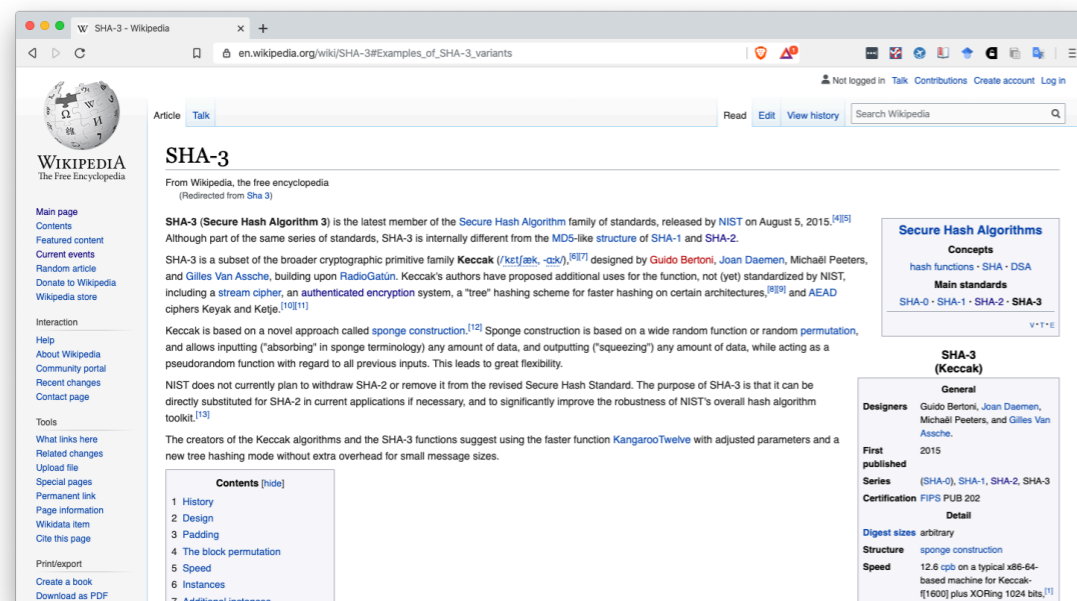
Model



The (Q)ROM

Reality

Model



The image shows a screenshot of the Wikipedia article for SHA-3. The browser address bar shows the URL: en.wikipedia.org/wiki/SHA-3#Examples_of_SHA-3_variants. The article title is "SHA-3" and it is noted as being redirected from "Sha 3". The main text of the article states: "SHA-3 (Secure Hash Algorithm 3) is the latest member of the Secure Hash Algorithm family of standards, released by NIST on August 5, 2015. Although part of the same series of standards, SHA-3 is internally different from the MD5-like structure of SHA-1 and SHA-2. SHA-3 is a subset of the broader cryptographic primitive family Keccak (/kɛtʃ/æk-, ɔk/) designed by Guido Bertoni, Joan Daemen, Michaël Peeters, and Gilles Van Assche, building upon RadioGatún. Keccak's authors have proposed additional uses for the function, not (yet) standardized by NIST, including a stream cipher, an authenticated encryption system, a "tree" hashing scheme for faster hashing on certain architectures, and AEAD ciphers Keyak and Ketje. Keccak is based on a novel approach called sponge construction. Sponge construction is based on a wide random function or random permutation, and allows inputting ("absorbing" in sponge terminology) any amount of data, and outputting ("squeezing") any amount of data, while acting as a pseudorandom function with regard to all previous inputs. This leads to great flexibility. NIST does not currently plan to withdraw SHA-2 or remove it from the revised Secure Hash Standard. The purpose of SHA-3 is that it can be directly substituted for SHA-2 in current applications if necessary, and to significantly improve the robustness of NIST's overall hash algorithm toolkit. The creators of the Keccak algorithms and the SHA-3 functions suggest using the faster function KangarooTwelve with adjusted parameters and a new tree hashing mode without extra overhead for small message sizes.

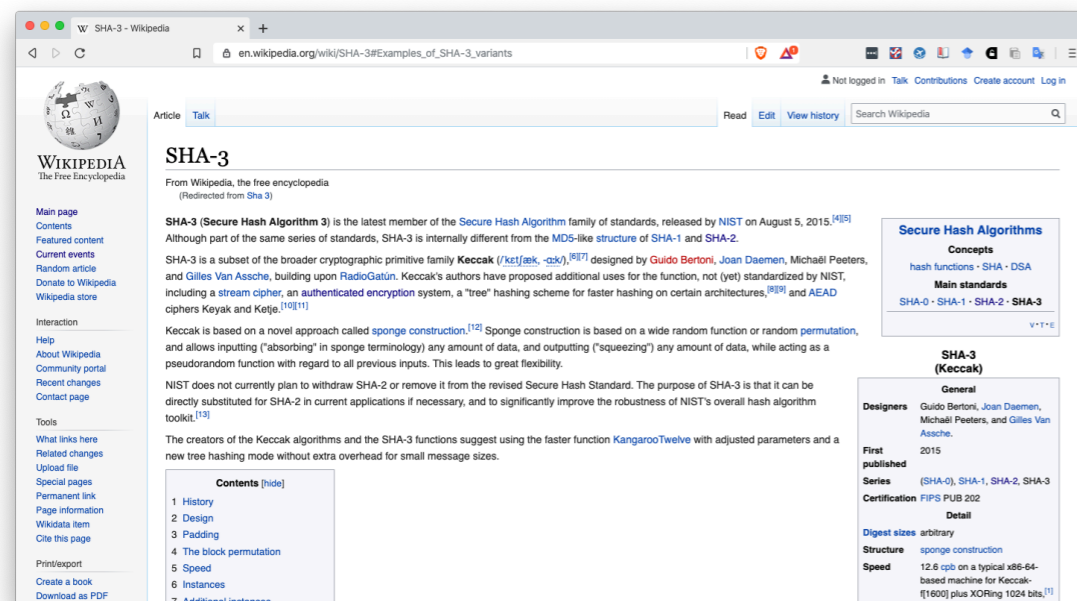
On the right side of the article, there are two summary boxes. The first is titled "Secure Hash Algorithms" and lists "Concepts" (hash functions - SHA - DSA) and "Main standards" (SHA-0 - SHA-1 - SHA-2 - SHA-3). The second box is titled "SHA-3 (Keccak)" and lists "General" information: Designers (Guido Bertoni, Joan Daemen, Michaël Peeters, and Gilles Van Assche), First published (2015), Series ((SHA-0), SHA-1, SHA-2, SHA-3), Certification (FIPS PUB 202), Digest sizes (arbitrary), Structure (sponge construction), and Speed (12.6 cpb on a typical x86-64-based machine for Keccak-[1600] plus XORing 1024 bits).

On the left side of the article, there is a "Contents" table of contents with the following items: 1 History, 2 Design, 3 Padding, 4 The block permutation, 5 Speed, 6 Instances, and 7 Additional instances.

The (Q)ROM

Reality

Model

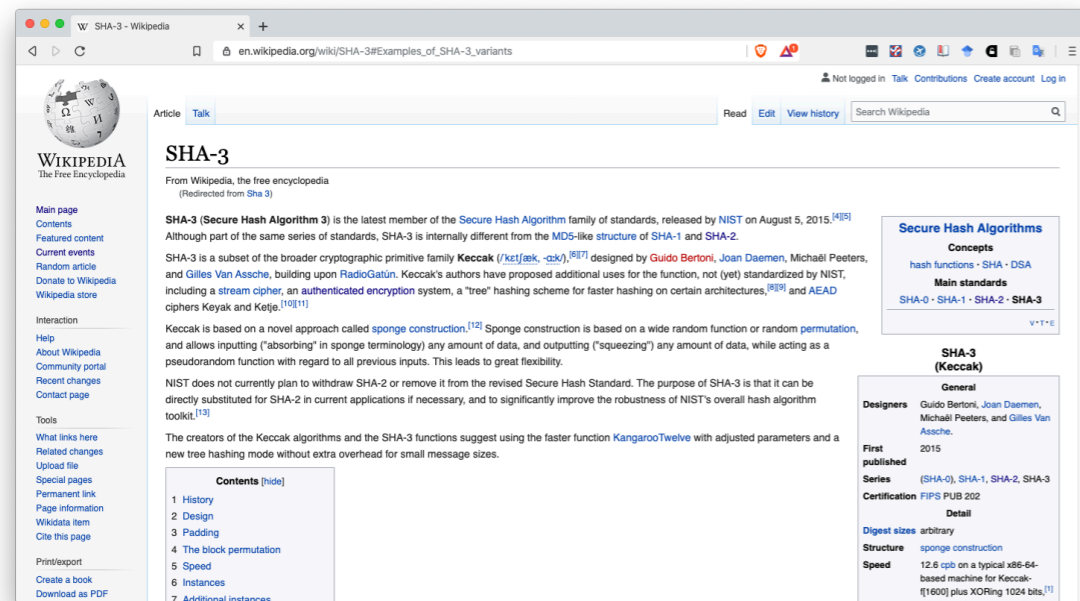


$$H : \{0,1\}^* \rightarrow \{0,1\}^n$$

Uniformly random

The (Q)ROM

Reality



Model

$$H : \{0,1\}^* \rightarrow \{0,1\}^n$$

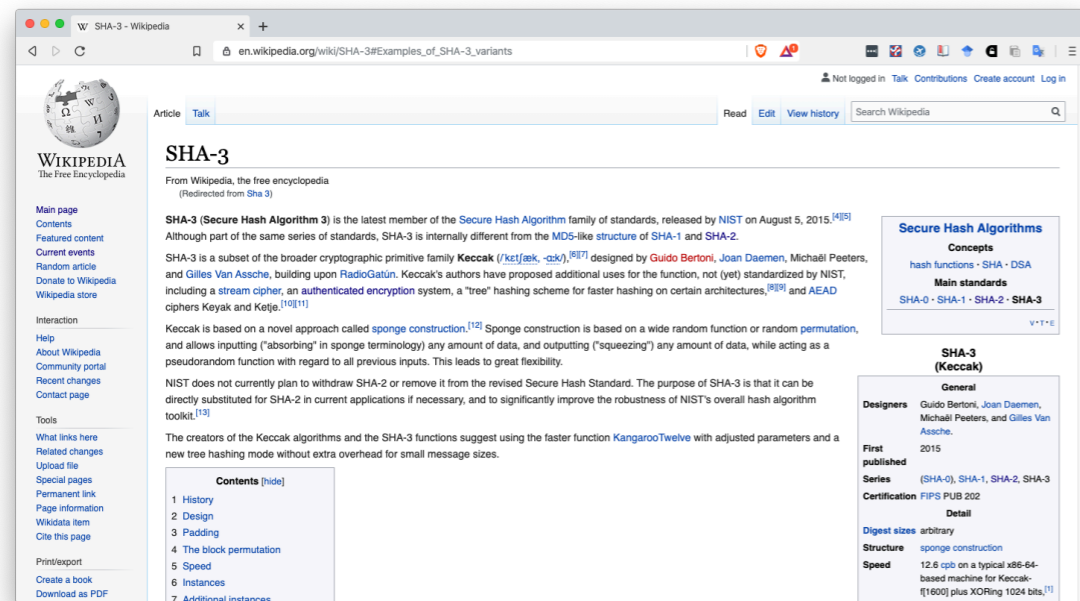
Uniformly random

All agents have (quantum)
oracle access to H

$$(x, y) \mapsto (x, y \oplus H(x))$$

The (Q)ROM

Reality



Model

$$H : \{0,1\}^* \rightarrow \{0,1\}^n$$

Uniformly random

All agents have (quantum)
oracle access to H

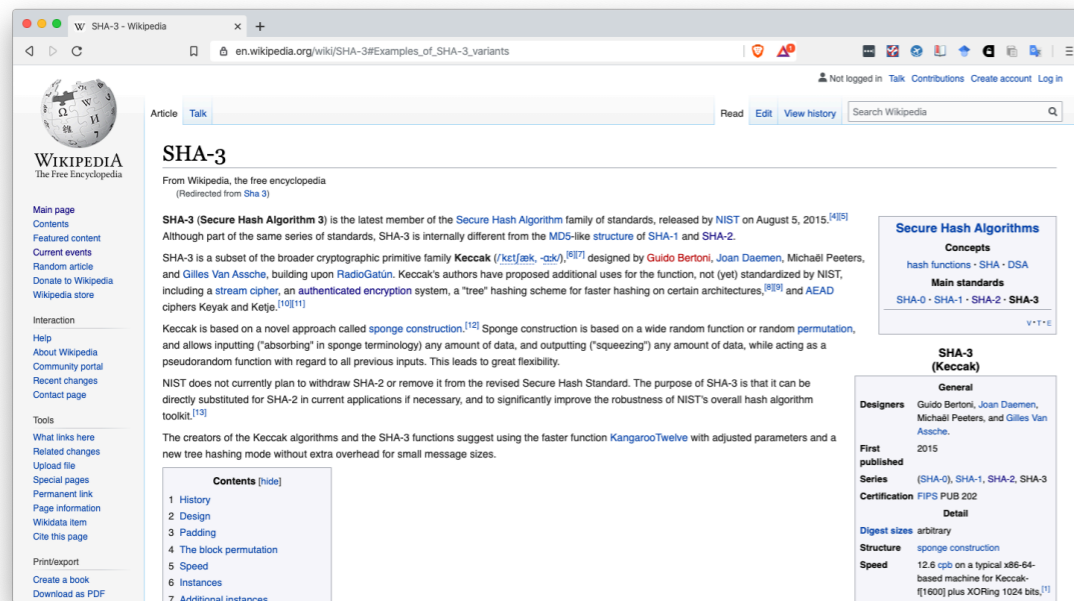
$$(x, y) \mapsto (x, y \oplus H(x))$$

► Outrageously optimistic

The (Q)ROM

Reality

Model



$$H : \{0,1\}^* \rightarrow \{0,1\}^n$$

Uniformly random

All agents have (quantum)
oracle access to H

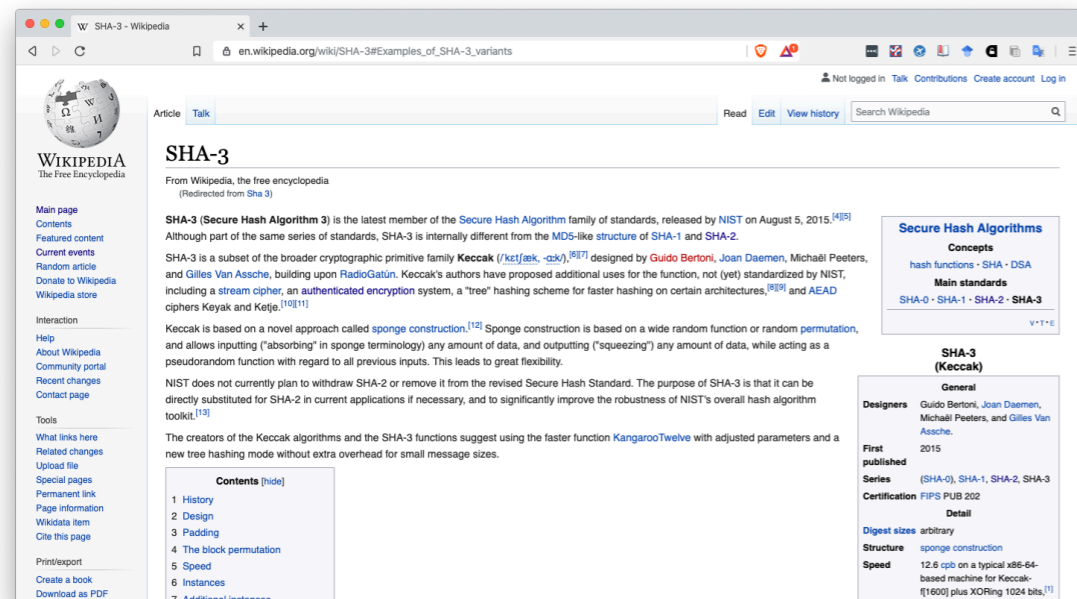
$$(x, y) \mapsto (x, y \oplus H(x))$$

- ▶ Outrageously optimistic
- ▶ Seems to work in practice

The (Q)ROM

Reality

Model



$$H : \{0,1\}^* \rightarrow \{0,1\}^n$$

Uniformly random

All agents have (quantum)
oracle access to H

$$(x, y) \mapsto (x, y \oplus H(x))$$

- ▶ Outrageously optimistic
- ▶ Seems to work in practice
- ▶ Enables very efficient crypto

QROM challenges

QROM challenges

ROM techniques:

1. Query transcripts

QROM challenges

ROM techniques:

1. Query transcripts
2. Rewinding

QROM challenges

ROM techniques:

1. Query transcripts
2. Rewinding
3. Reprogramming

QROM challenges

ROM techniques:

1. Query transcripts
2. Rewinding
3. Reprogramming

Quantum theory makes things difficult! No-cloning,
Measurement disturbance

QROM challenges

ROM techniques:

1. Query transcripts
2. Rewinding
3. Reprogramming

Quantum theory makes things difficult! No-cloning,
Measurement disturbance

QROM:

QROM challenges

ROM techniques:

1. Query transcripts
2. Rewinding
3. Reprogramming

Quantum theory makes things difficult! No-cloning,
Measurement disturbance

QROM:

- ~~1. Query transcripts~~

QROM challenges

ROM techniques:

1. Query transcripts
2. Rewinding
3. Reprogramming

Quantum theory makes things difficult! No-cloning,
Measurement disturbance

QROM:

1. ~~Query transcripts~~
2. ~~Rewinding~~ \implies 3 specialized rewinding techniques that don't cover all applications

QROM challenges

ROM techniques:

1. Query transcripts
2. Rewinding
3. Reprogramming

Quantum theory makes things difficult! No-cloning,
Measurement disturbance

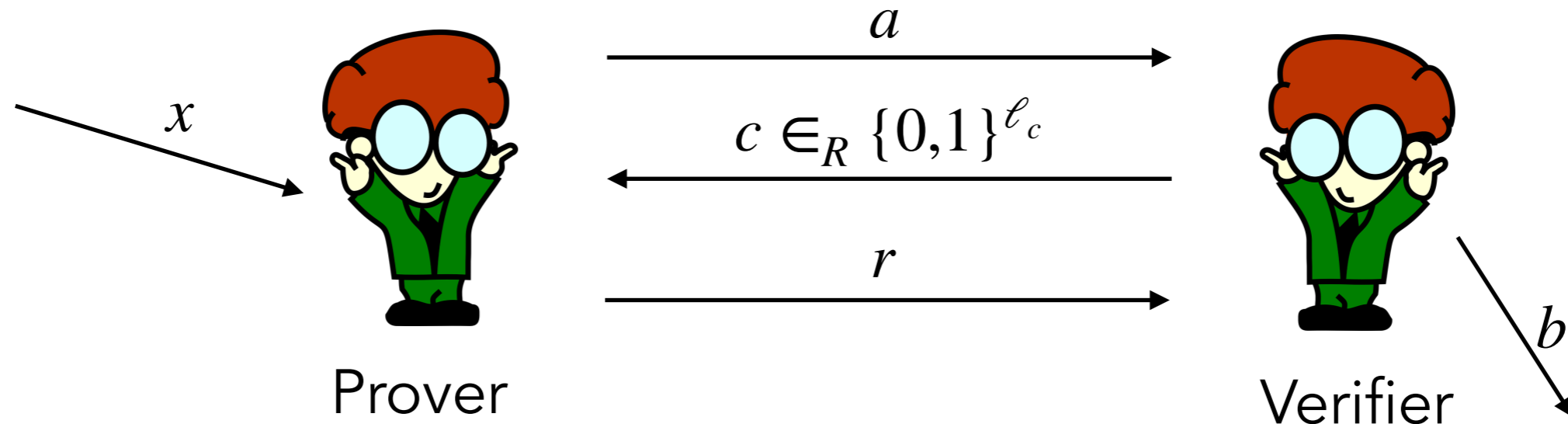
QROM:

1. ~~Query transcripts~~
2. ~~Rewinding~~ \implies 3 specialized rewinding techniques that don't cover all applications
3. Reprogramming: Sure, if you know how without 1. and 2.

The Fiat Shamir transformation

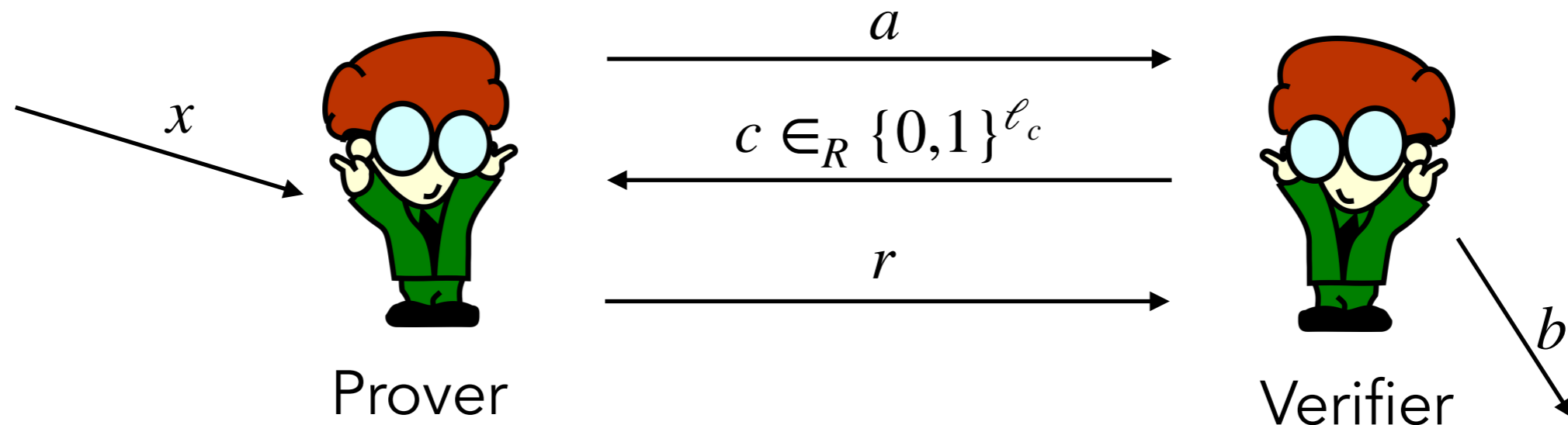
The Fiat Shamir transformation

Σ -protocol: Interactive proof system



The Fiat Shamir transformation

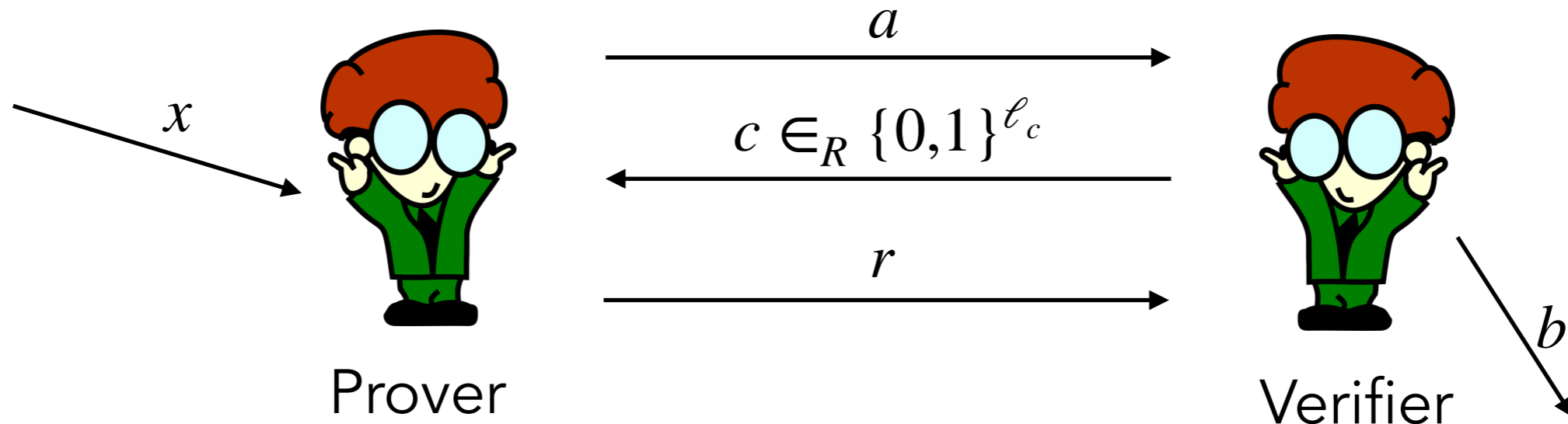
Σ -protocol: Interactive proof system



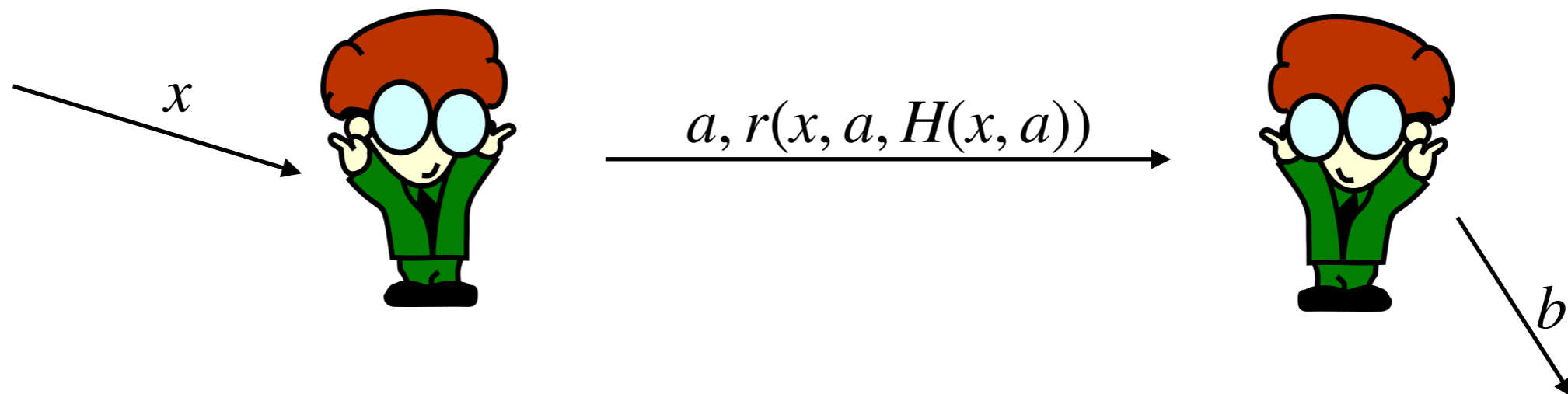
Fiat Shamir (FS) transformation: $c = H(x, a)$

The Fiat Shamir transformation

Σ -protocol: Interactive proof system

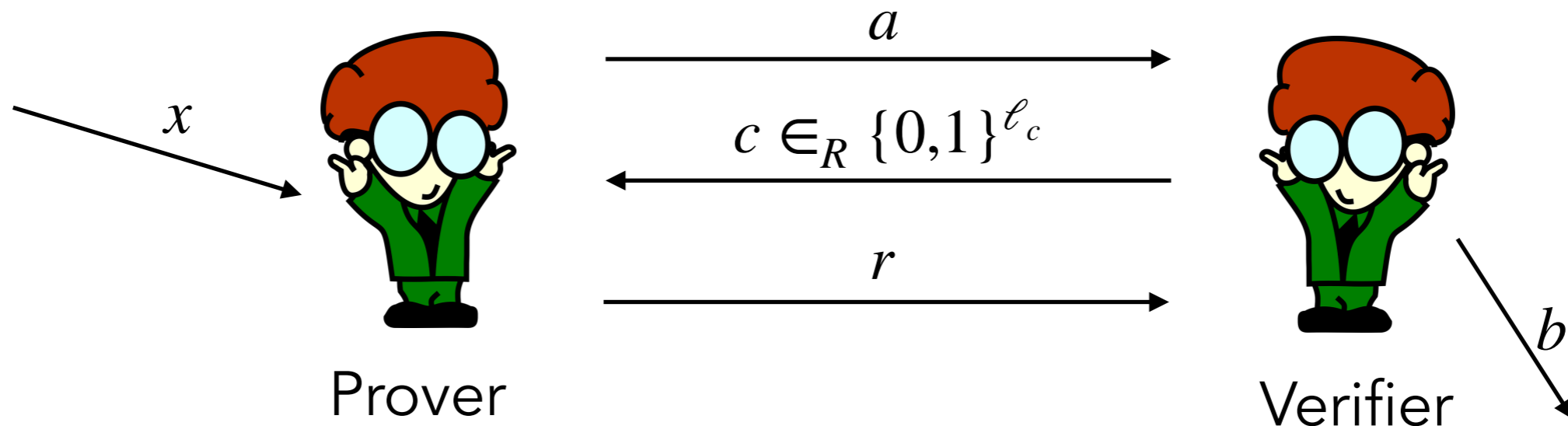


Fiat Shamir (FS) transformation: $c = H(x, a)$

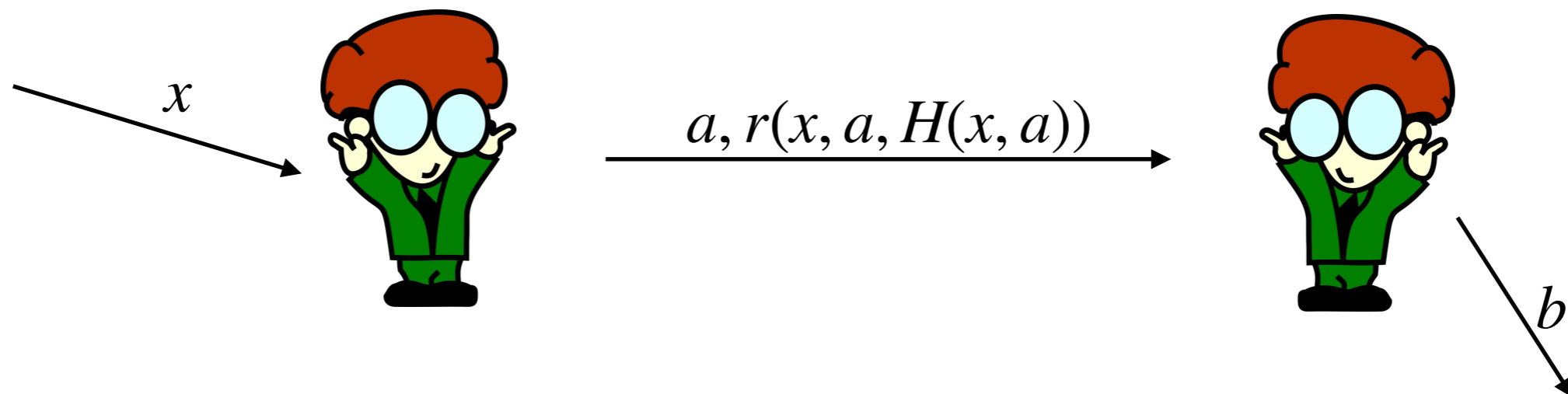


The Fiat Shamir transformation

Σ -protocol: Interactive proof system



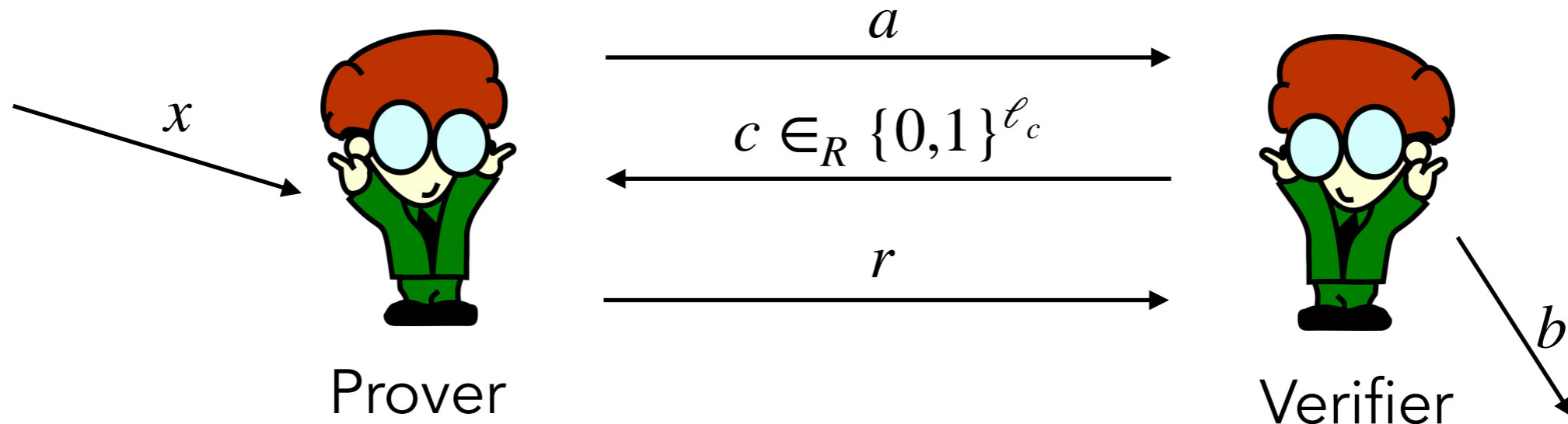
Fiat Shamir (FS) transformation: $c = H(x, a)$



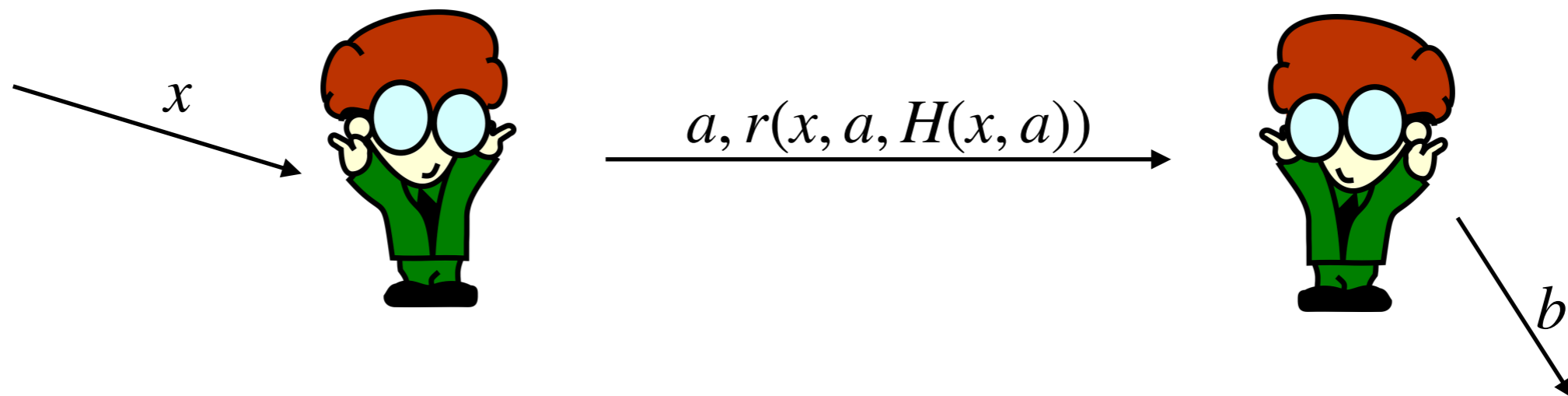
non-interactive!!!

The Fiat Shamir transformation

Σ -protocol: Interactive proof system



Fiat Shamir (FS) transformation: $c = H(x, a)$



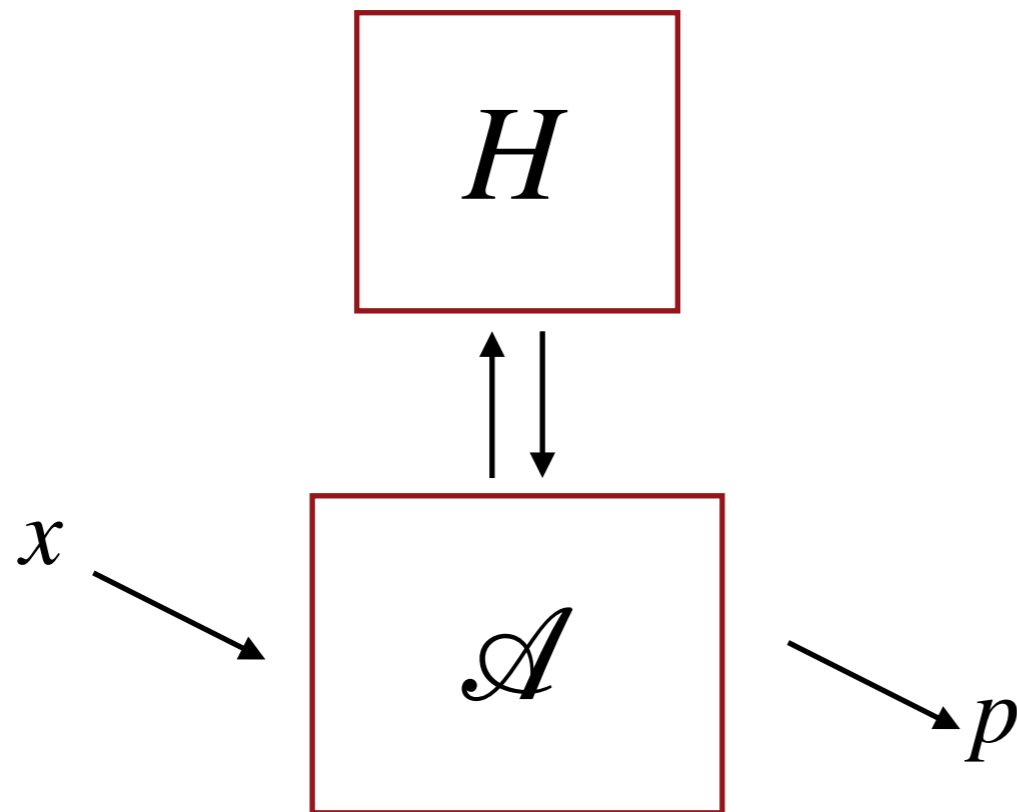
non-interactive!!! \implies used for efficient digital signatures

ROM security

The FS transformation is secure in the ROM (Pointcheval, Stern 96):

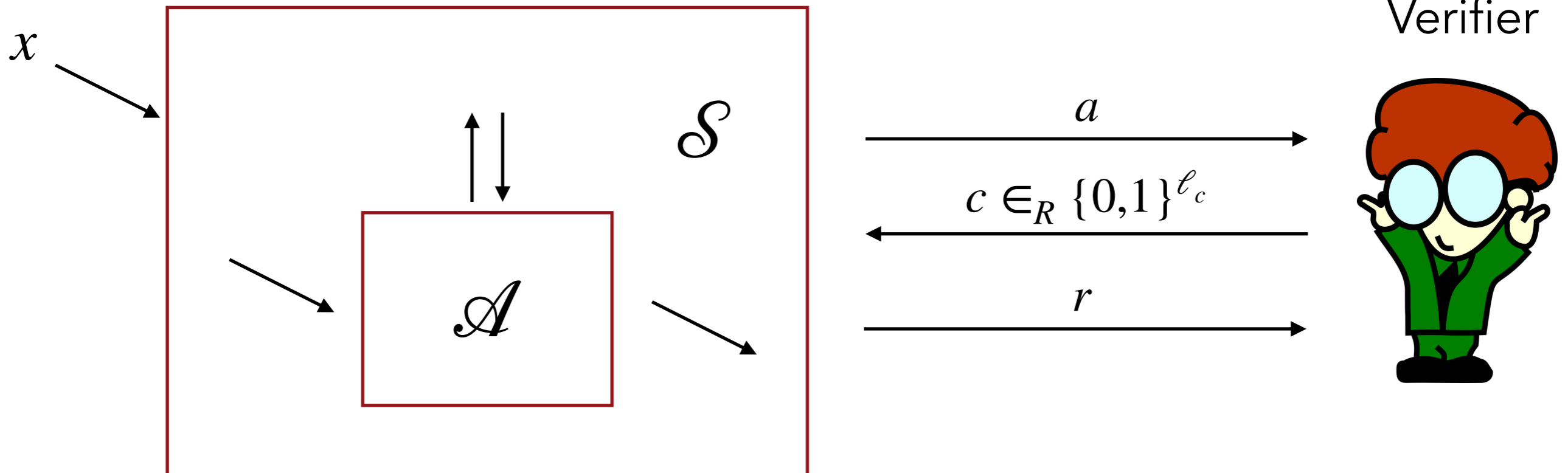
ROM security

The FS transformation is secure in the ROM (Pointcheval, Stern 96):



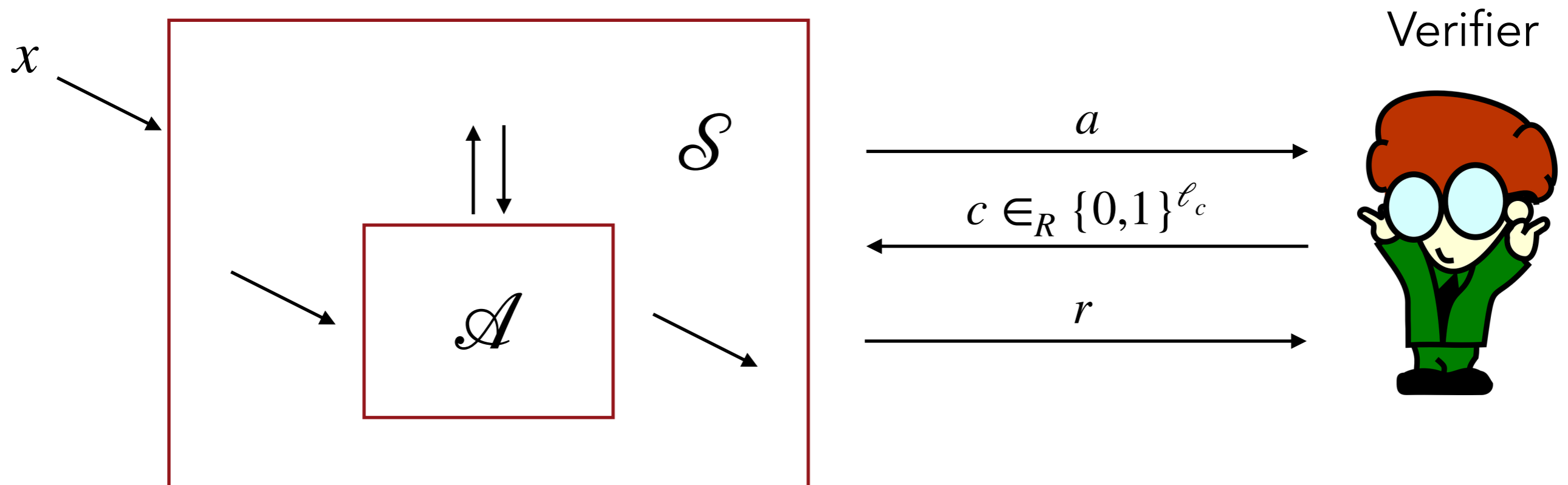
ROM security

The FS transformation is secure in the ROM (Pointcheval, Stern 96):



ROM security

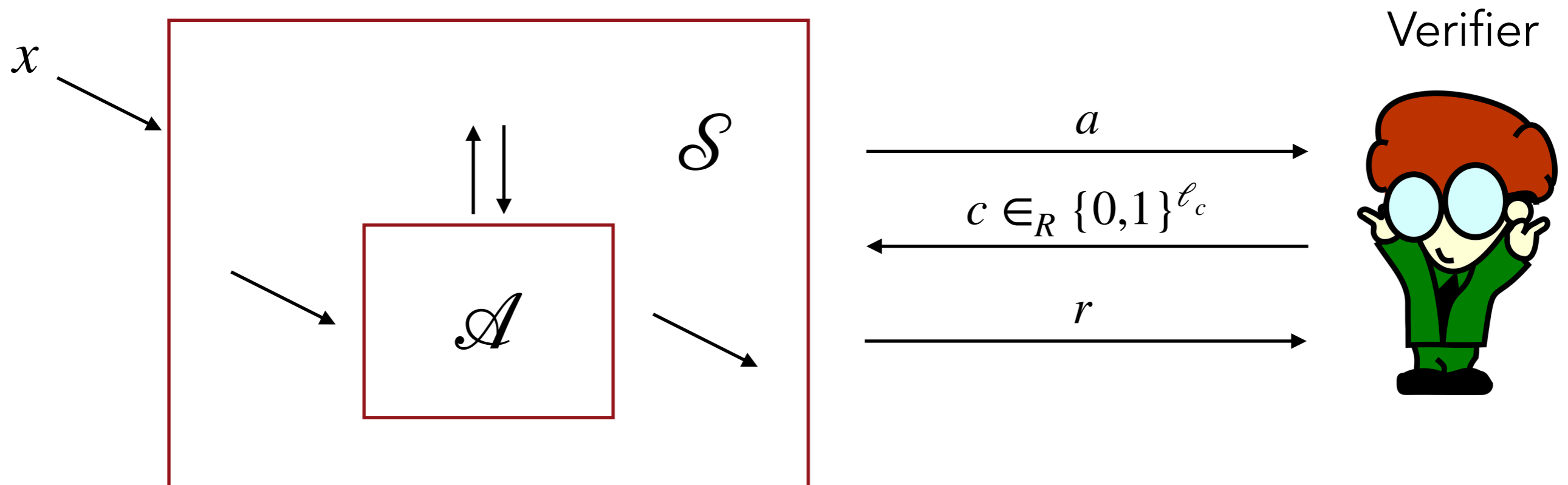
The FS transformation is secure in the ROM (Pointcheval, Stern 96):



Success probability: $\varepsilon(\mathcal{S}[\mathcal{A}]) \geq \frac{\varepsilon(\mathcal{A})}{O(q)}$

ROM security

The FS transformation is secure in the ROM (Pointcheval, Stern 96):

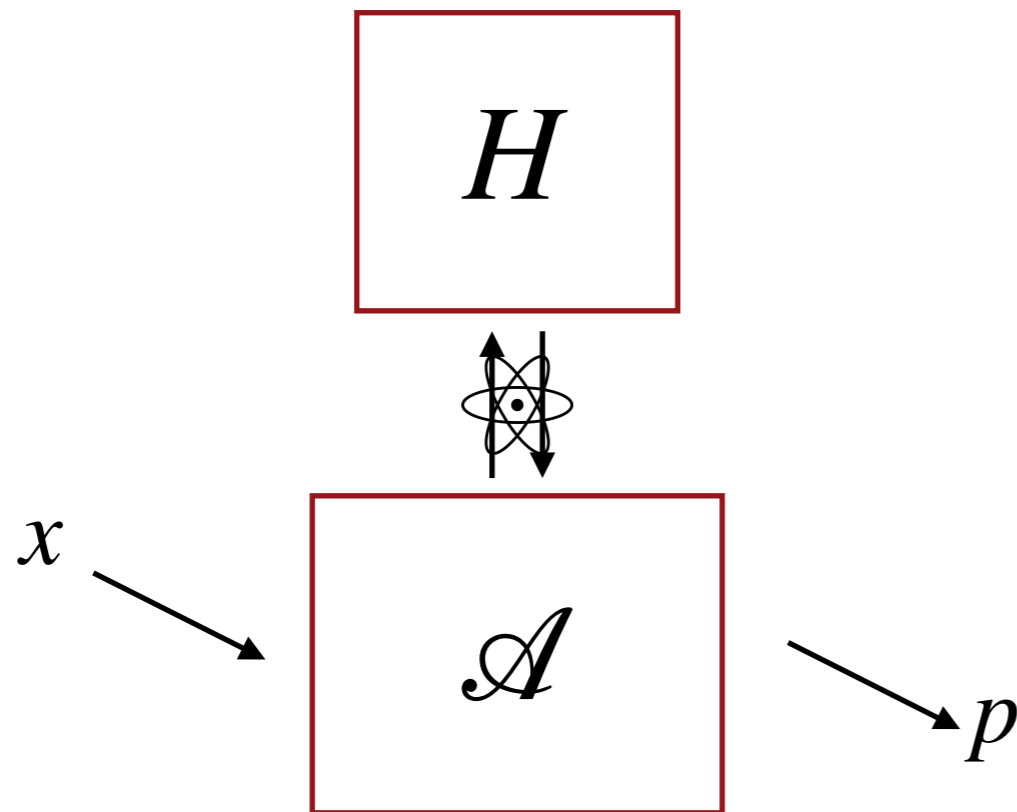


Success probability: $\varepsilon(\mathcal{S}[\mathcal{A}]) \geq \frac{\varepsilon(\mathcal{A})}{O(q)}$

of queries \mathcal{A} makes to H

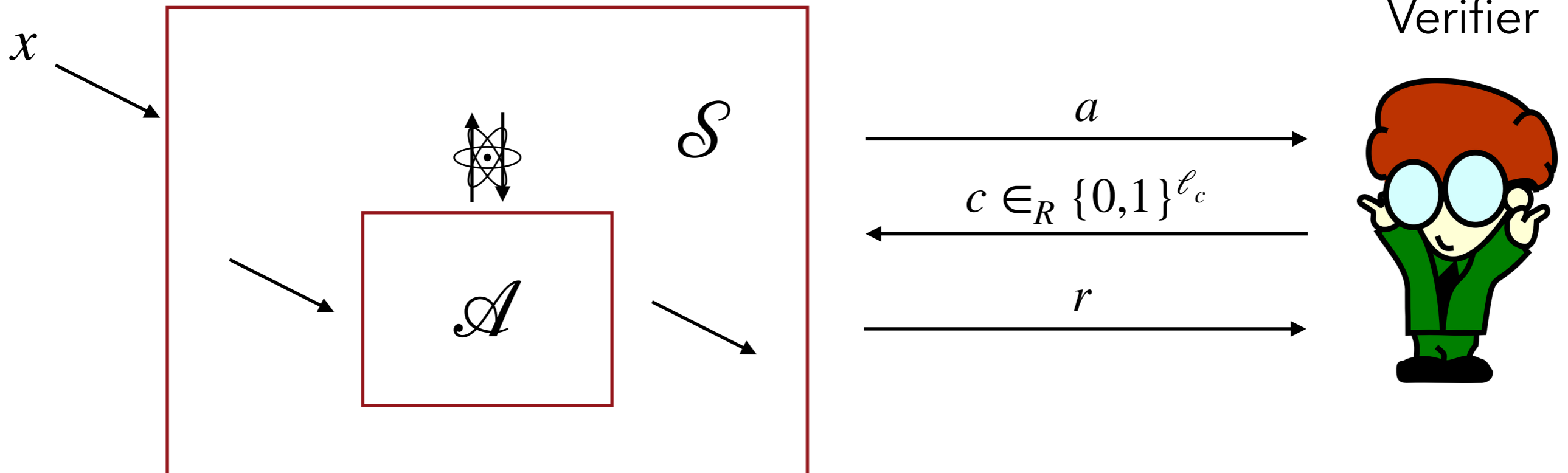
QRROM security

The FS transformation is secure in the QRROM (Don, Fehr, M, Schaffner '19):



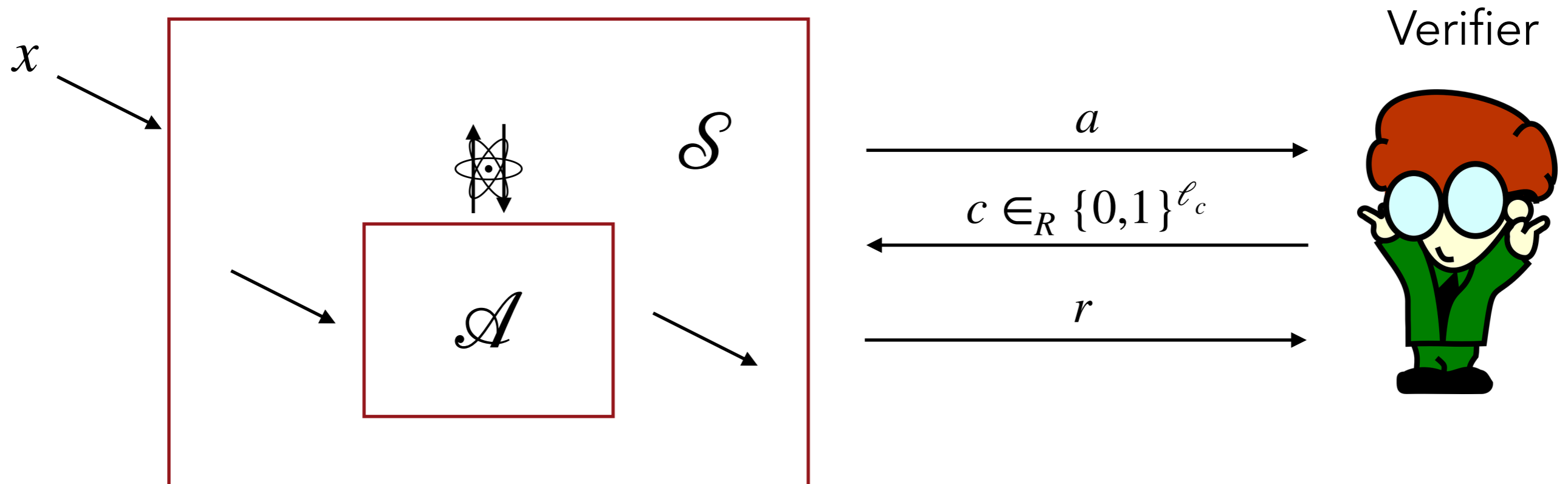
QRROM security

The FS transformation is secure in the QRROM (Don, Fehr, M, Schaffner '19):



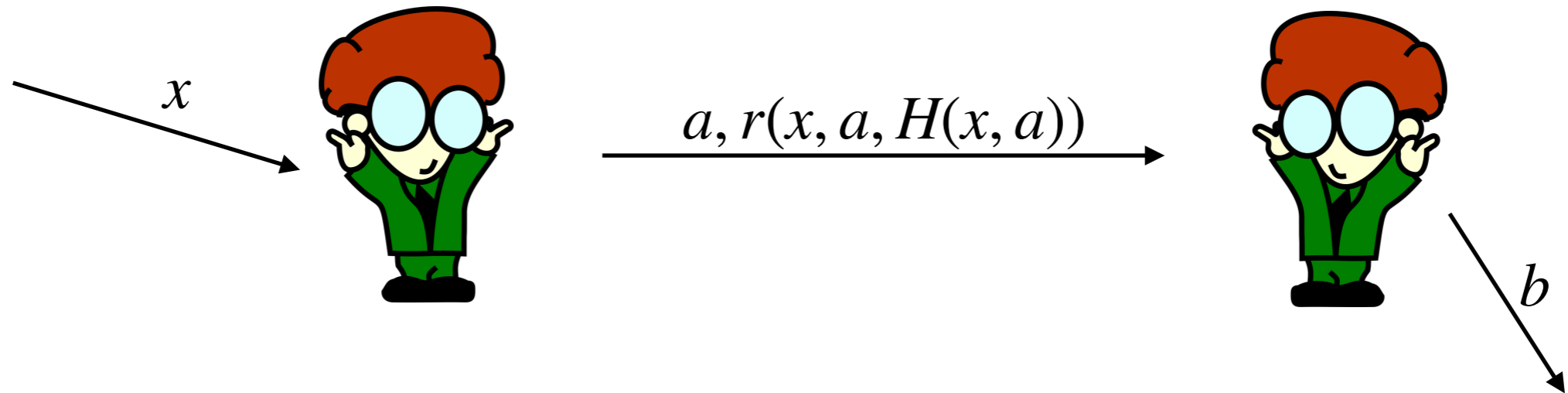
QROM security

The FS transformation is secure in the QROM (Don, Fehr, M, Schaffner '19):

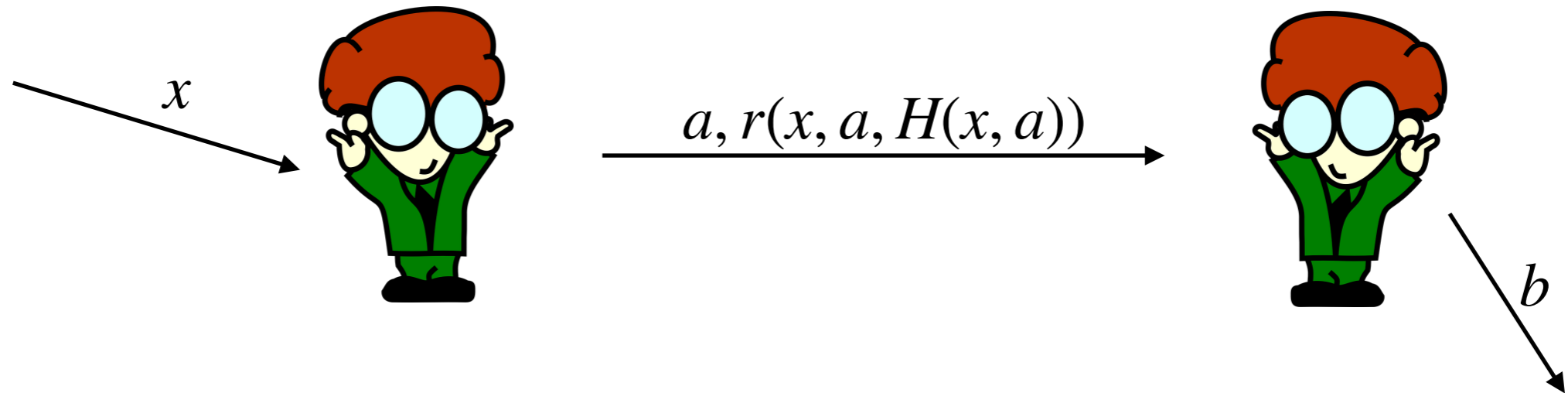


$$\text{Success probability: } \varepsilon(\mathcal{S}[\mathcal{A}]) \geq \frac{\varepsilon(\mathcal{A})}{O(q^2)}$$

Technique

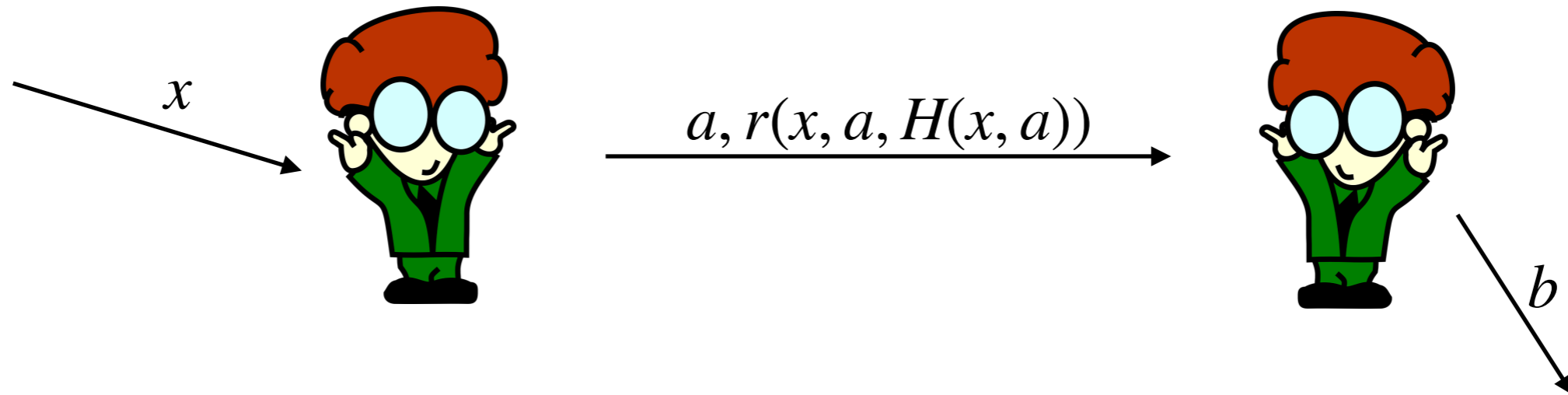


Technique



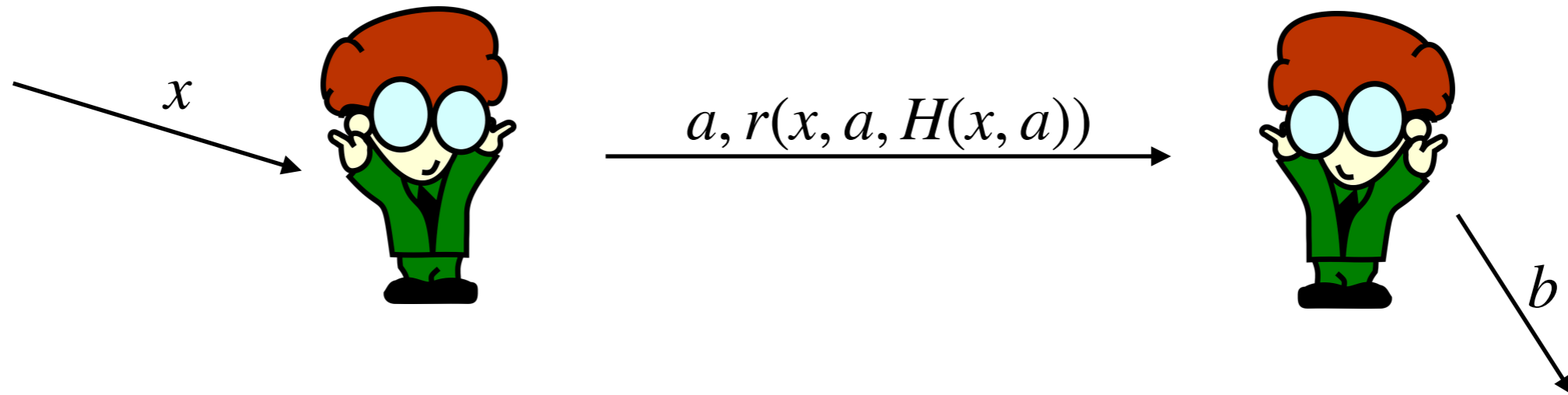
Suppose r was injective $\implies \mathcal{A}$ essentially needs to classically query H on (x, a) .

Technique



Suppose r was injective $\implies \mathcal{A}$ essentially needs to classically query H on (x, a) .

Technique



Suppose r was injective $\implies \mathcal{A}$ **essentially** needs to classically query H on (x, a) .

Measure-and-Reprogram: Pick a random query, measure it and reprogram with c from the Σ -protocol.

Long term goal

Popular belief about QROM: Grover speed-up is as good as it gets.

Long term goal

Popular belief about QROM: Grover speed-up is as good as it gets.

⇒ Dream: QROM-to-ROM reduction should solve all our problems!

Long term goal

Popular belief about QROM: Grover speed-up is as good as it gets.

⇒ Dream: QROM-to-ROM reduction should solve all our problems!

Reality:

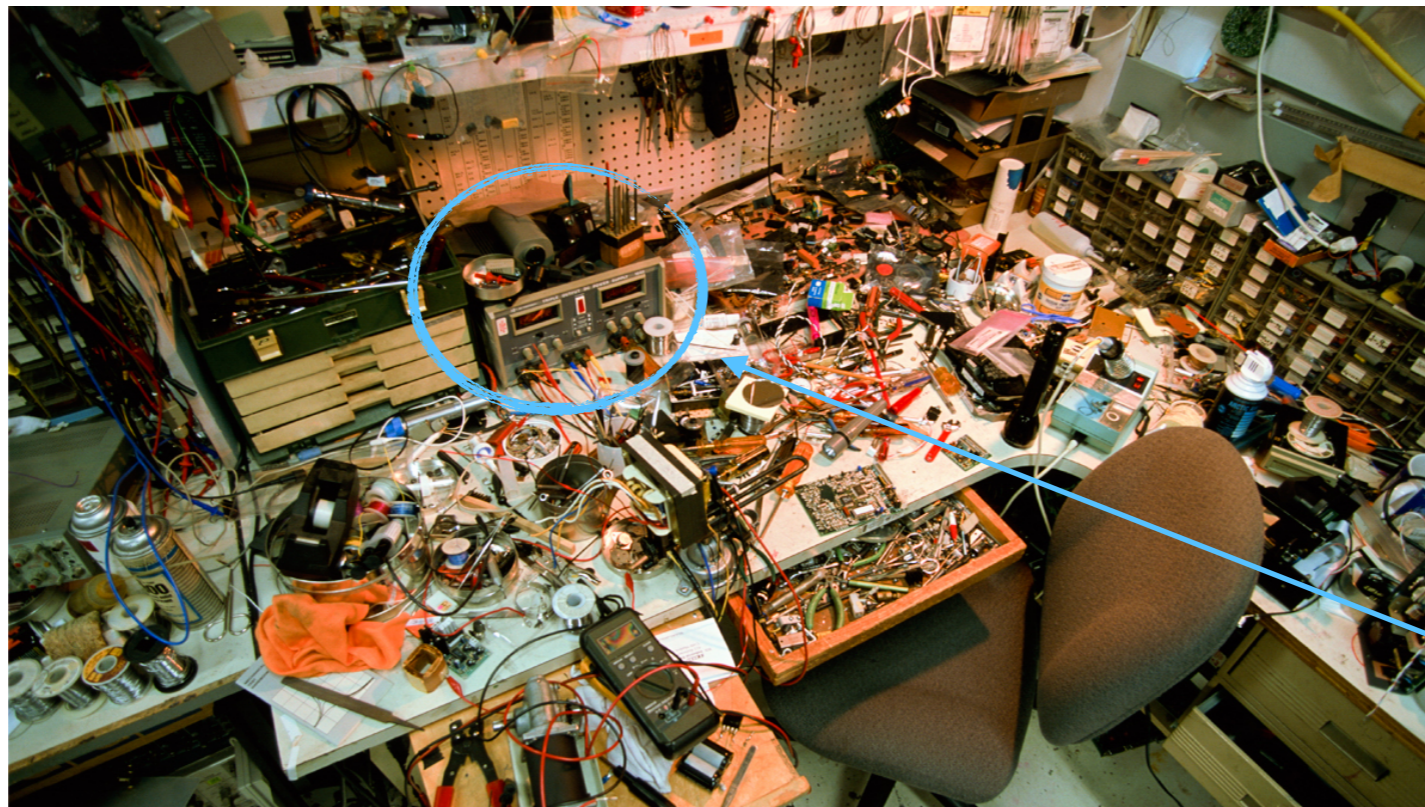


Long term goal

Popular belief about QROM: Grover speed-up is as good as it gets.

⇒ Dream: QROM-to-ROM reduction should solve all our problems!

Reality:



QROM security
of FS

Summary

- ▶ The (Q)ROM is extremely useful for efficient cryptography
- ▶ Quantum theory complicates things, much less coherent picture of QROM security
- ▶ Important cases solved, e.g. Fiat Shamir
- ▶ General reduction from QROM to ROM would be nice to have!