Hash functions in post-quantum cryptography

Christian Majenz
CWI

Cryptography is everywhere

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DigiD

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Quantum computers

Quantum computers

- Accelerating effort to build a quantum computer

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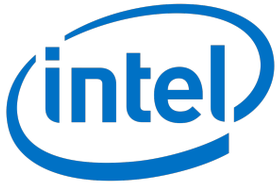
- Major investments:



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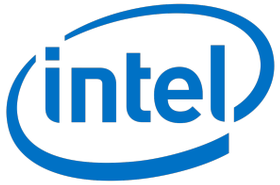

We need to prepare cryptography for the arrival of quantum computers!

Quantum computers

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Google IBM

intel Microsoft

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- Security against quantum attackers

- Quantum cryptography

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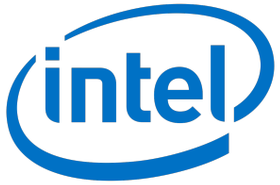

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- **This talk: Security against quantum attackers (post-quantum cryptography)**

Elements of post-quantum crypto

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- Quantum Cryptanalysis

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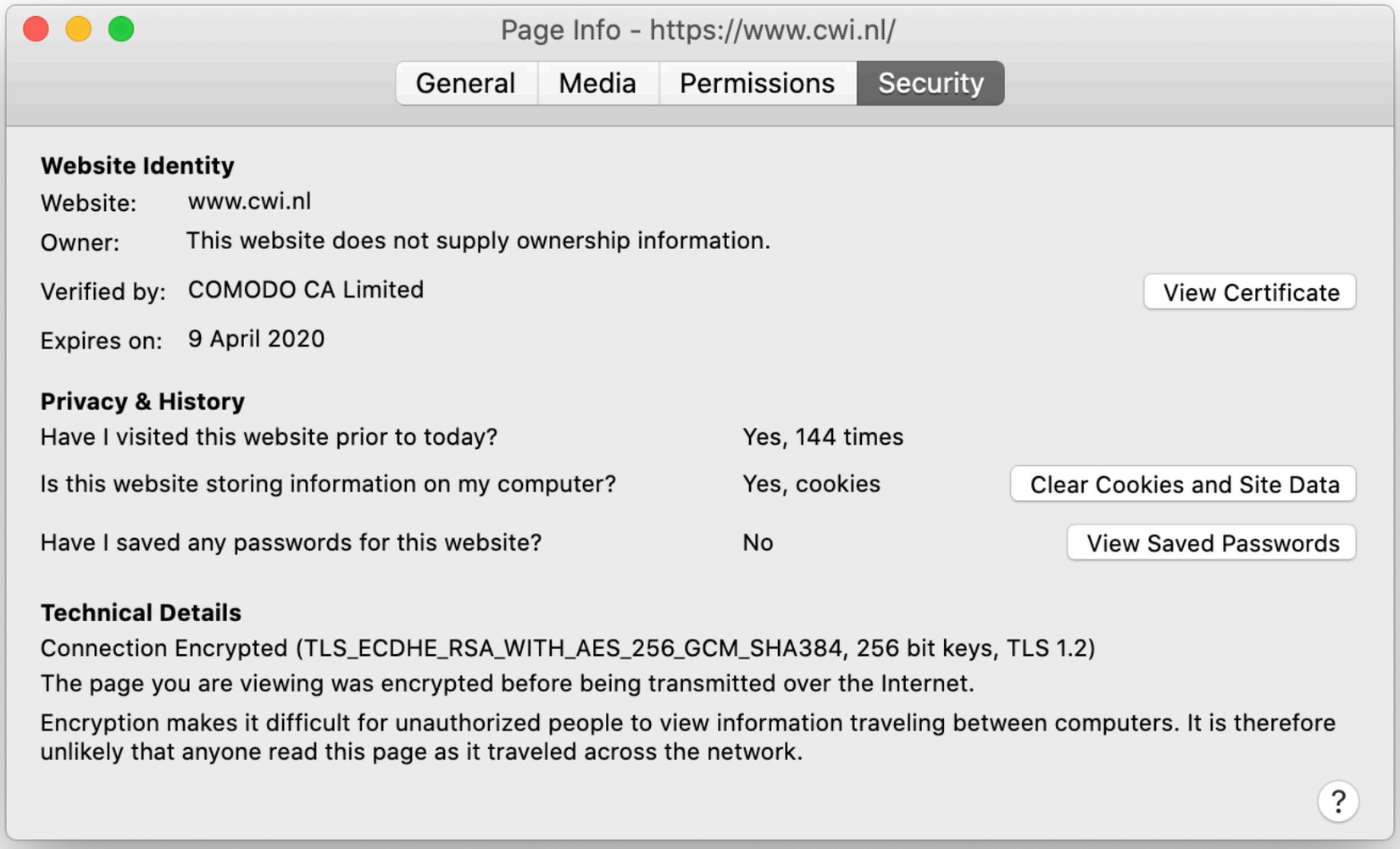

Elements of post-quantum crypto

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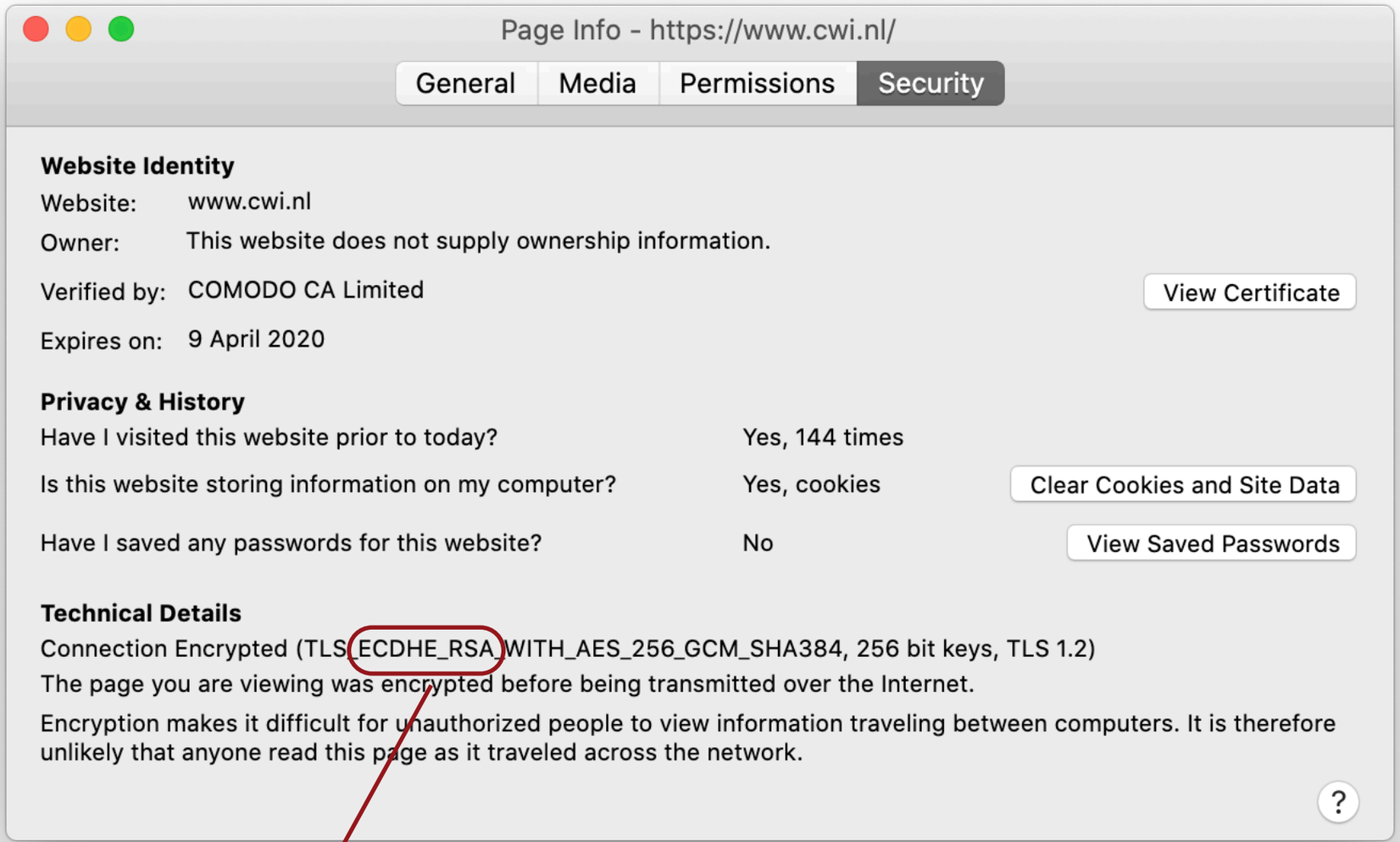

Shor's algorithm:
Complete break

Elements of post-quantum crypto

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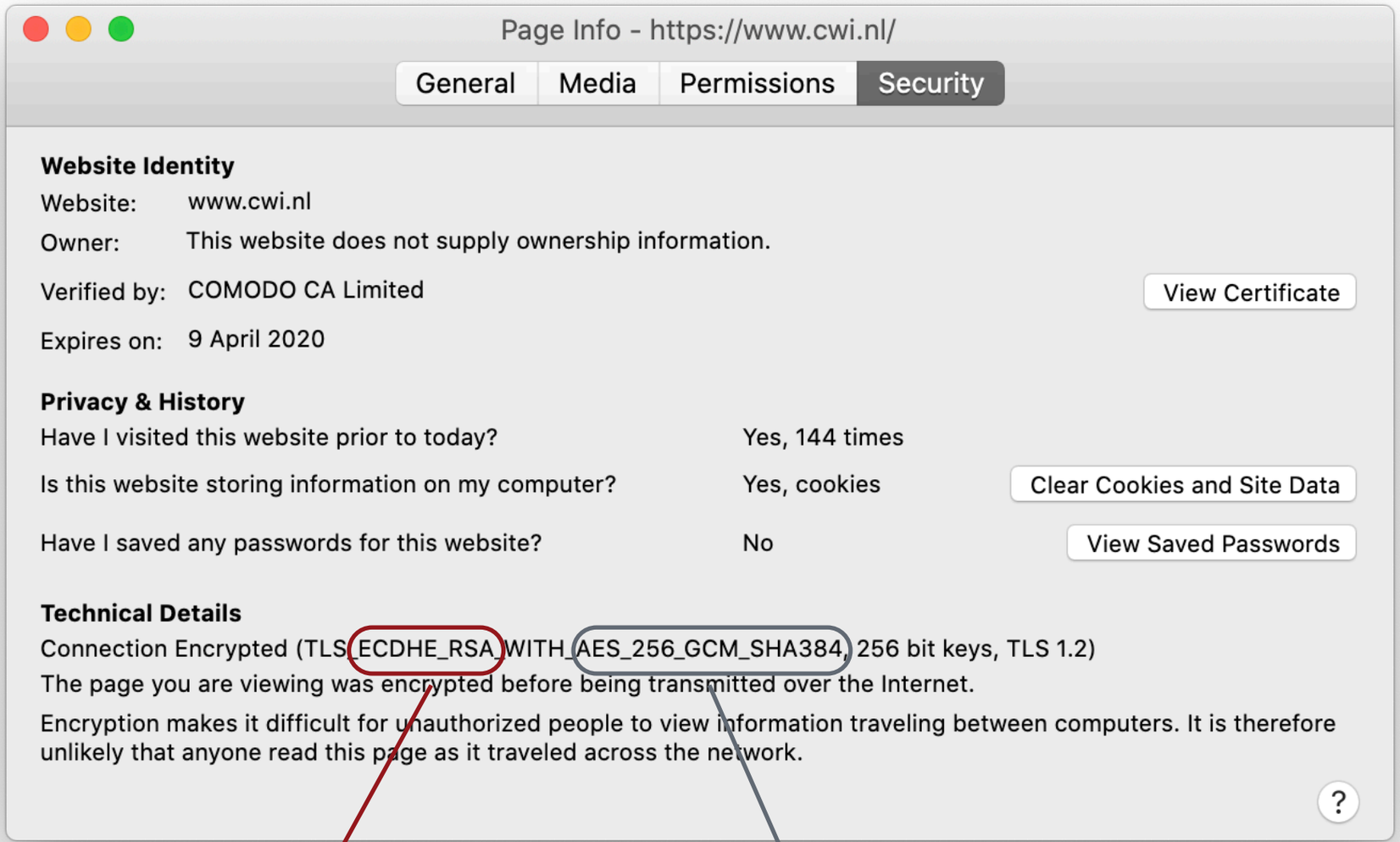

Shor's algorithm:
Complete break

Grover's algorithm:
Might necessitate increased key lengh

Elements of post-quantum crypto

- Quantum Cryptanalysis: Shor, Grover
- Quantum-secure computational assumptions
  - Lattice problems
  - Decoding random codes
  - Inverting multivariate polynomials
  - Secure hash functions
  - Supersingular isogeny Diffie-Hellman

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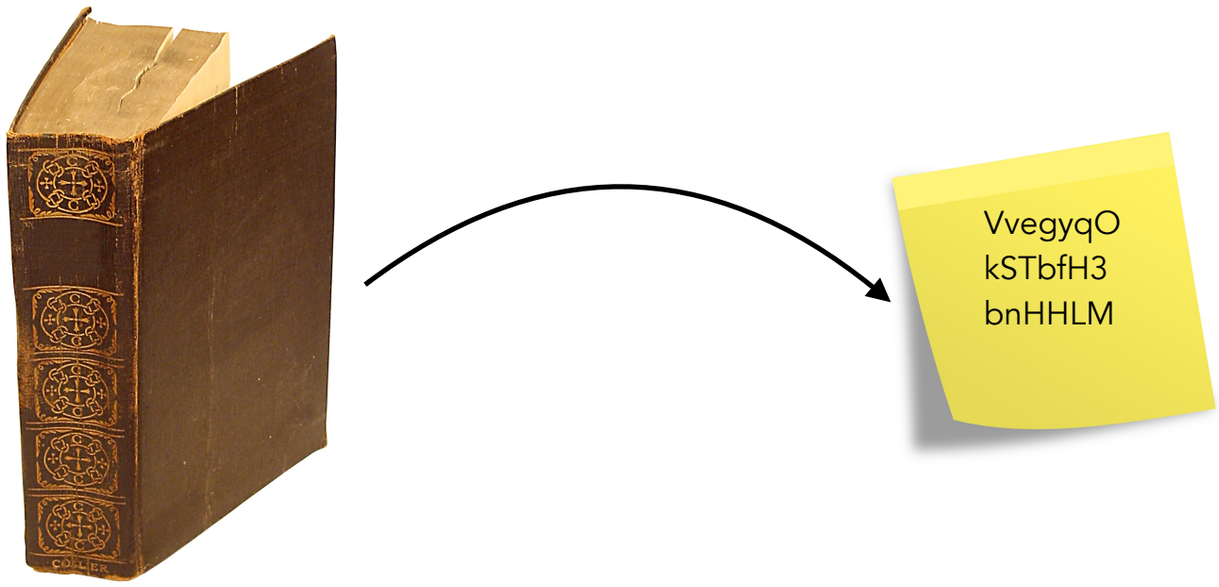
- Models: Quantum Random Oracle Model (QROM)

Hash functions



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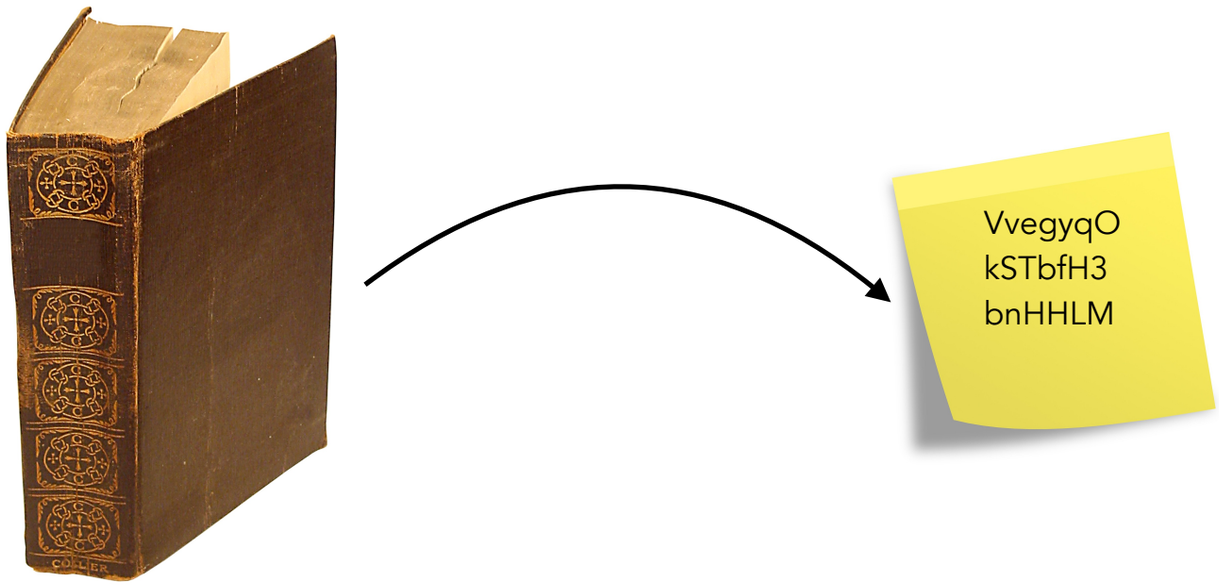


Ubiquitous in cryptography. Example: digital signatures

The (Q)ROM

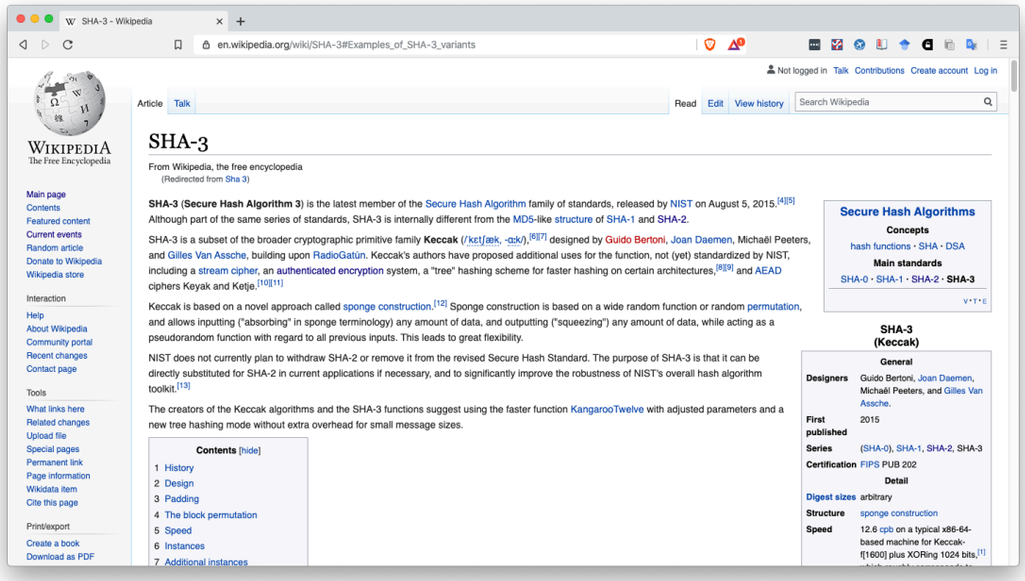
The (Q)ROM

Reality

Model

The (Q)ROM

Reality



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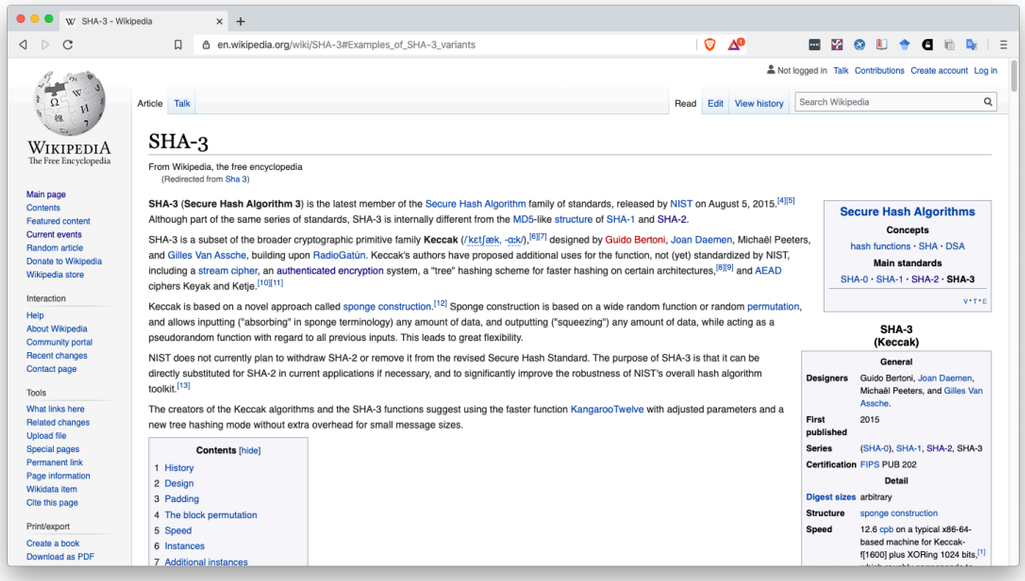

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$$H : \{0,1\}^* \rightarrow \{0,1\}^n$$

Uniformly random

The (Q)ROM

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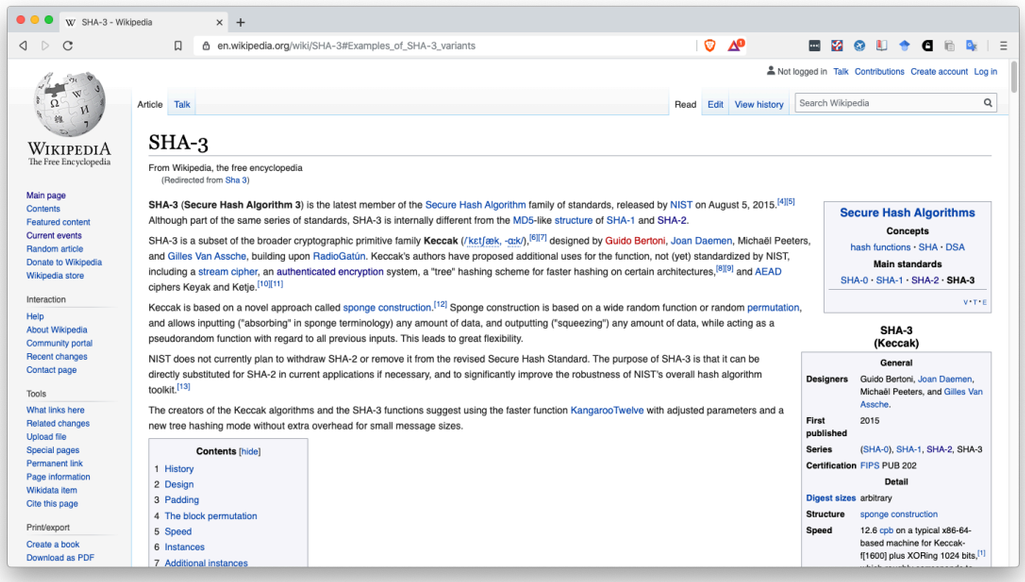

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All agents have (quantum) oracle access to H

$$(x, y) \mapsto (x, y \oplus H(x))$$

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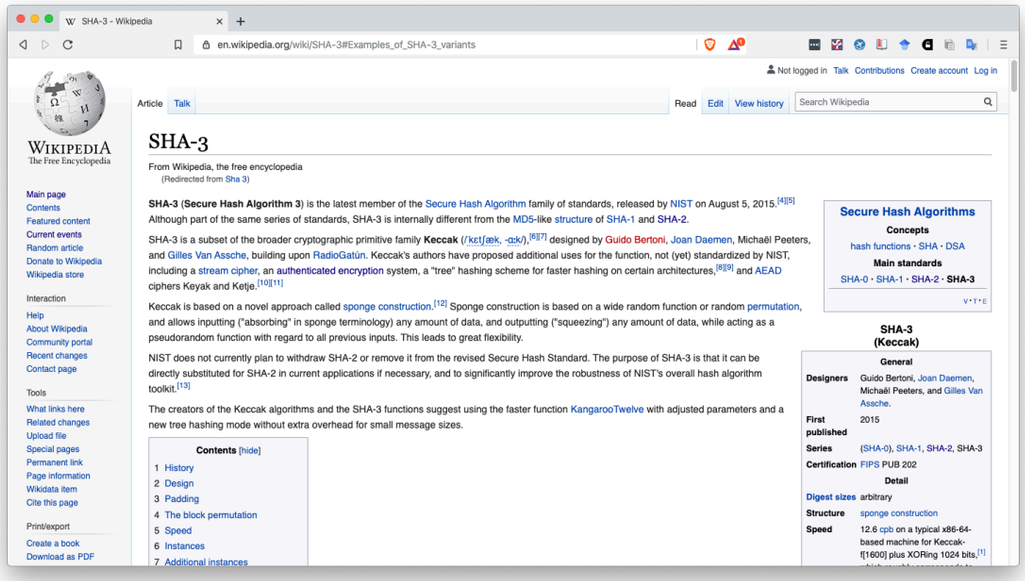

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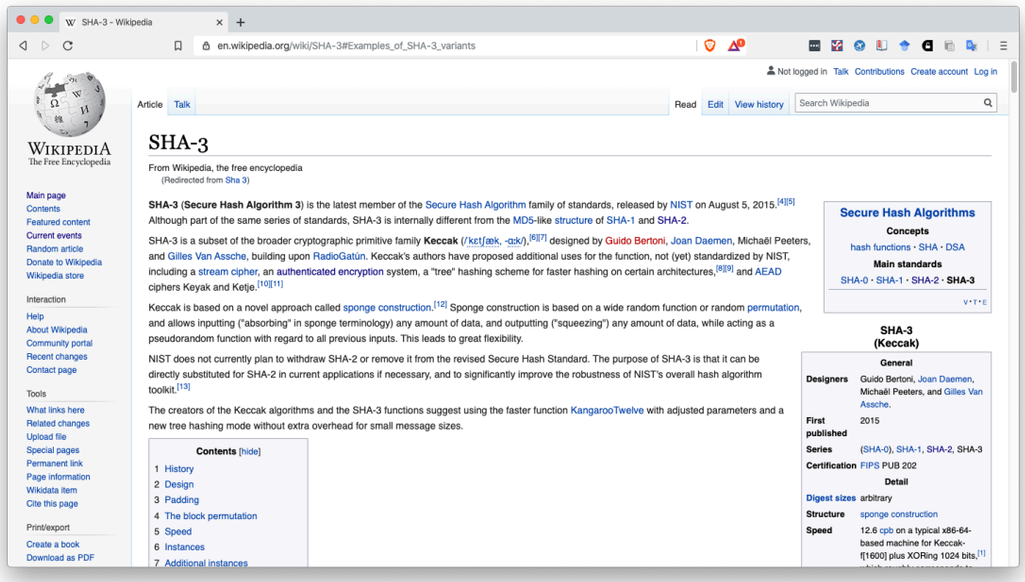

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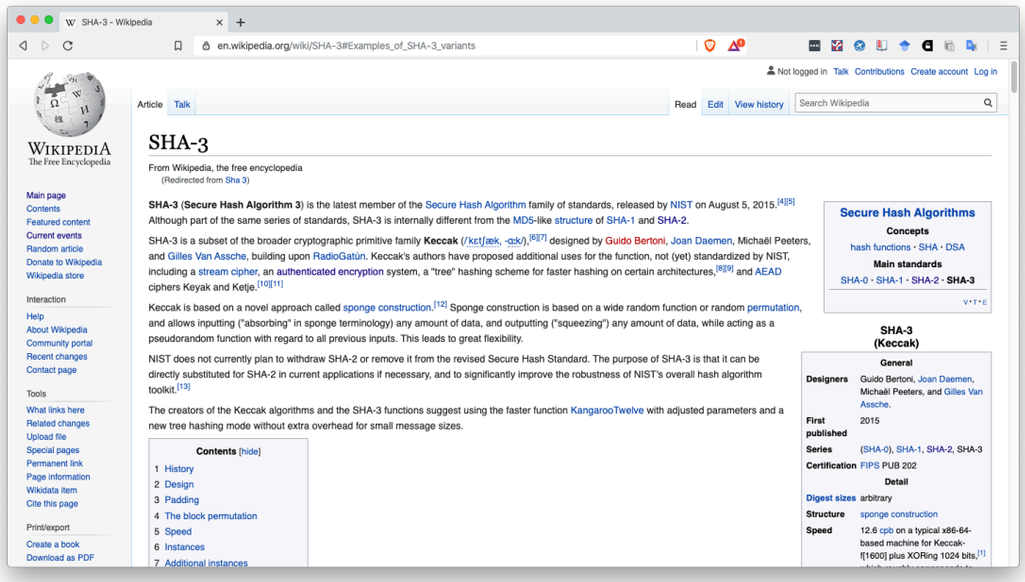

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- Seems to work in practice
- Enables very efficient crypto

QROM challenges

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ROM techniques:

1. Query transcripts

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QROM:

1. ~~Query transcripts~~
2. ~~Rewinding~~ $\Longrightarrow$ 3 specialized rewinding techniques that don't cover all applications
3. Reprogramming: Sure, if you know how without 1. and 2.

The Fiat Shamir transformation

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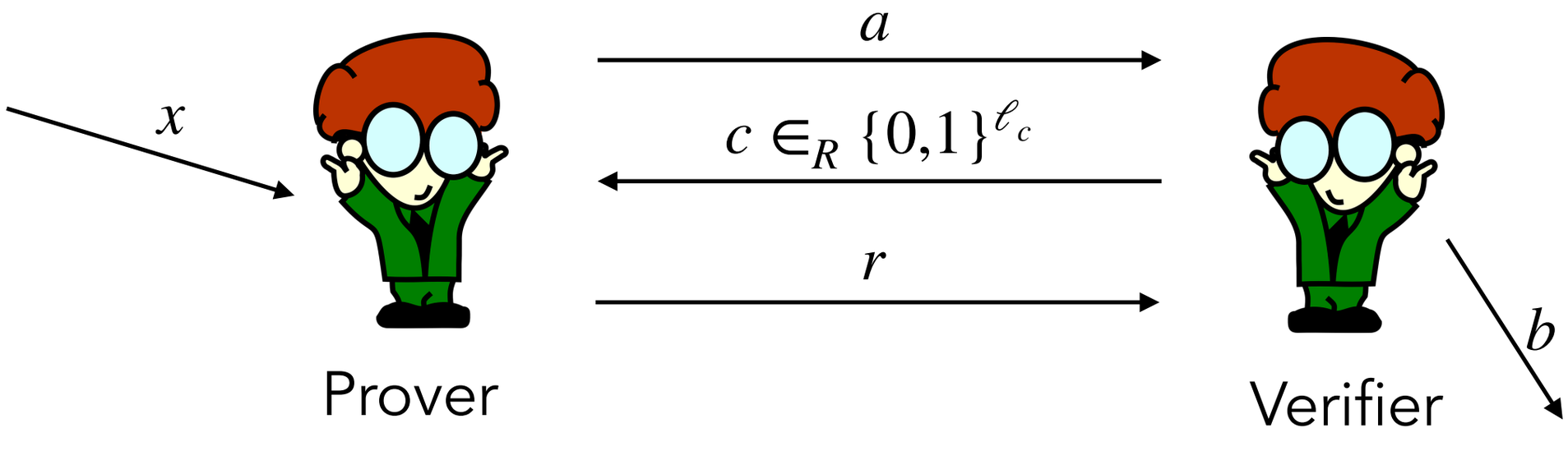
Σ-protocol: Interactive proof system



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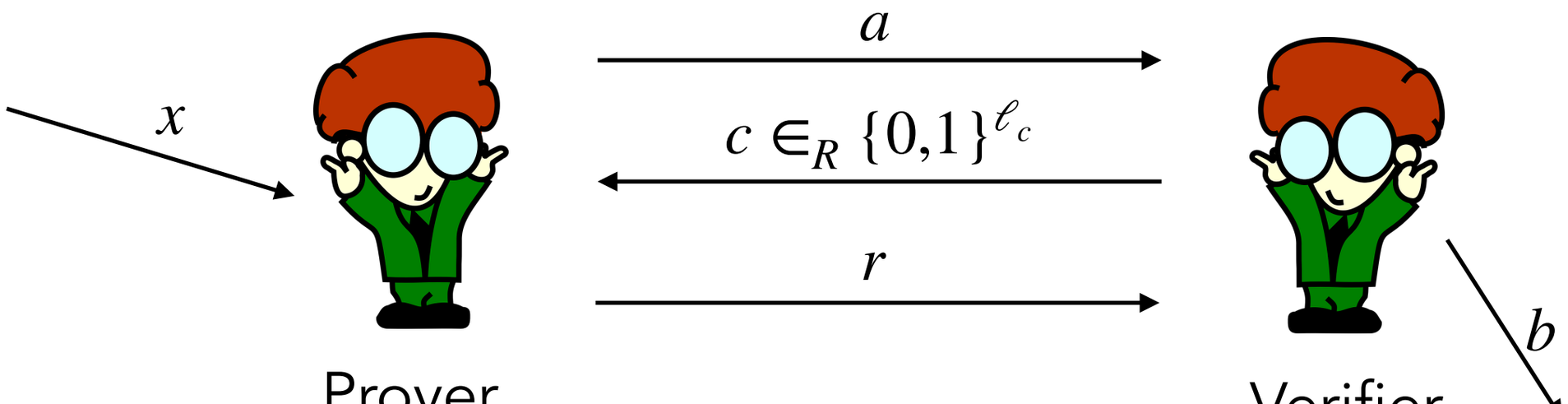

Fiat Shamir (FS) transformation: $c = H(x, a)$

The Fiat Shamir transformation

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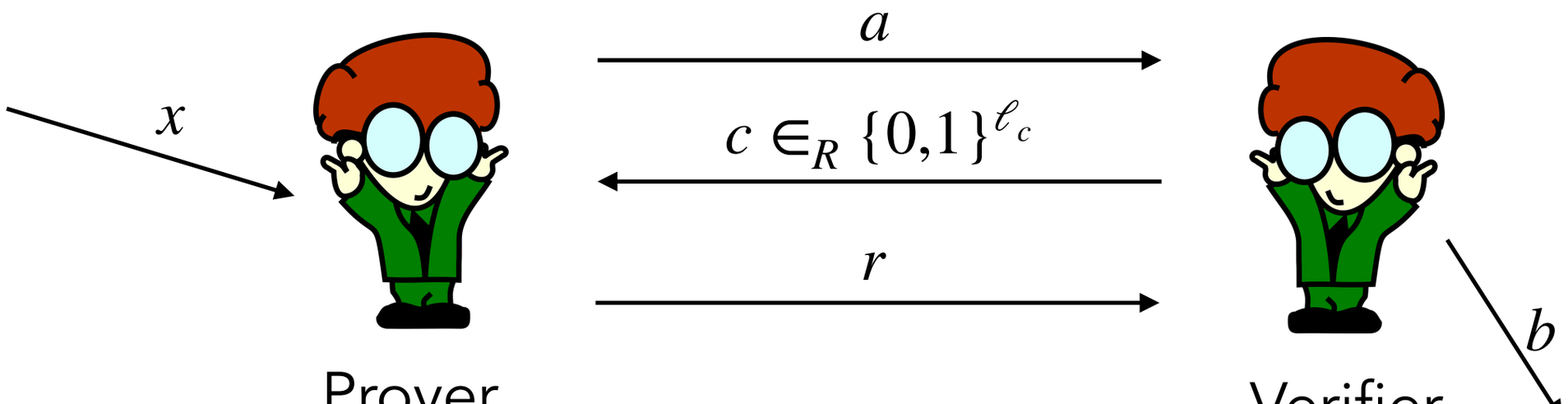

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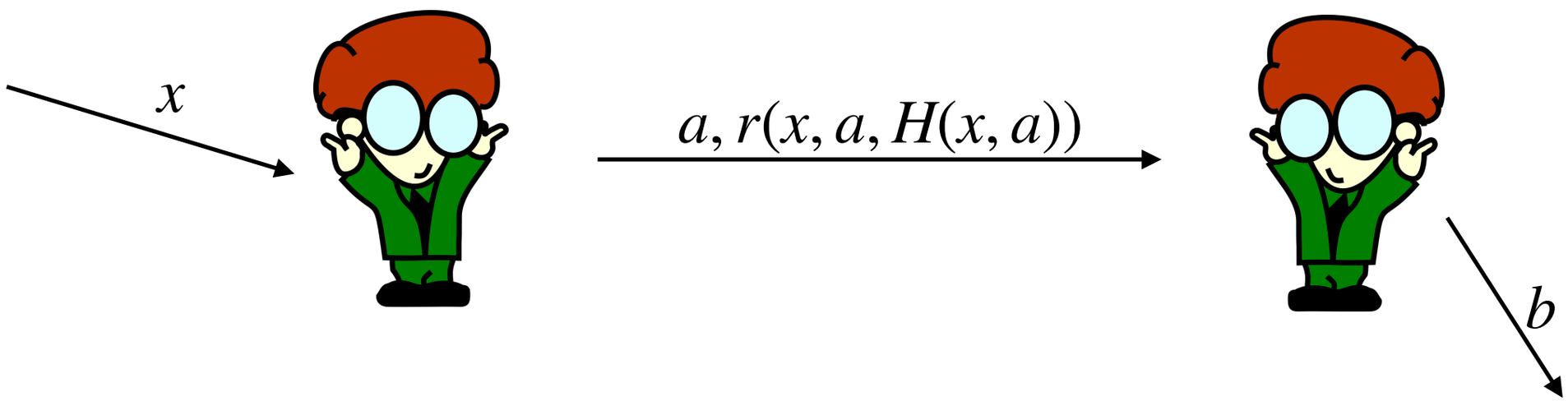

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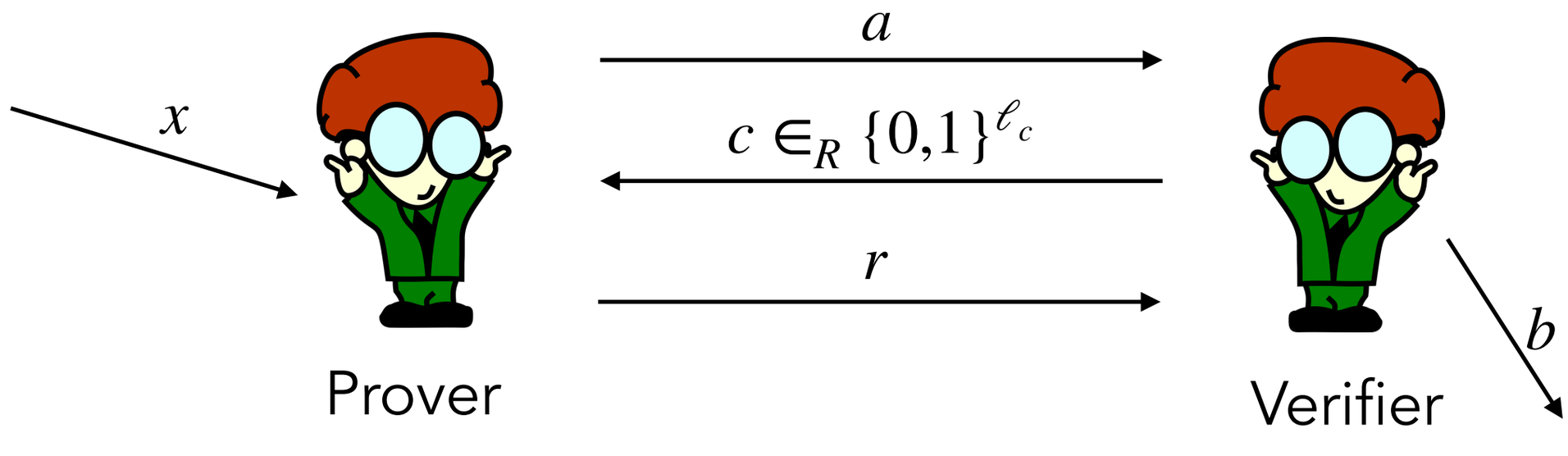

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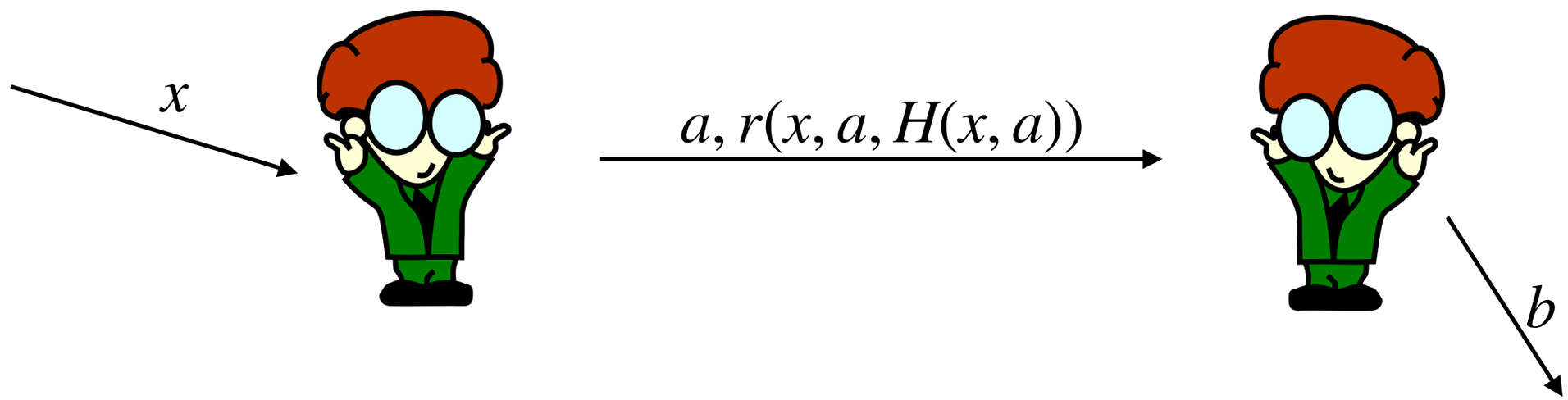

non-interactive!!!

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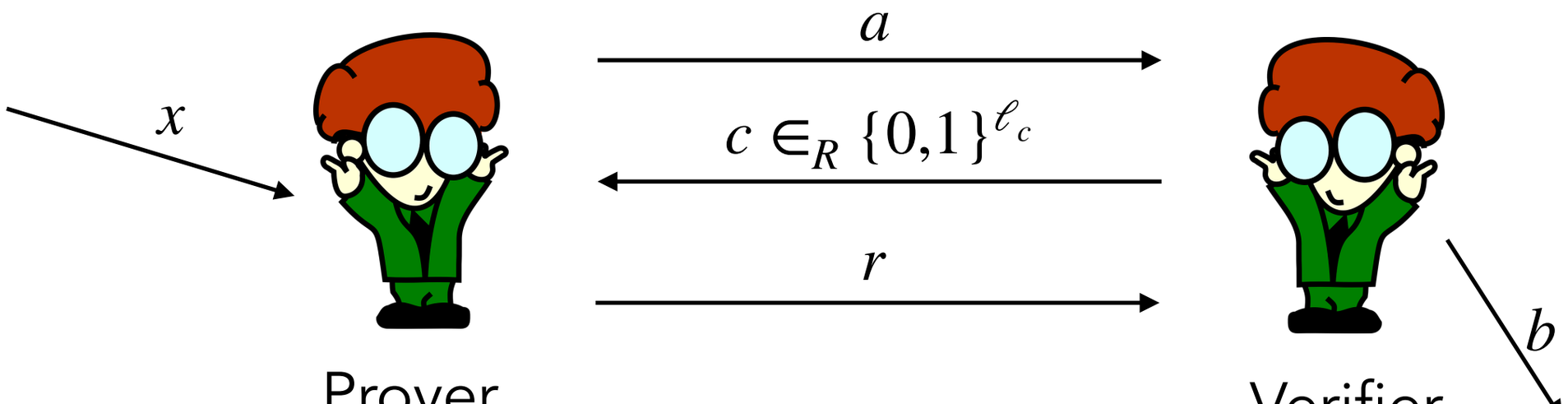

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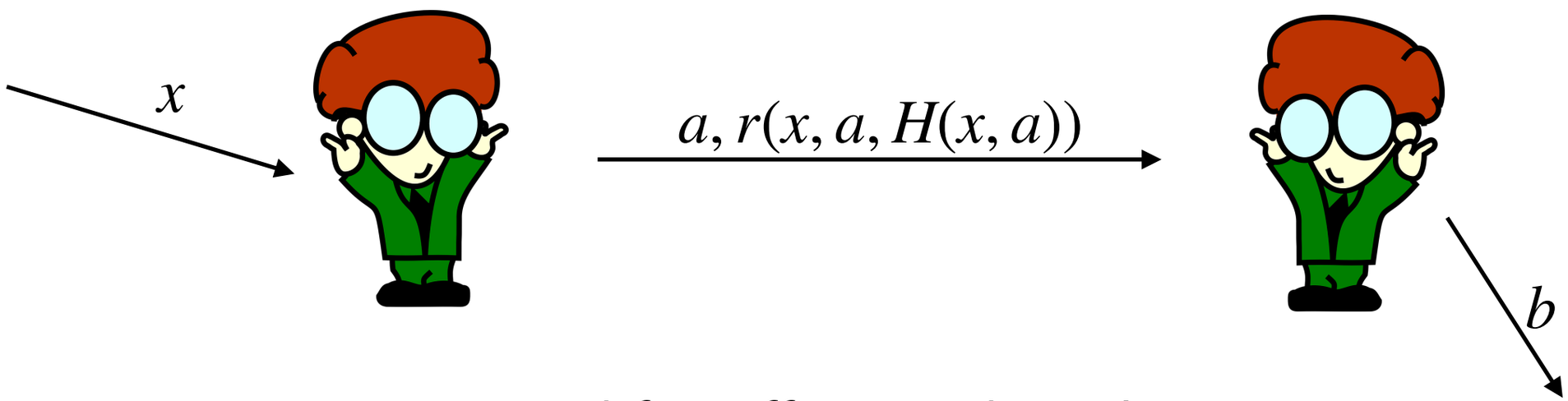

non-interactive!!! $\Longrightarrow$used for efficient digital signatures

ROM security

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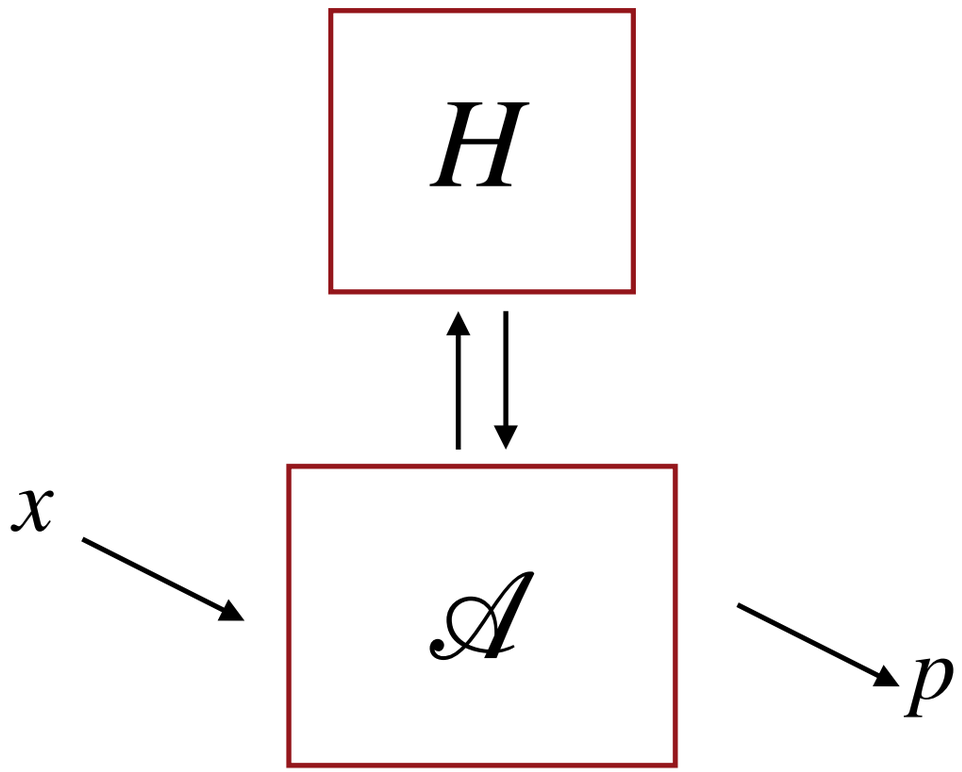

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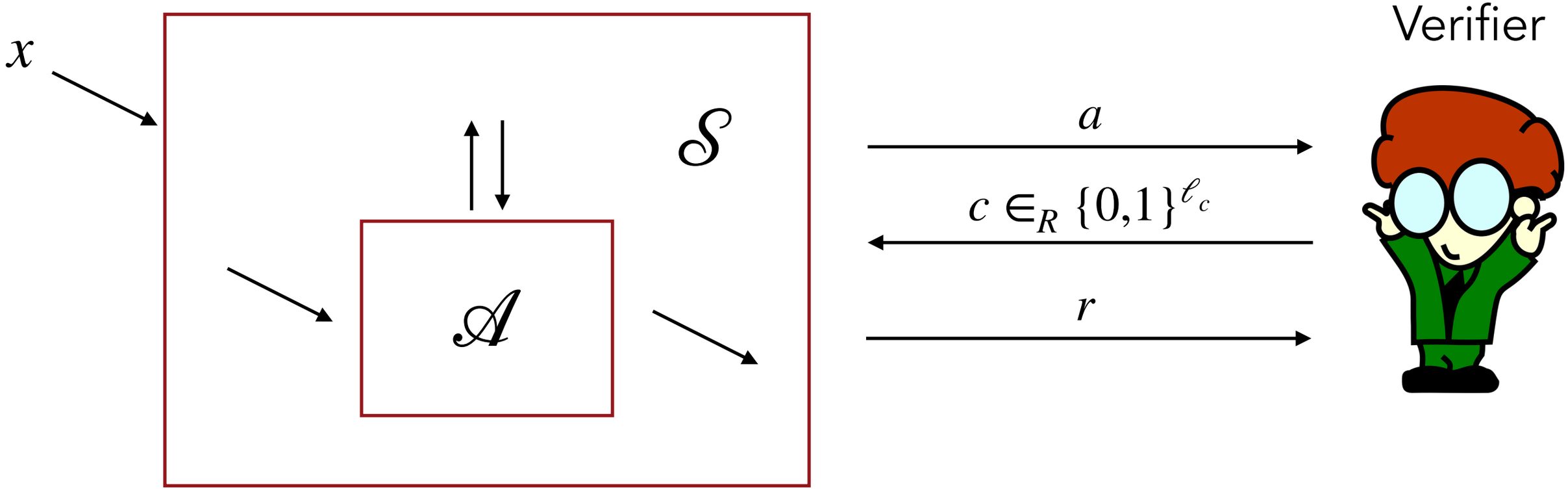

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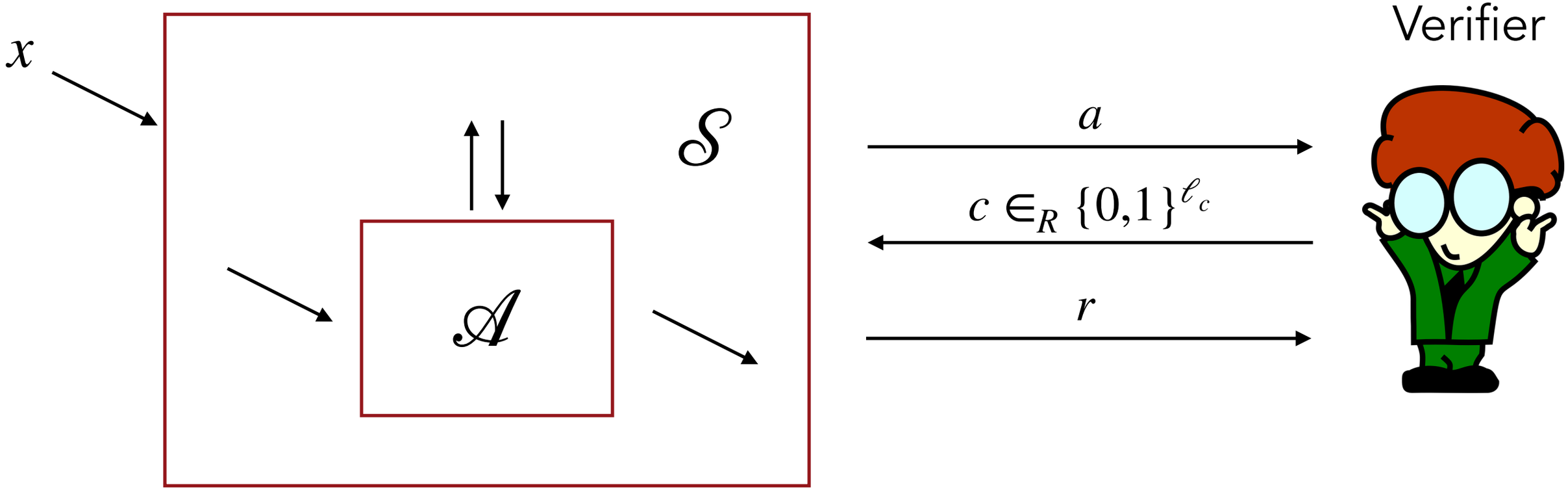

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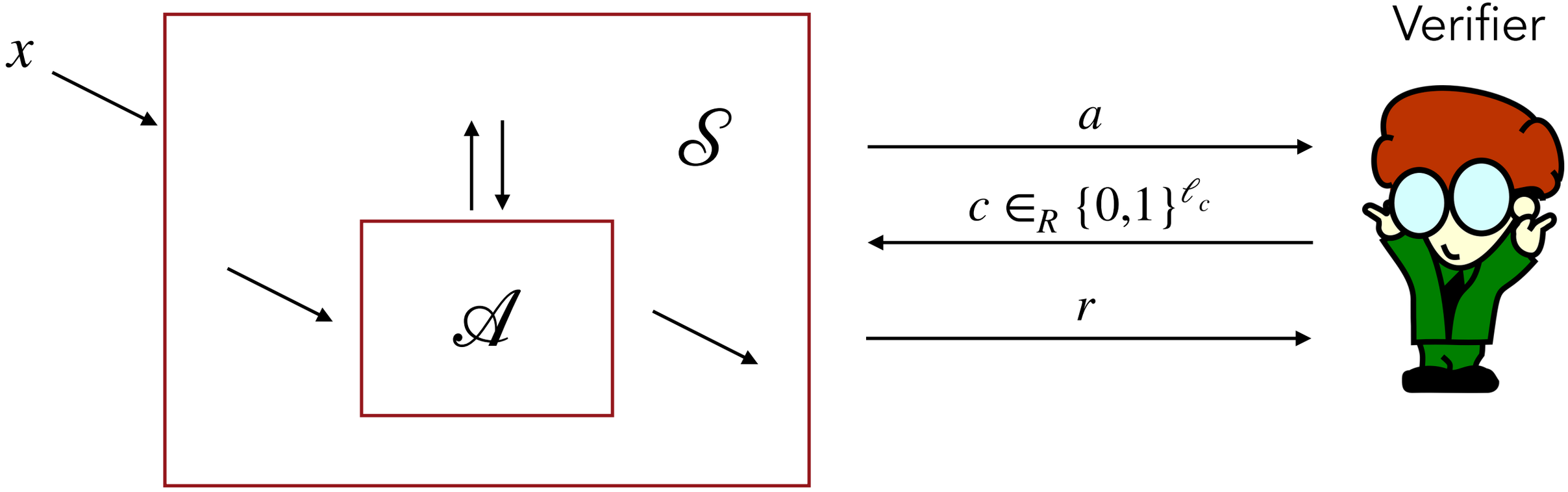

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# of queries $\mathcal{A}$ makes to H

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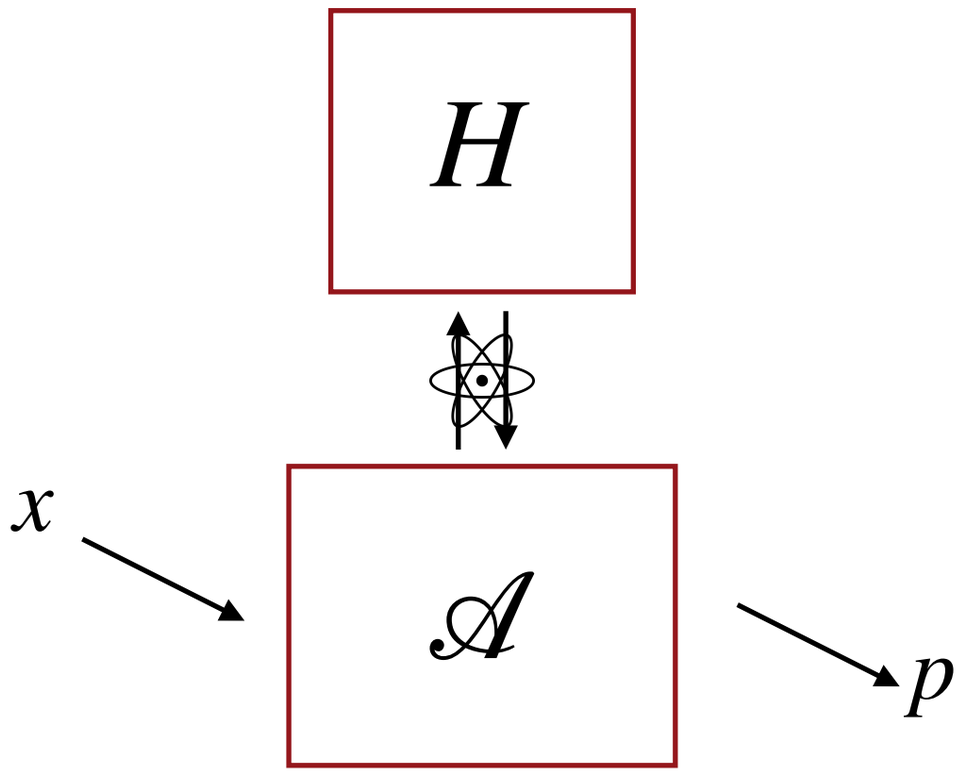

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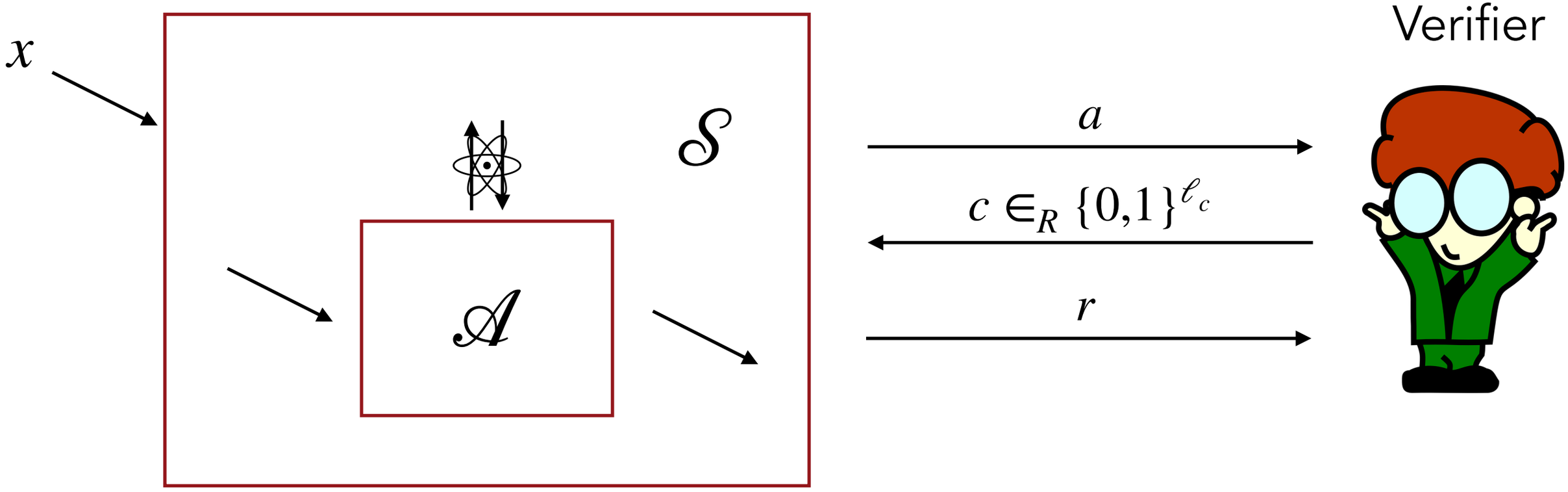

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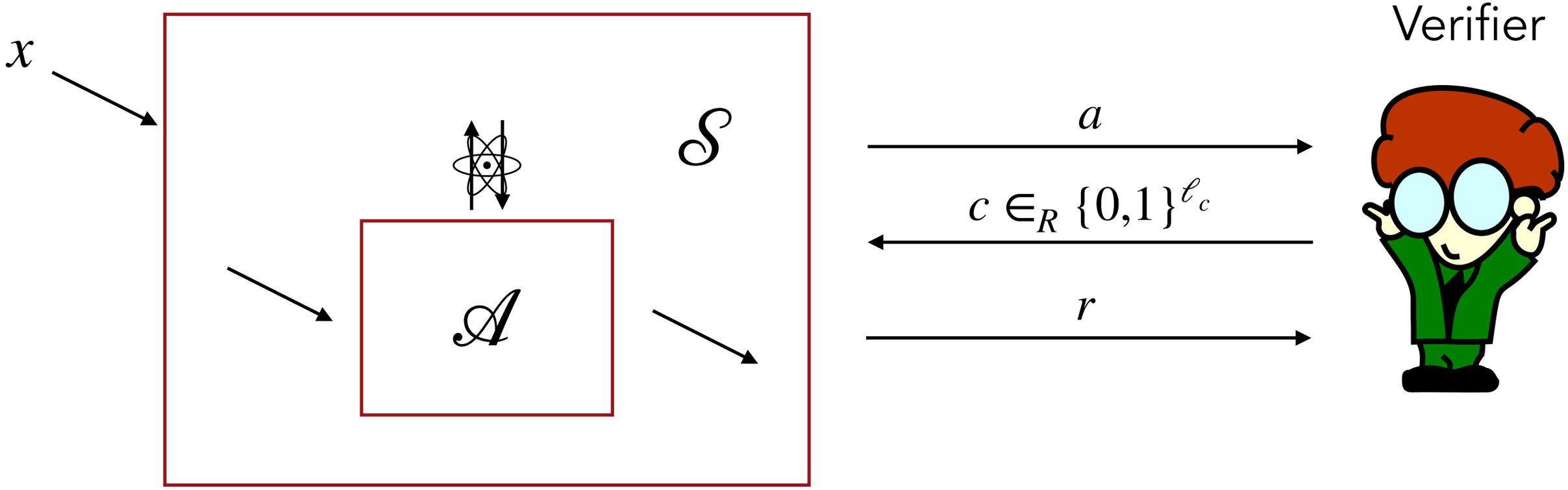
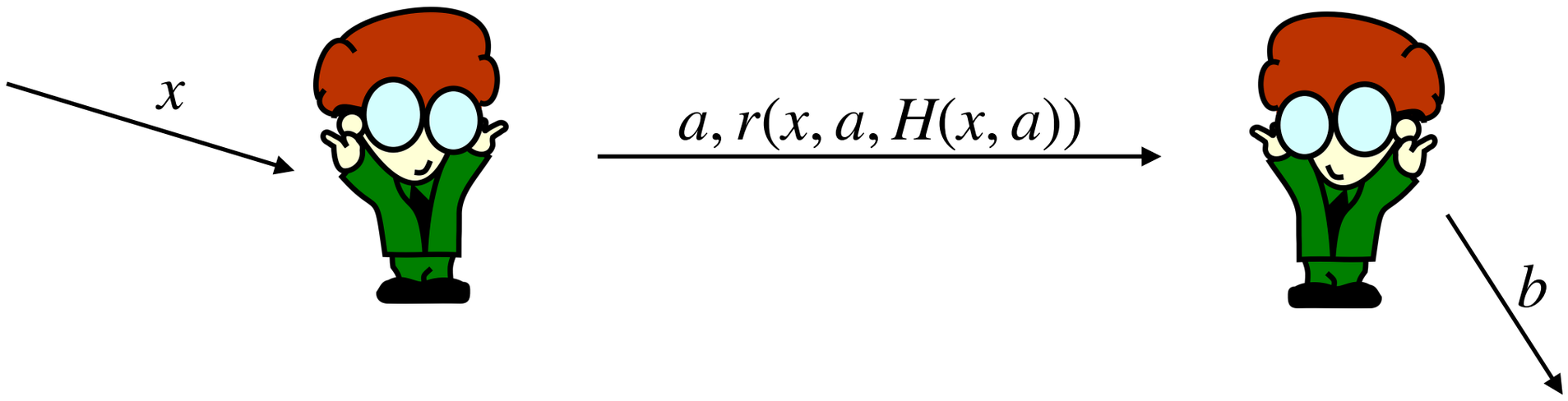


Success probability: $\varepsilon(\mathcal{S}[\mathcal{A}]) \geq \dfrac{\varepsilon(\mathcal{A})}{O(q^2)}$

Technique



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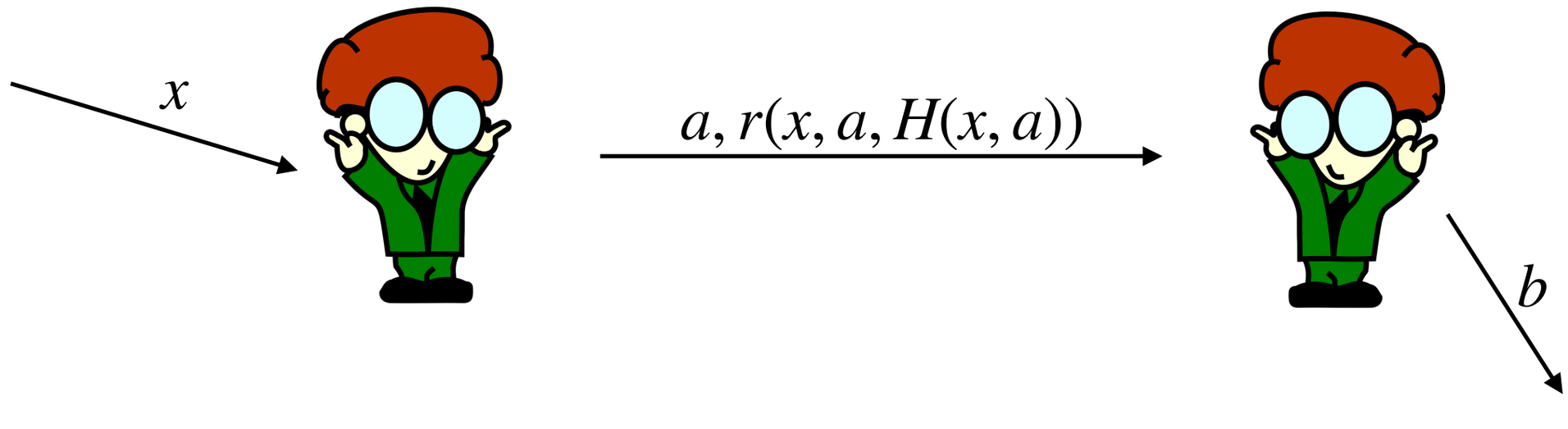

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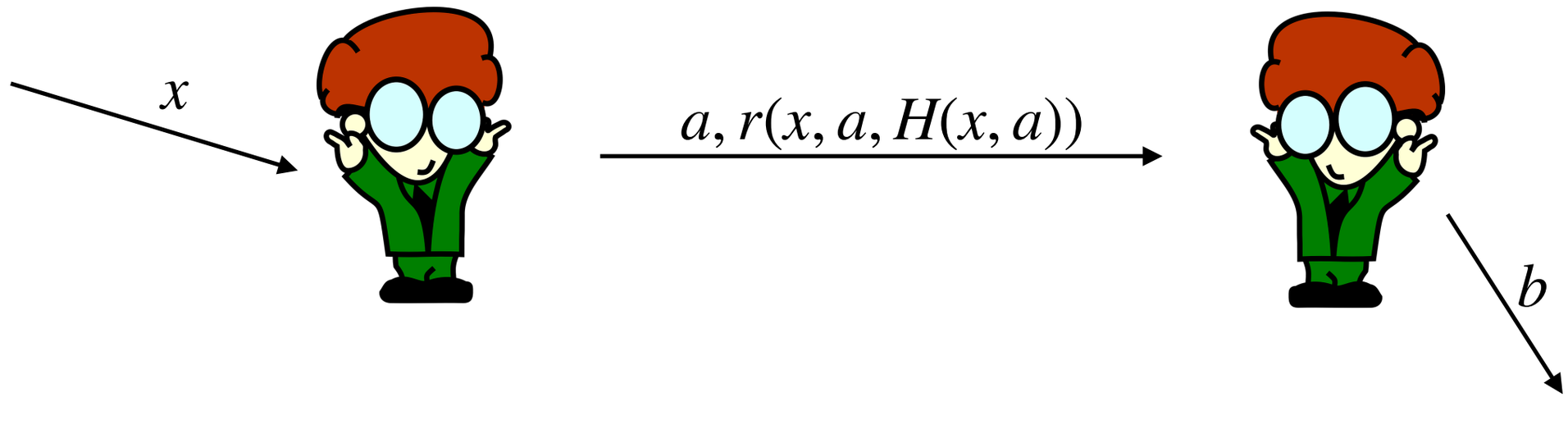

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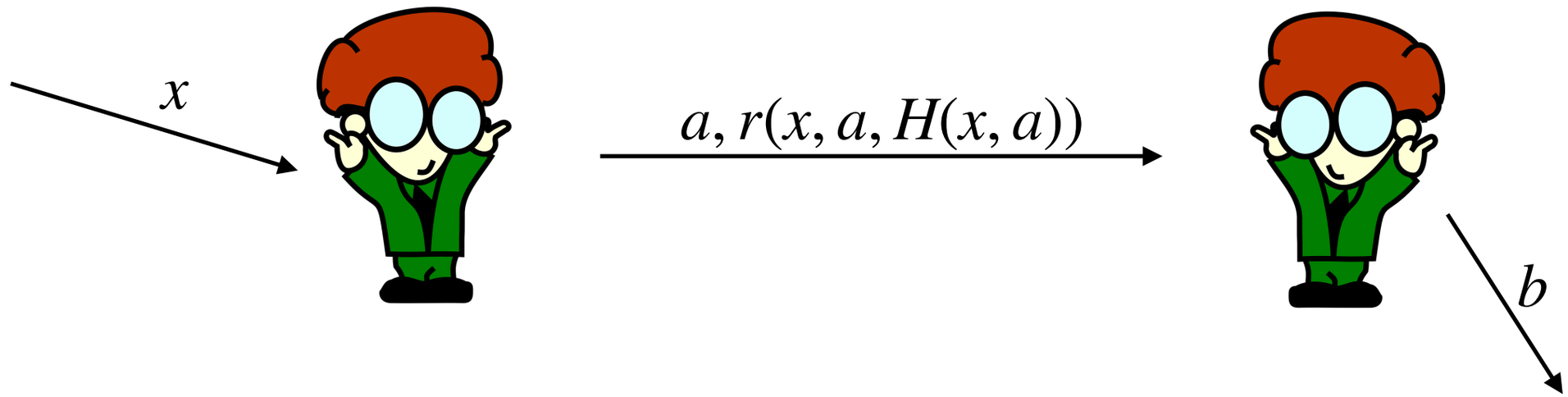

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Measure-and-Reprogram: Pick a random query, measure it and reprogram with c from the Σ-protocol.

Long term goal

Popular belief about QROM: Grover speed-up is as good as it gets.

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Reality:

Summary

- The (Q)ROM is extremely useful for efficient cryptography
- Quantum theory complicates things, much less coherent picture of QROM security
- Important cases solved, e.g. Fiat Shamir
- General reduction from QROM to ROM would be nice to have!