Frozen percolation on the triangular lattice

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Motivation

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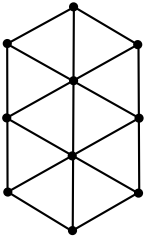
- vertices $\leftrightarrow$ atoms
- clusters $\leftrightarrow$ molecules
- Dynamics: small molecules merge, while big ones do not interact with other particles

The model: N-parameter frozen percolation



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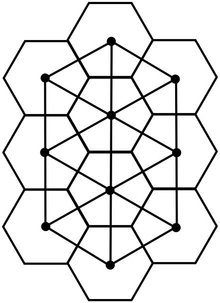

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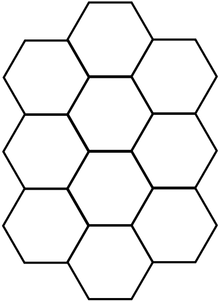

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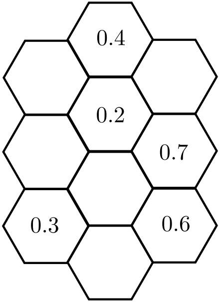

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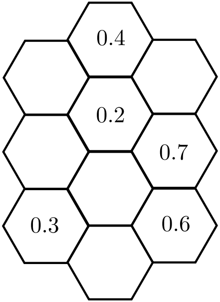

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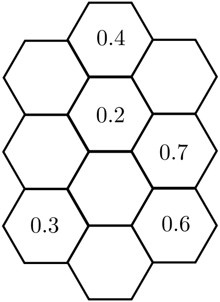

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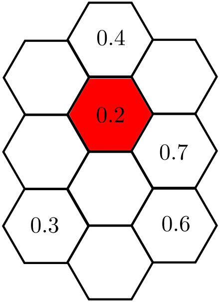

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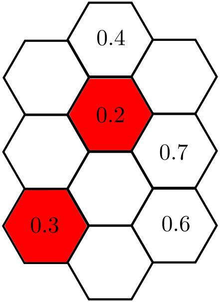

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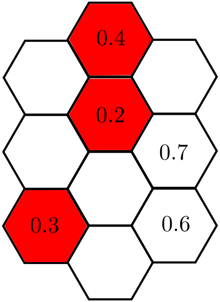

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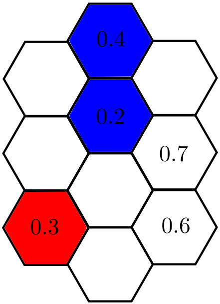

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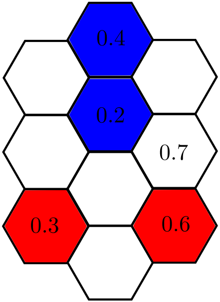

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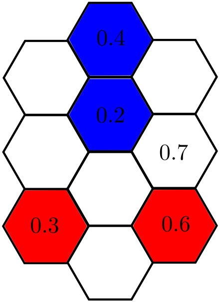

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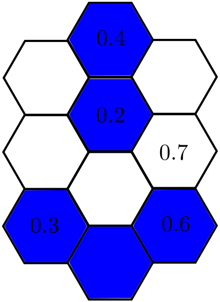

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Corollary

For all $v \in V$

$$\mathbb{P}_N\,(v \textit{ freezes at some time in } [0,1]) \to 0 \textit{ as } N \to \infty.$$

Scaling limit

Conjecture

Scale the space by N and the time close to $1/2$ appropriately. Then the N-parameter frozen percolation processes converge to a (continuum) limiting process.
Moreover, the limiting process completely describes the frozen clusters.

Thank you for your attention!