IMPROVING FDK RECONSTRUCTIONS BY DATA-DEPENDENT FILTERING

Real time tomography project

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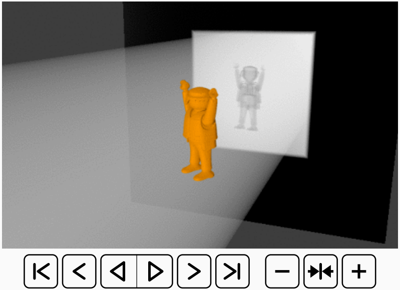
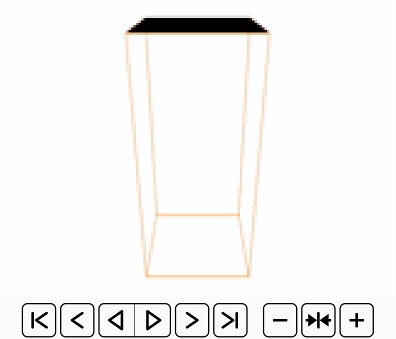
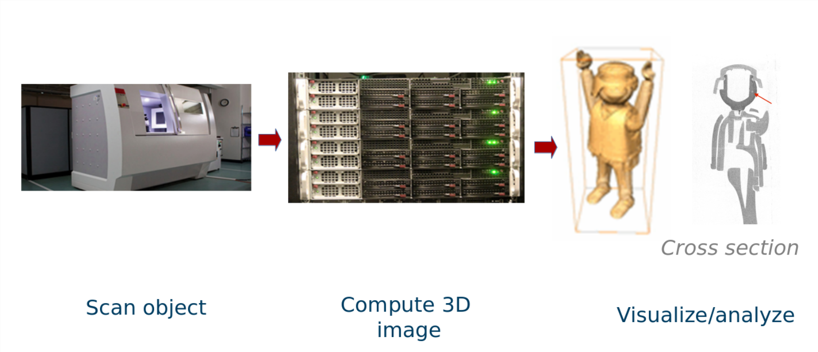
Computational Imaging
Centrum voor Wiskunde en Informatica, Amsterdam

CT SCAN





SCANNING PROCESS



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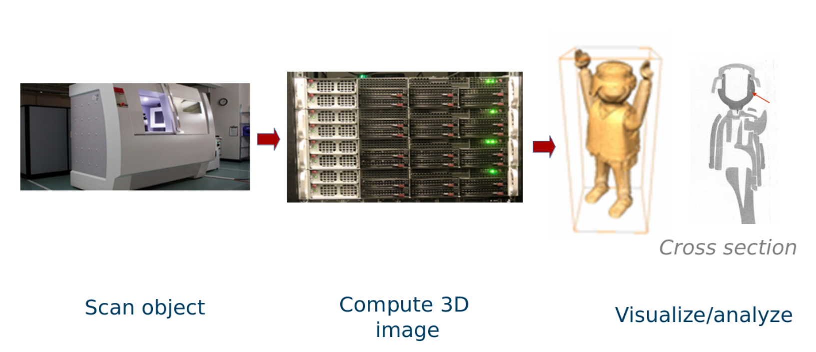

- Fast measurements
- Few projection angles

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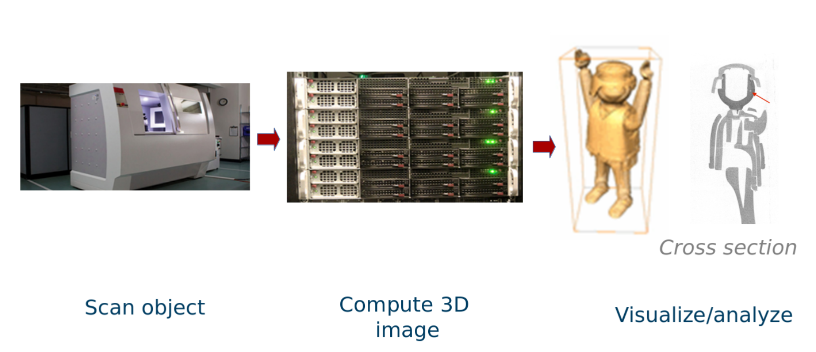

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- Few projection angles
- Analytic methods

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COMPLICATION

Analytic methods produce accurate results if the input data has...

- many projection angles
- low noise

ALGEBRAIC FILTER FDK METHOD

Consider the inverse problem:

$$\mathcal{D}\mathrm{f} = \mathrm{g},$$

where

- $\mathcal{D}$, the linear 3D cone-beam transform, or forward projection,
- g, the measured data,
- f, the unknown object.

FDK reconstruction:

$$f_{\mathrm{FDK}} = \mathbb{F}_h(g) = \mathcal{D}^*(\tilde{g} * h)_{1D},$$

with

- $\mathcal{D}^*$, the adjoint of $\mathcal{D}$, or the backprojection,
- $\tilde{g}$, a weighted version of the data g
- $(g * h)_{1D}$, a one dimensional convolution,
- h, a one dimensional filter.

Fix the data and compute the data-dependent[1] filter:

$$\hat{h} = \underset{h}{\operatorname{argmin}} \|\mathcal{D}\mathbb{F}_g h - g\|_2^2 + \lambda \|Th\|_2^2,$$

with

- $F_g h$, FDK reconstruction on fixed data g, with filter h,
- T, the Tikhonov operator that imposes the type of regularization,
- λ, the regularization parameter.

[1]Similar strategy as [D.M. Pelt, 2014] for 2D FBP.

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LS-FDK:
Setting, $T = 0$, gives the least squares problem, i.e.

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T-FDK:
Setting $T = \mathrm{Id}$, gives the original Tikhonov, i.e.

$$\hat{h} = \underset{h}{\operatorname{argmin}} \|\mathbf{D}\mathbb{F}_g h - g\|_2^2 + \lambda\|h\|_2^2.$$

RESULTS

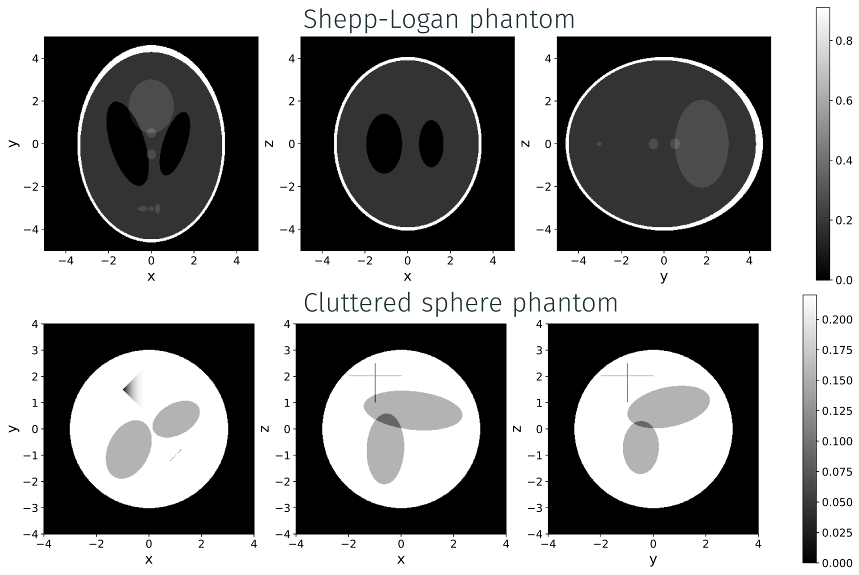


Figure: Simulated data phantoms.

VARYING NUMBER OF PROJECTION ANGLES

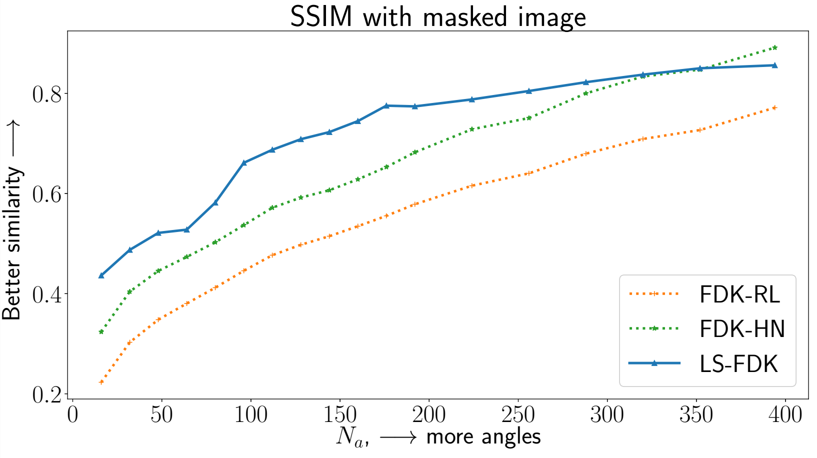


Figure: Results for Shepp-Logan phantom.

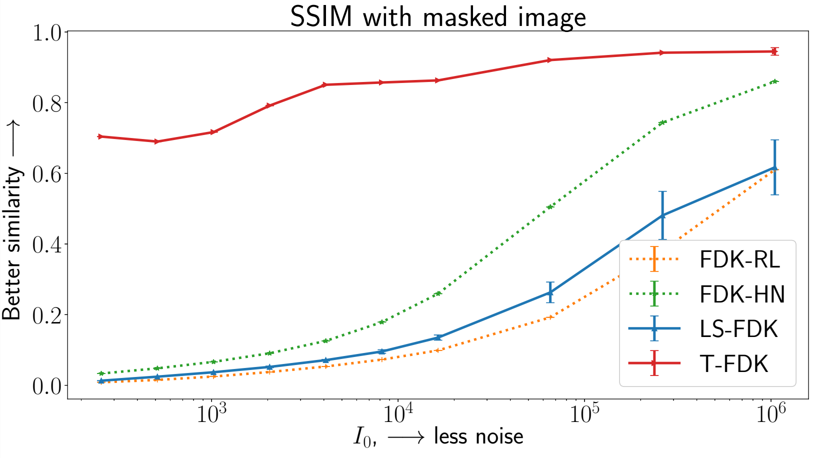


Figure: Results for cluttered sphere phantom.

RECONSTRUCTIONS NOISY DATA

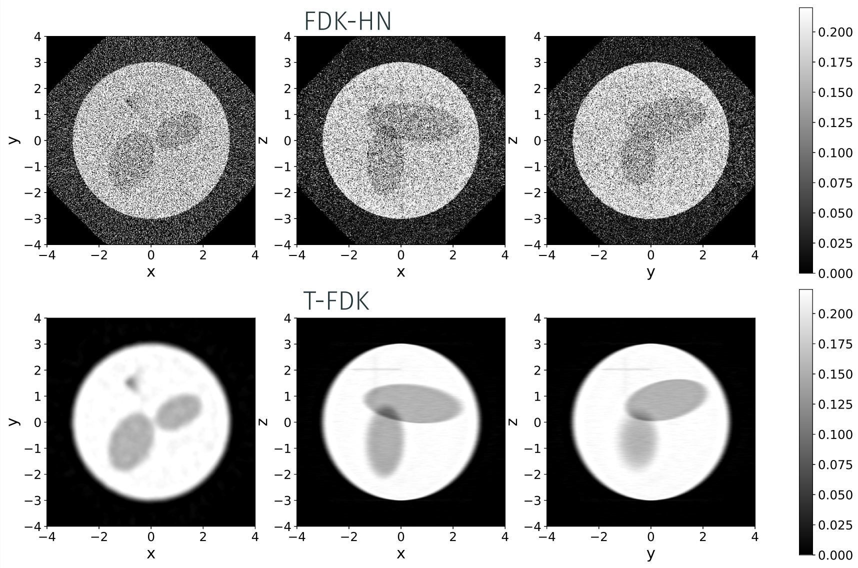


Figure: Cluttered sphere phantom, 360 equidistant projection angles, $I_0 = 256$.

High-dose scan:
70 keV, 45 W, 500 ms per projection.

Low-dose scan:
70 keV, 20 W, 100 ms per projection.

Figure: Scanned objects

Gold standard reconstruction:
SIRT-300, high-dose data, 500 equidistant projection angles

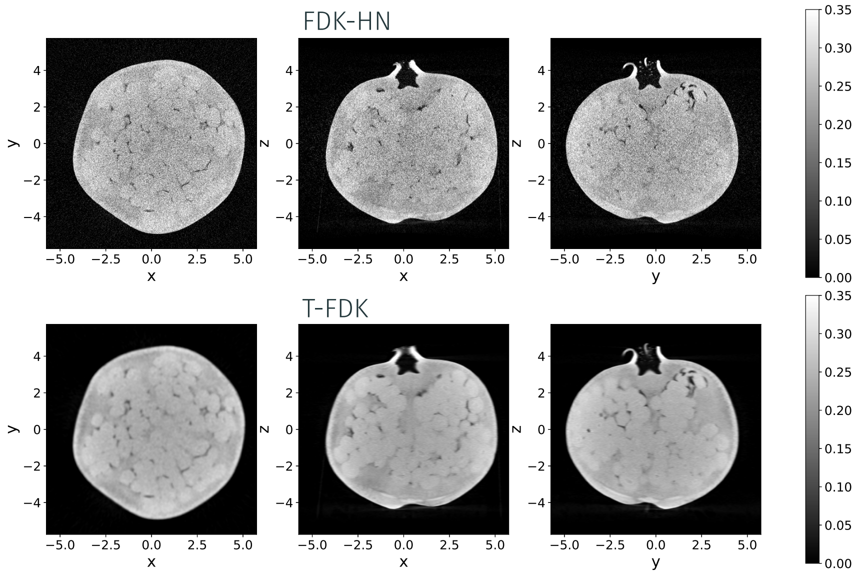


Figure: Low-dose, 500 equidistant projection angles.

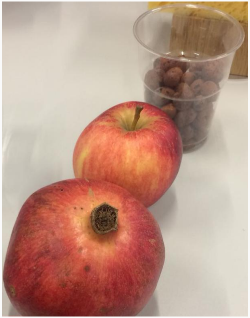
Compute an algebraic filter

h_{Pom1}

$\Longrightarrow$

Use algebraic filter to reconstruct

$f_{\text{Pom1 filter}} = \mathbb{F}_g(h_{\text{Pom1}})$



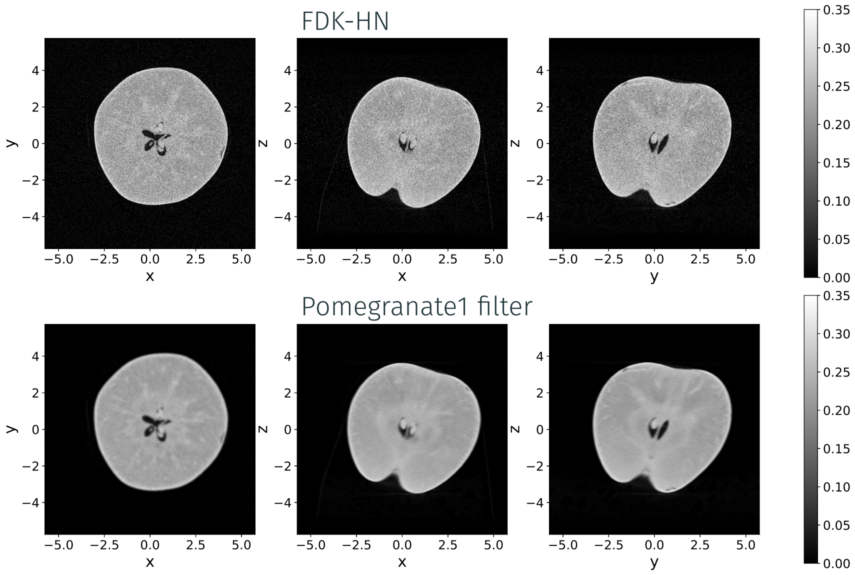


Figure: Low-dose apple, 500 equidistant projection angles.

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Paper in preparation:
M.J. Lagerwerf et al., "Improving FDK reconstructions by data-dependent filtering", 2018.

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