Coordination Games on Graphs

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joint work with:

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Motivation

Model coordination in a local setting.

Examples: People choose...

- ... which mobile phone provider to use
- ... which social network (e.g., Facebook, Google+) to use
- ... to which location to go on holiday

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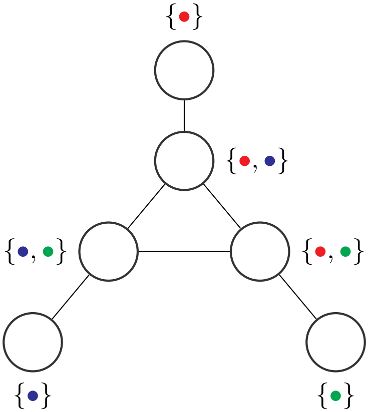
Coordination Games

Given:

- Undirected graph,
- finite set of colours,
- for every node v: a nonempty set of available colours (depending on v)

Every node chooses a colour; his payoff is the number of neighbours choosing the same colour.

Example



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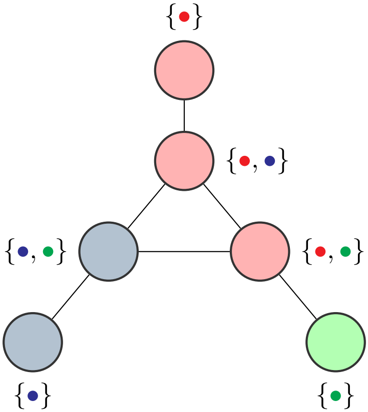

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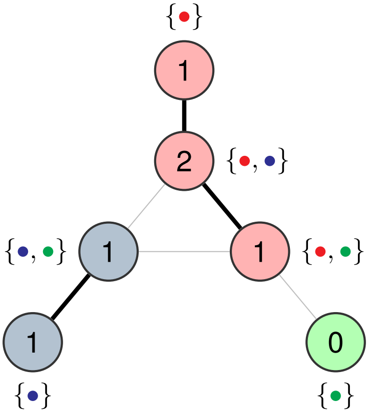

Equilibria

Definition

A Nash equilibrium is a joint strategy from which no player wants to unilaterally deviate.
A strong equilibrium is a joint strategy from which no *group* of players wants to unilaterally deviate.

Strong equilibrium $\Rightarrow$ Nash equilibrium.
"$\Leftarrow$" does not hold:

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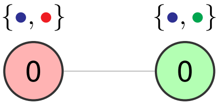

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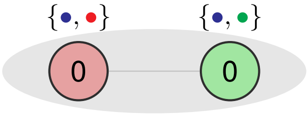

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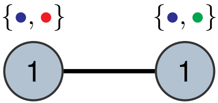

Questions

Regarding equilibria:

- Existence?
- Convergence of improving move sequences?
- Quality?
- Computation?

Regarding the model:

- Extensions? (Edge weights / Choosing multiple colours)

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Nash equilibria always exist. If players deviate "one after the other" while respectively improving their payoff, a Nash equilibrium is reached in polynomial time.

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Strong equilibria may not exist. If groups of players deviate "one after the other" while respectively improving their payoff, this process may cycle.

However: We can guarantee existence of strong equilibria in certain graph classes (among them: trees, complete graphs).

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Equilibria: Quality

Worst case analysis with respect to *social welfare* (= sum of players' payoffs):

Nash equilibria can be arbitrarily bad compared to the optimum:

SW = 0, OPT = 2

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Strong equilibria have at least half of the optimal social welfare.

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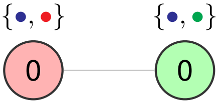

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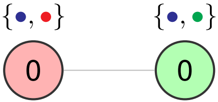

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Nash equilibria are efficiently computable.

Proof idea: Use best-response dynamics.

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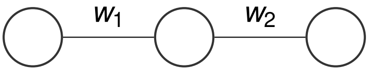
Edge-weighted model.



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Players are allowed to choose multiple colors; payoff is the number of neighbours sharing at least one color.

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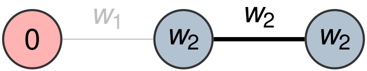

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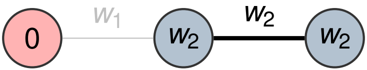

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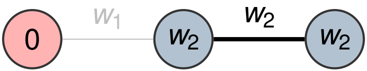

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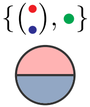

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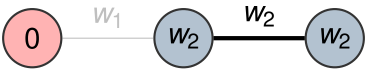

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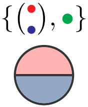

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Conclusion

- Introduced coordination games on graphs.
- Analyzed equilibria: Existence, convergence, quality, computability.
- Generalized some results to broader models.

The end.

Thank you!