On the application of spectral filters in a Fourier option pricing technique

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CWI Scientific Meeting
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Introduction

- PhD student since October 2010 (Prof.dr.ir. C.W. Oosterlee)
- Group Scientific Computing
- Computational finance

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  - Fourier option pricing techniques
  - Stochastic process for asset price
  - Expected value
  - Non-smooth functions
  - Spectral filters

CWI

Asset price S_t

Real world market

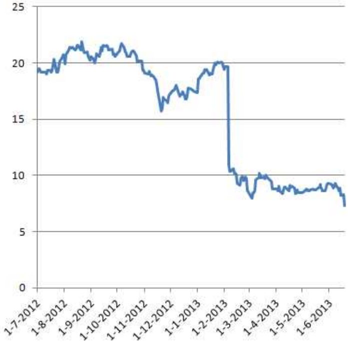

Figure 1: Asset price Imtech.

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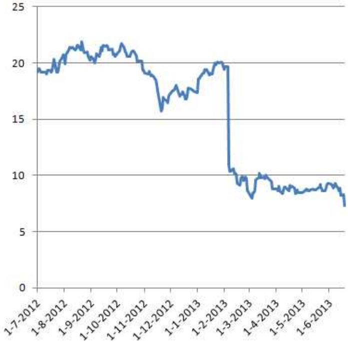

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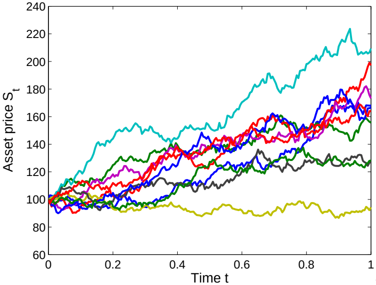
Model:

- Geometric Brownian motion

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$



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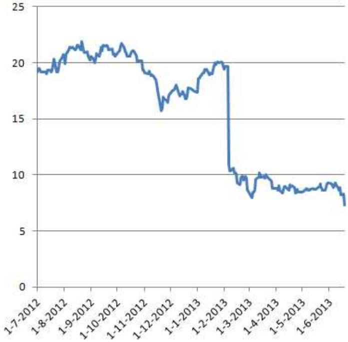

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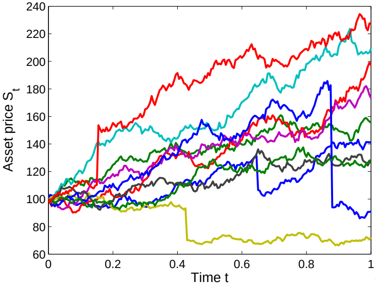

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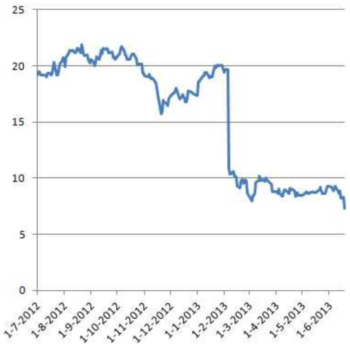

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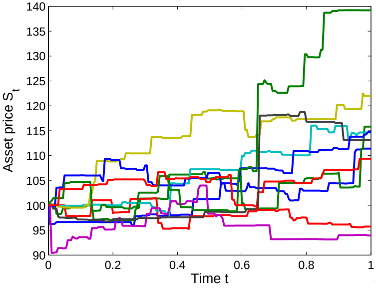
- Merton's jump diffusion
- Variance Gamma process



Option valuation techniques

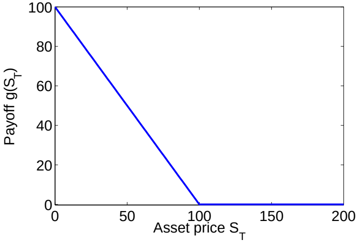
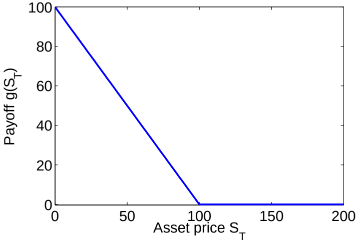


Figure 2: Put payoff function g.

Option value

$$v(t_0, S_0) = \mathbb{E}\left[g(S_T)\right]$$

Option valuation techniques



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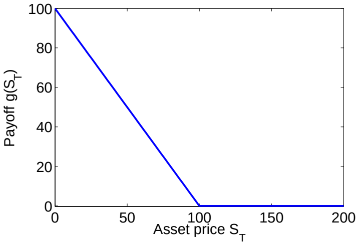

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- Monte Carlo simulation
- Numerical integration
- Fourier methods (Fourier transform density p is known)

Fourier-cosine series

The *Fourier-cosine series* of $f(y)$ is defined as

$$f(y) = {\sum_{n=0}^{\infty}}' \hat{f}_n \cos(ny), \quad y \in [0, \pi]$$

with Fourier-cosine coefficients

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The partial sum is given by

$$f_N(y) = {\sum_{n=0}^{N}}' \hat{f}_n \cos(ny).$$

Put option payoff $g(y)$

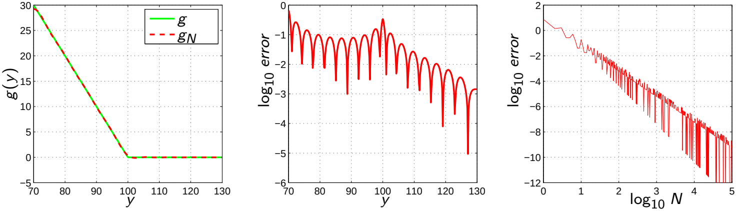
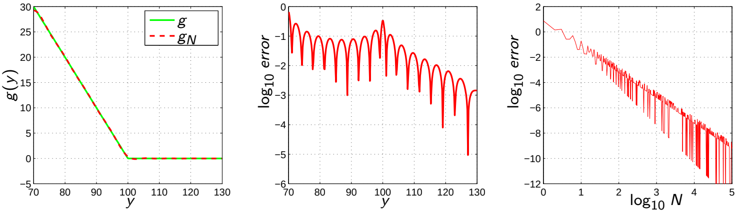


Figure 3: Put payoff and error ($N = 16$).

Put option payoff $g(y)$



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Option value

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Density function $p(y)$

Its Fourier-cosine series expansion reads

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\end{aligned}$$

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$\Rightarrow$ COS method

Variance Gamma asset price process

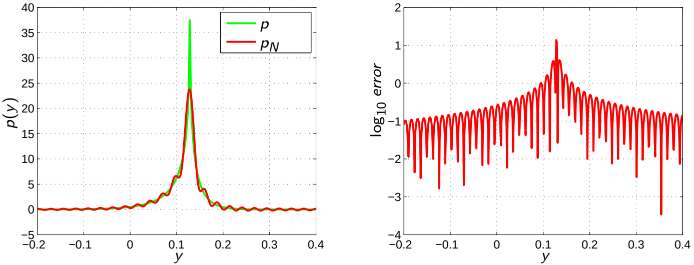


Figure 4: Density function VG process ($N = 128$).

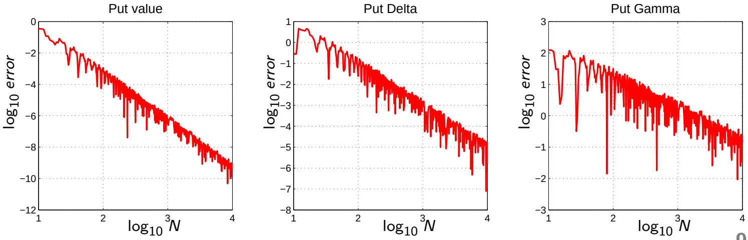
Put option

$$\text{Option value } v(t_0, S_0)$$

$$\text{Delta } \frac{\partial v(t_0, S_0)}{\partial S}$$

$$\text{Gamma } \frac{\partial^2 v(t_0, S_0)}{\partial S^2}$$

They are approximated by the COS method with N coefficients.



Spectral filter

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The *filtered* partial sum is defined by

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We can rewrite this as a *convolution* in the physical space:

$$f_{N,\sigma}(y) = \frac{2}{\pi}\int_0^{\pi} S(y-t)f(t)dt.$$

Put option payoff $g(y)$

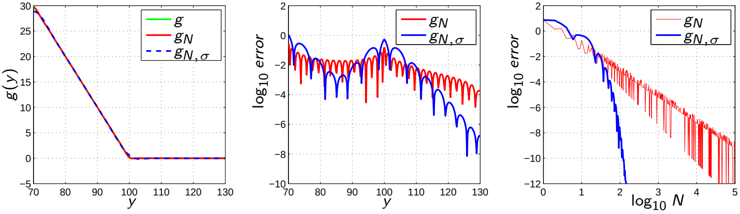

Figure 6: Put payoff and error, exponential filter ($q = 4$).

Variance Gamma asset price process

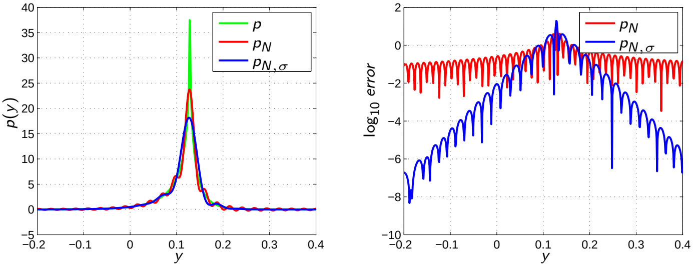

Figure 7: Density function VG process, exponential filter ($q = 4$).

Put option

Exponential filter with order q

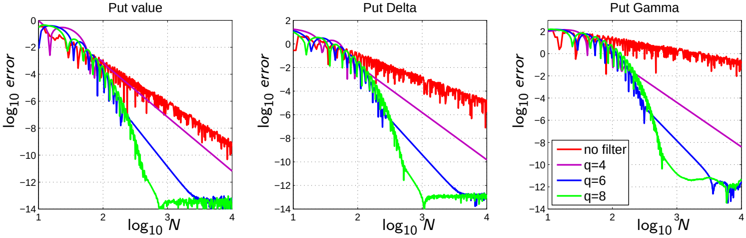

Figure 8: Error option value and the Greeks.

Summary and conclusion

- Stochastic process for asset price
- Fourier option pricing technique
- Slow convergence (Gibbs phenomenon)
- Spectral filters

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