CWI

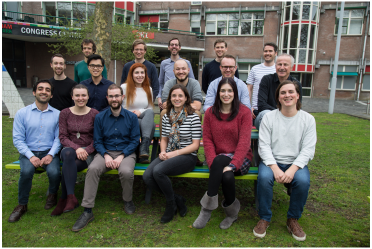
Centrum Wiskunde & Informatica

UCL

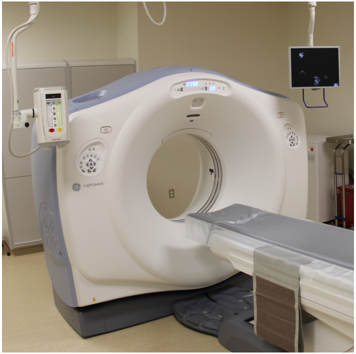
# Image Reconstruction:
# A Playground for Curious Applied Mathematicians

Felix Lucka, Computational Imaging Group

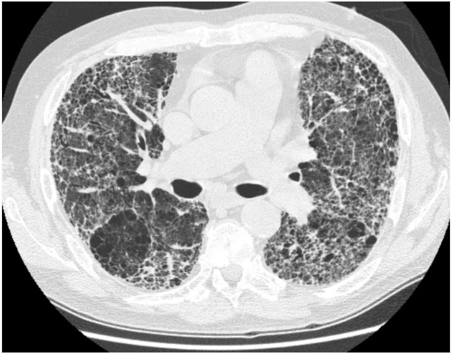
**CWI Scientific Meeting**
14 Feb 2020



# And what do you do for a living?



**(a)** Modern CT scanner



**(b)** CT scan of a patient's lung

Source: Wikimedia Commons

# Imaging Across Disciplines

**Observational astronomy**

**Life and material science**

microscopy

**Medical imaging**

CT, MRI, US, PET, SPECT...

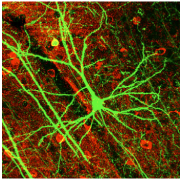
**Geophysical imaging**

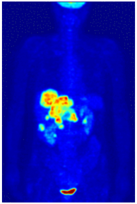
(electrical) resistivity, seismic (ground-penetrating) radar...

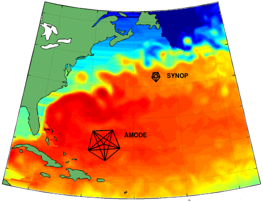
**Remote sensing**

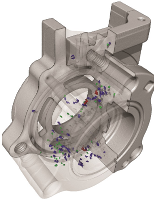
military/intelligence, earth/climate science

**Industrial process imaging**







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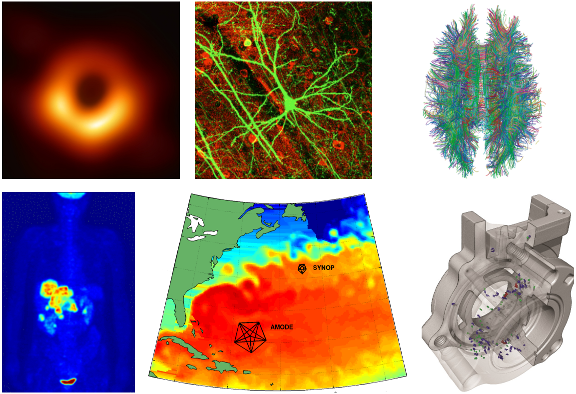



Source: Wikimedia Commons

**Mathematical Imaging:** *Reconstruct spatially distributed of quantities of interest from indirect observations through algorithms derived from rigorous mathematics.*

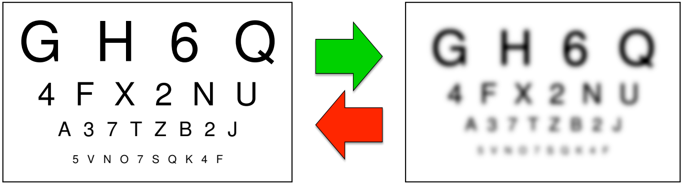
# Imaging: An Inverse Problem

**Inverse problem**: Recover **unknowns** $u$ (image) from **data** $f$ via

$$\boxed{f = A(u) + \varepsilon}$$

- **Forward operator** $A$ solution of **PDE** modelling underlying physics.

# Imaging: An Inverse Problem




$u$ $\qquad$ $A(u)$

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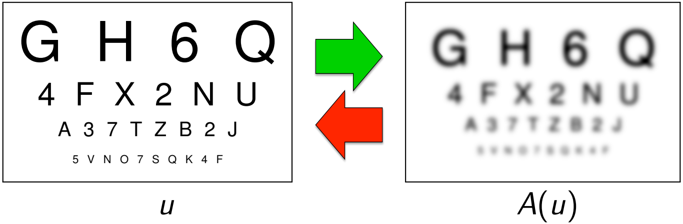


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$$f = A(u) + \varepsilon$$

- **Forward operator** $A$ solution of **PDE** modelling underlying physics.
- Typical inverse problems are **ill-posed**.
- Stable solution requires **a-priori information** on $u$.

# Overview Inverse Problems / Imaging Workflow

**mathematical modeling:**
physics, PDEs, approximations

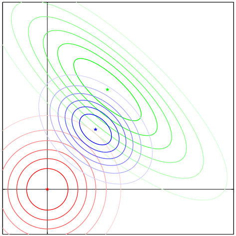
**reconstruction/inference approach:**
regularization, statistical inference, machine learning

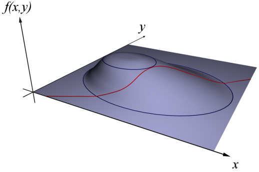
**theoretical analysis:**
uniqueness, recovery conditions, stability

**reconstruction algorithm:**
PDEs, numerical linear algebra, optimization, MCMC

**large-scale computing:**
parallel computing, GPU computing

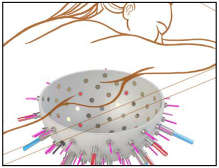
$$(s \cdot \nabla + \mu_a(x) + \mu_s(x))\, \phi(x, s) = q(x, s) + \mu_s(x) \int \Theta(s, s')\phi(x, s')ds'$$





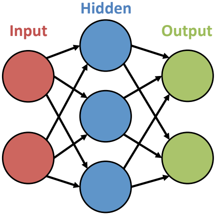
# Current Challenges in Computational Imaging

**core development for new modalities:**
hybrid imaging



**more from more:**
multi-spectral, multi-modal, high resolution

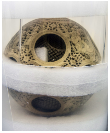
**same from less:**
low-dose, limited-view, compressed, dynamic

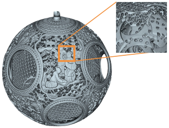


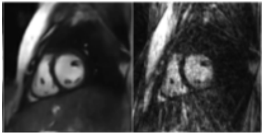

**break the routine:**
real-time, adaptive, explorative

**uncertainty quantification & quantitative imaging**

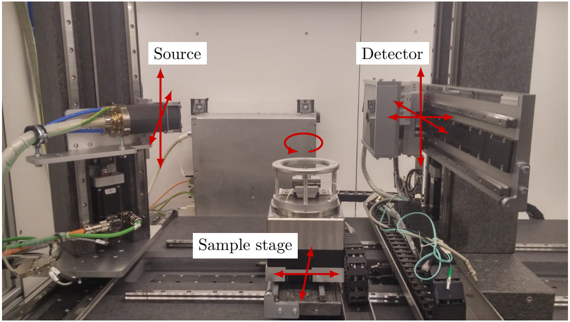
**machine learning:**
embedding, networks for 3D/4D, clinical training data





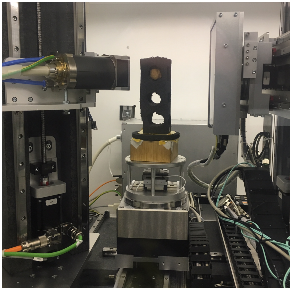
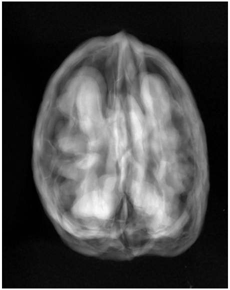
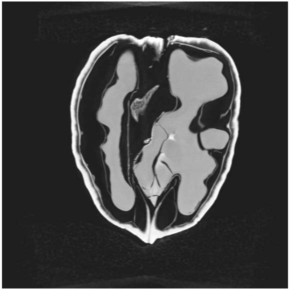


# FleX-ray Lab


- custom-built, fully-automated, highly flexible
- **Aim: Proof-of-concept** experiments directly accessible to mathematicians and computer scientists.

# X-Ray Scan of Static Object

**Der Sarkissian, L, van Eijnatten, Colacicco, Coban, Batenburg, 2019.**
A Cone-Beam X-Ray CT Data Collection Designed for Machine Learning,
*arXiv:1905.04787.*

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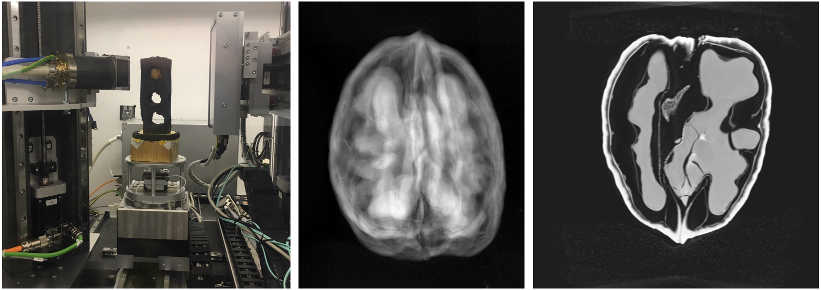

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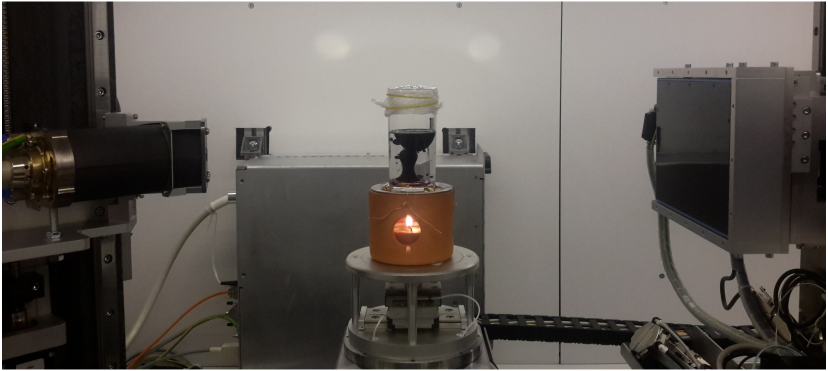
**What about dynamic processes?**

# X-Ray Scan of Dynamic Object



canonical example of temperature-driven **two-phase flow instability**

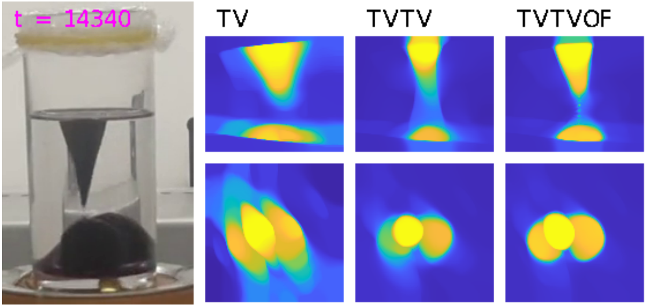
# Joint Image Reconstruction and Motion Estimation

**reconstruct image sequence $u$ and motion fields $v$ simultaneously**

$$\min_{u,v} \sum_t \|A_t u_t - f_t\|_2^2 + \mathcal{J}(u_t) + \mathcal{M}(u,v) + \mathcal{H}(v)$$

- data discrepancy
- spatial assumptions on image
- motion model (PDE)
- spatial assumptions on motion

**large-scale, non-convex, non-smooth optimization**



# Dynamic Compressed Sensing via Deep Learning

**Hauptmann, Arridge, L, Muthurangu, Steeden, 2018.** Real-time cardiovascular MR with spatio-temporal artifact suppression using deep learning - proof of concept in congenital heart disease, *Magnetic Resonance in Medicine.*

# Novel Modalities: Photoacoustic Tomography (PAT)

- hybrid imaging: "**light in, sound out**"
- quantitative images of optical properties
- **high res + high contrast**
- non-ionizing radiation

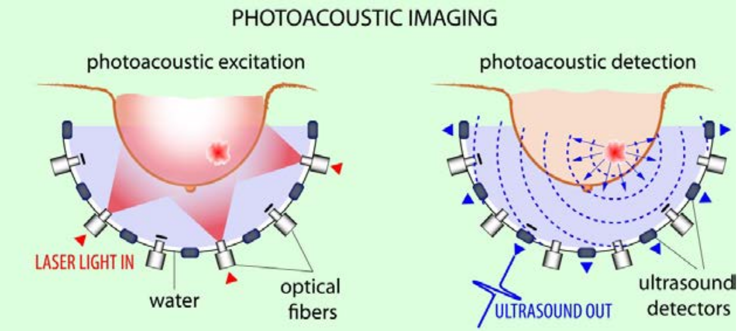
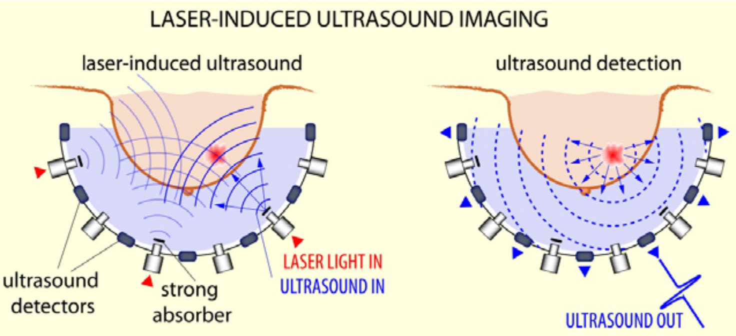



image reconstruction: **two coupled inverse problems** in high res 3D

- initial value problem for wave equation
- parameter identification for radiative photon transfer

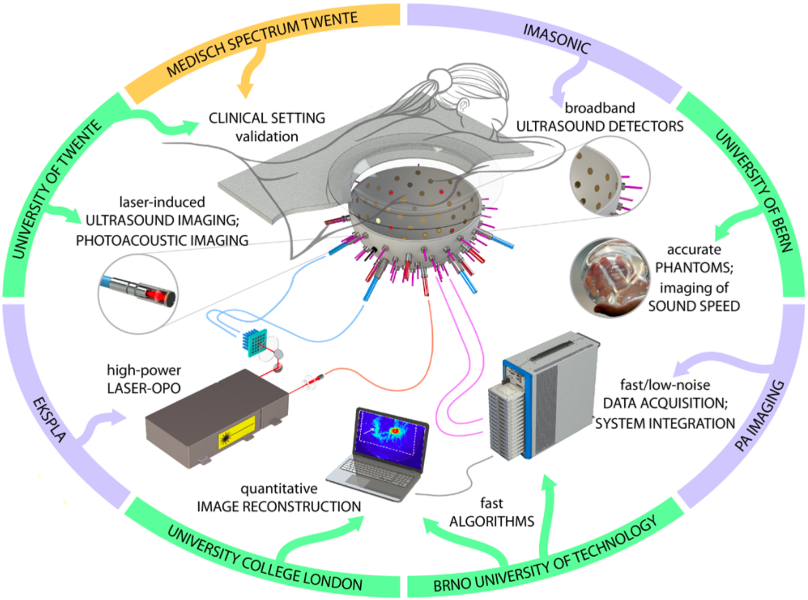
# Novel Modalities: Ultrasound Computed Tomography (USCT)

- "**sound in, sound out**"
- safe as conventional US
- quantitative images of acoustic properties
- novel diagnostic information




- parameter-identification for wave equation
- multiple sources, boundary recordings
- 3D full waveform inversion: non-convex, large-scale optimization

# H2020 Project: PAT + USCT Breast Scanner



# Summary

- imaging has broad range of applications
- mathematically: **inverse problem** of reconstructing distributed quantities from indirect observations
- **mathematical modeling**, (solving) **PDEs**, **numerical optimization**
- **challenges:** large-scale, optimization, uncertainty quantification, compressed sensing, dynamic/spectral imaging
- stable solution requires **a-priori information**
- hot topic: **deep learning**

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**Thank you for your attention!**

# Mathematical Modelling (simplified)

**Quantitative Photoacoustic Tomography (QPAT)**

radiative transfer equation (RTE) + acoustic wave equation

$$(v \cdot \nabla + \mu_a(x) + \mu_s(x))\,\phi(x,v) = q(x,v) + \mu_s(x)\int \Theta(v,v')\phi(x,v')dv',$$

$$p^{PA}(x,t=0) = p_0 := \Gamma(x)\mu_a(x)\int \phi(x,v)dv, \qquad \partial_t p^{PA}(x,t=0) = 0$$

$$(c(x)^{-2}\partial_t^2 - \Delta)p^{PA}(x,t) = 0, \qquad f^{PA} = Mp^{PA}$$

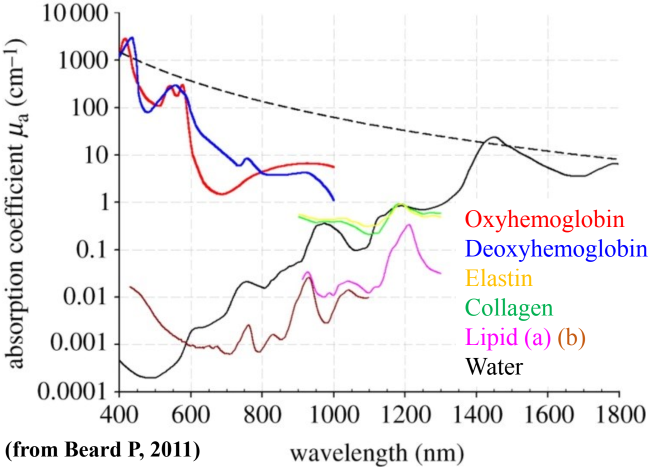
**Ultrasound Computed Tomography (USCT)**

$$(c(x)^{-2}\partial_t^2 - \Delta)p^{US}(x,t) = s(x,t), \qquad f^{US} = Mp^{US}$$

**Step-by-step inversion**

1. $f^{US} \to c$: acoustic parameter identification from boundary data.
2. $f^{PA} \to p_0$: acoustic initial value problem with boundary data.
3. $p_0 \to \mu_a$: optical parameter identification from internal data.

# Photoacoustic Imaging: Spectral Properties




**(from Beard P, 2011)**

- different wavelengths allow quantitative spectroscopic examinations.
- gap between oxygenated and deoxygenated blood.

# Lava Lamp: Image and Motion Estimation