# Uncertainty Quantification in Computational Fluid Dynamics


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CWI Scientific Meeting
Thursday, April 4, 13.00 - 14.00
Room Z009 (Euler)

# Fluid Dynamics

## Flow of fluids in contact with objects

*Air over airplane wings*


*Wind around turbines*

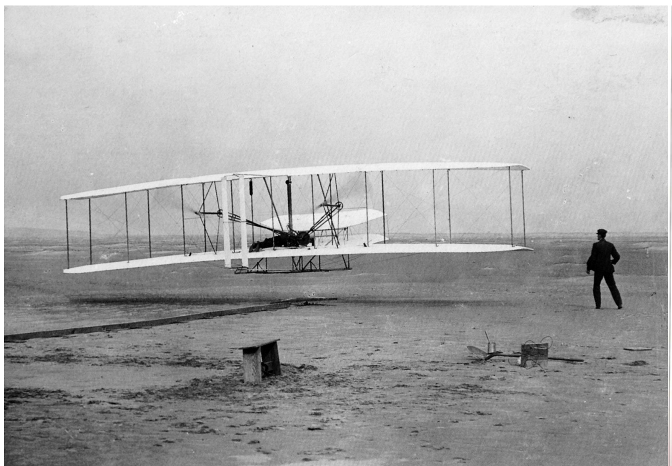
*Oil in reservoirs and pipes*

*Blood in veins*

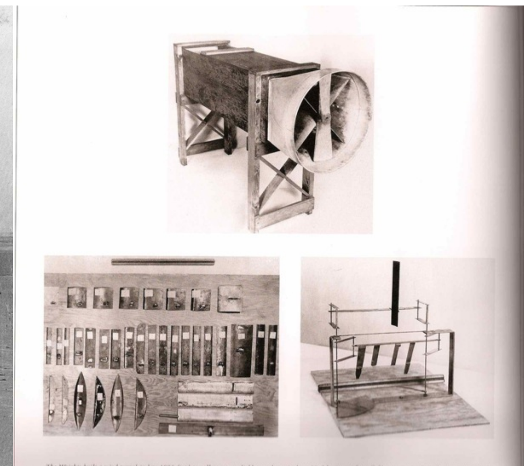
# Experimental Fluid Dynamics

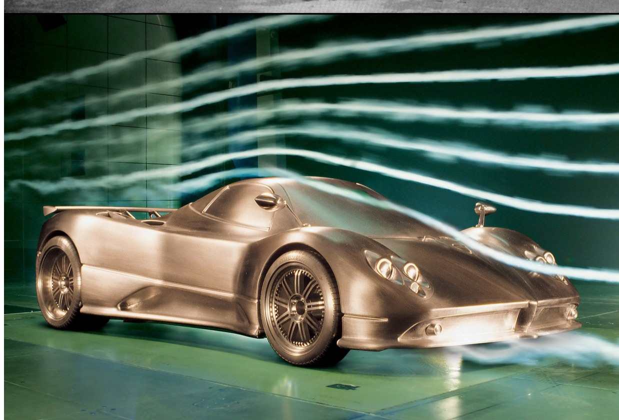
**Testing stationary objects in moving air flow is expensive**

*First flight by Wright brothers*



*Wright wind tunnel*





*Smoke visualization around car*

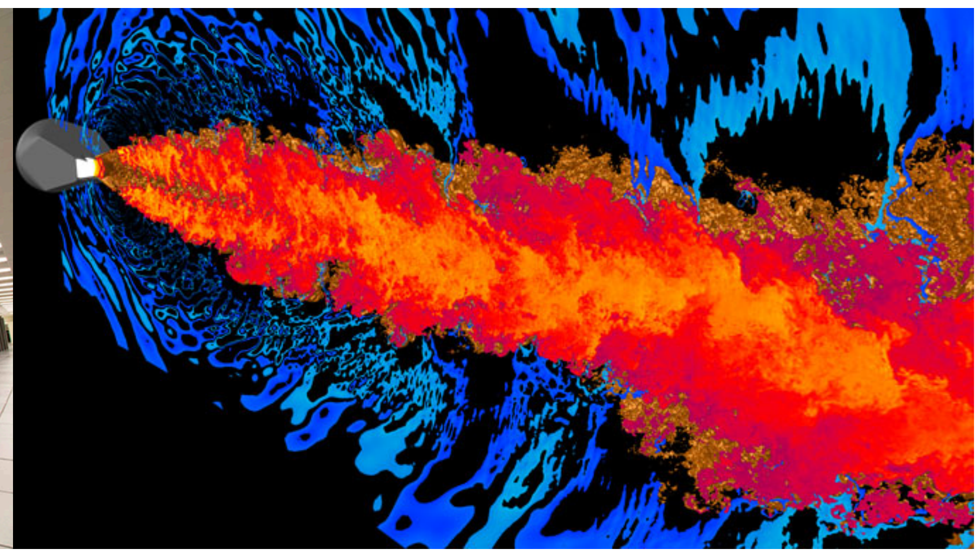
*Largest wind tunnel at NASA Ames*

# Computational Fluid Dynamics (CFD)

**Simulating fluid flow by solving equations on computers**

*Sequoia supercomputer at Lawrence Livermore National Laboratories*


*Jet engine exhaust simulation on 1 million computer processors*

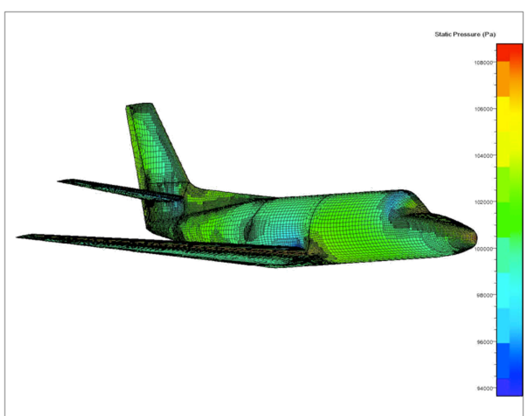


# CFD process

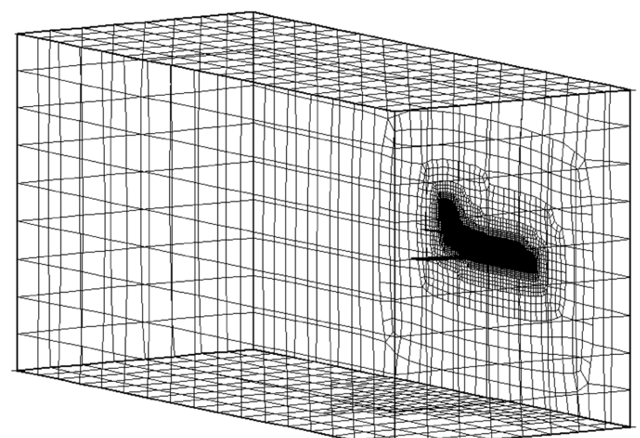
## Computer implementation of a discretized geometry

*Real-world object*


*Geometrical description*




*Computer solution*



*Spatial discretization*

# Navier-Stokes equations

**Mathematical model of the flow physics**

**Physical laws of classical mechanics:**

- Conservation of mass
- Conservation of energy
- Conservation of momentum:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\rho g_x,$$

*time* *inertia terms* *pressure* *viscous effects* *gravity*

A system of nonlinear, time-dependent, partial differential equations (PDEs)

# Robust discretizations for discontinuous solutions

**Computer algorithms based on mathematical methods from numerical analysis:**

- Finite difference
- Finite element
- Finite volume

*Example of temporal discretization*

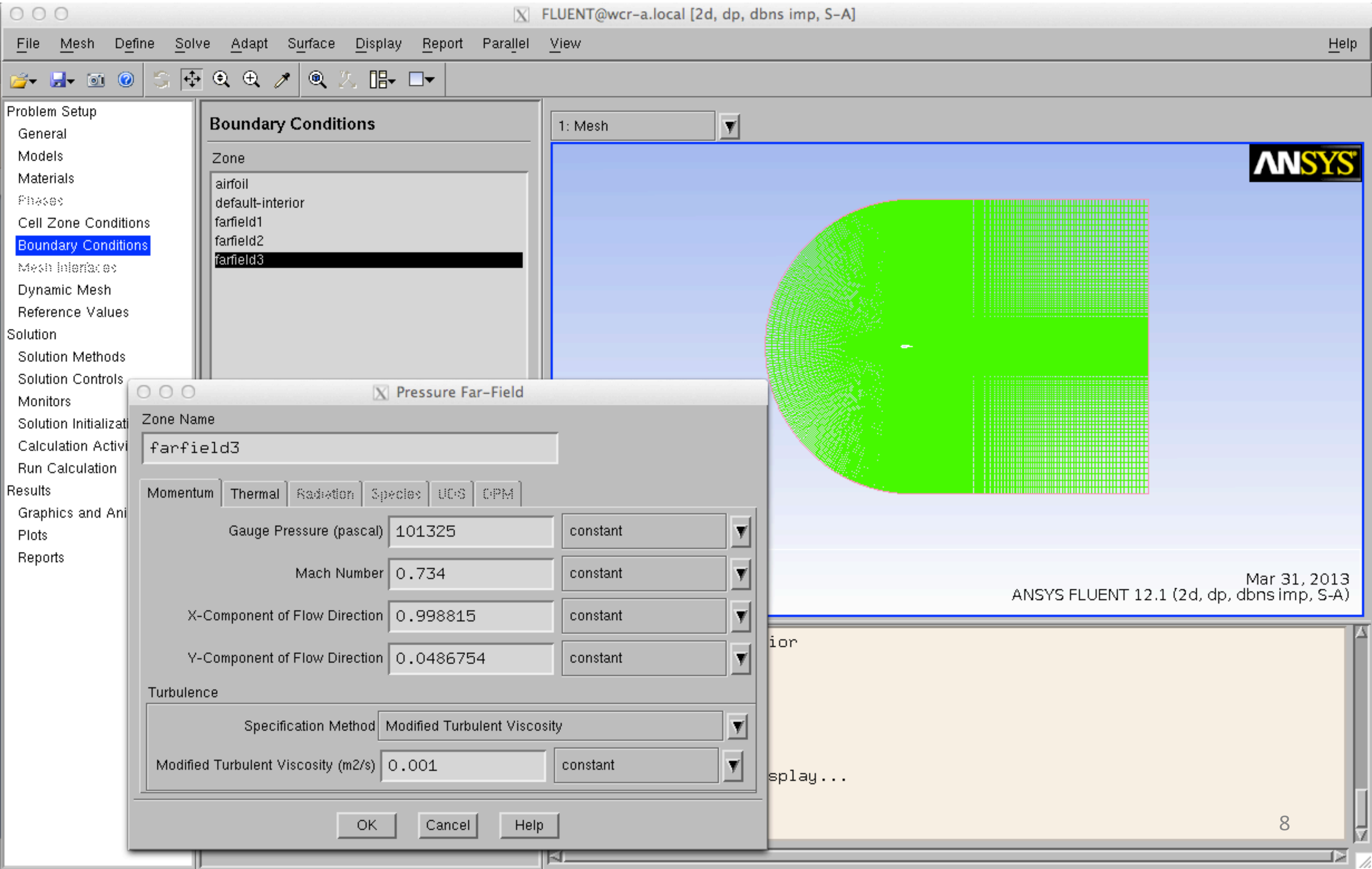
$$\frac{\partial u}{\partial t} = \frac{u_i - u_{i-1}}{\Delta t}$$

**Robustness concepts:**

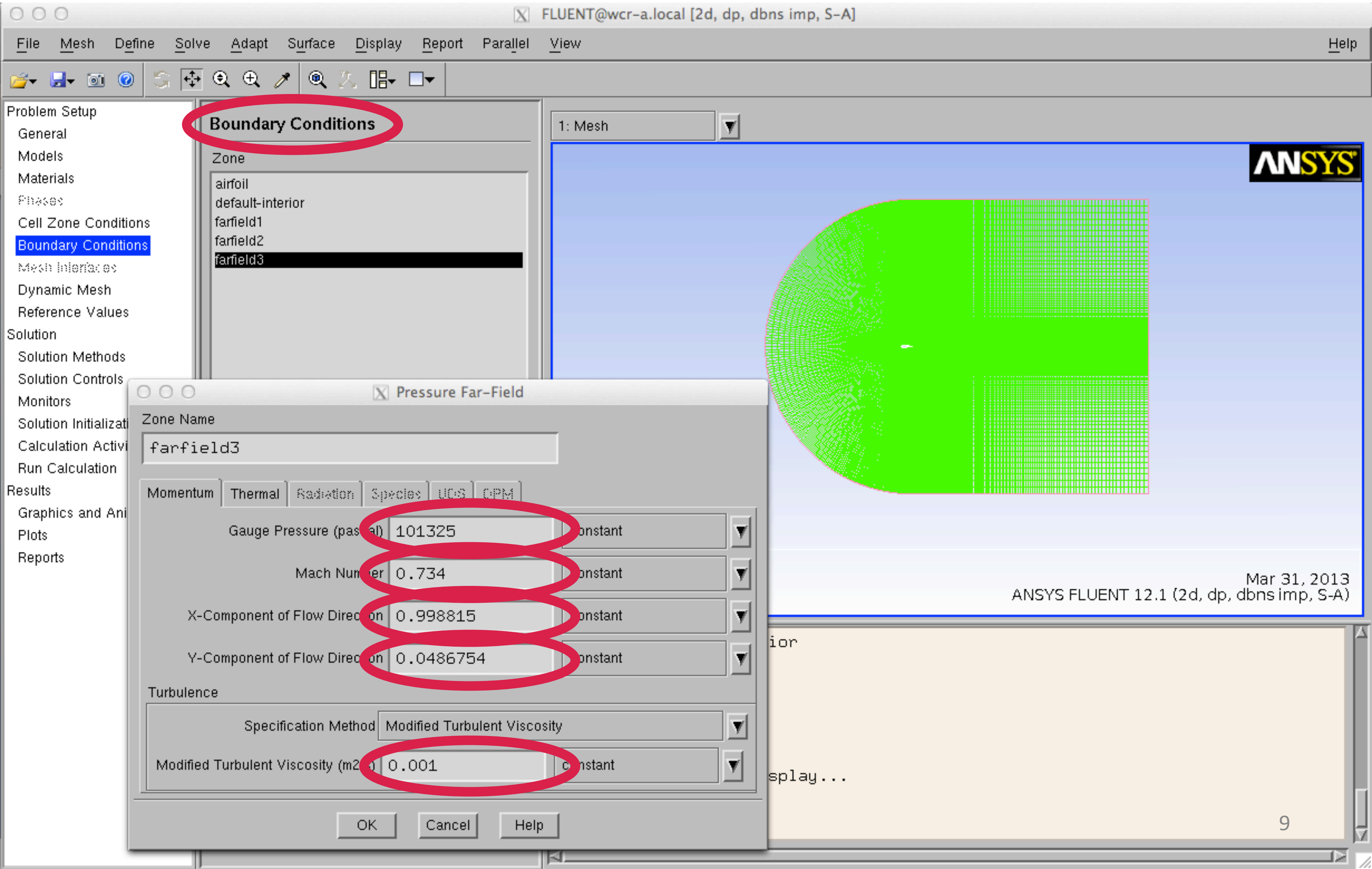
- Local Extremum Diminishing (LED)
- Total Variation Diminishing (TVD)
- Monotonicity Preserving (MP)
- Essentially Non-Oscillatory (ENO)
- Subcell Resolution (SR)

*Transonic shock wave at a jet fighter causes water vapor condensation*

# Graphical user interface of commercial software



# Specifying boundary conditions



# Input parameters not exactly known

**Sources of uncertainty:**

- Atmospheric fluctuating wind
- Manufacturing tolerances on the geometry
- Wear and tear of surface roughness
- Insufficient measurements

**Uncertain boundary and initial conditions, and model parameters:**

- Fluid velocity *u*, air density *ρ*, ambient pressure *p*
- viscosity *μ*

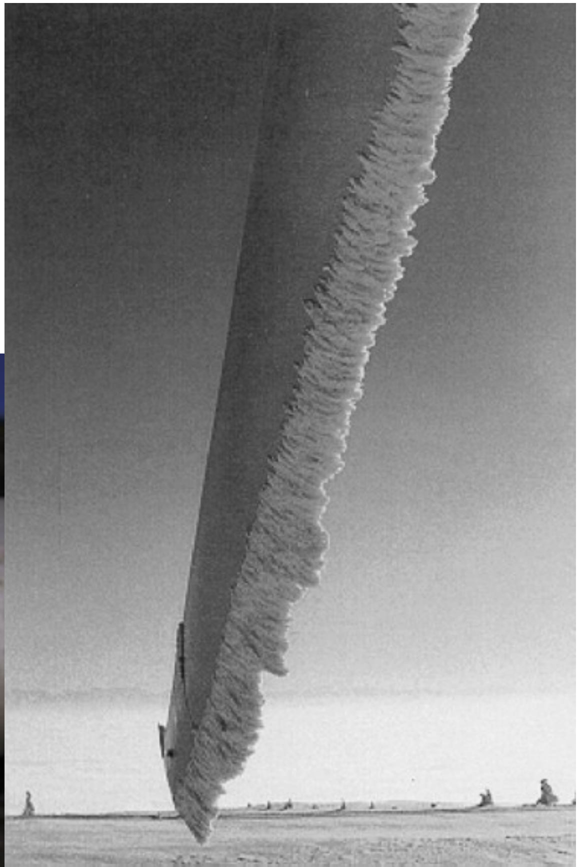
$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\rho g_x,$$

# Wind turbine performance degenerates faster than expected

**Surface roughness wear and tear of wind turbine blades:**

- Ice formation
- Sand blasting
- Insect contamination
- Bird and bat impact

*Ice formation*



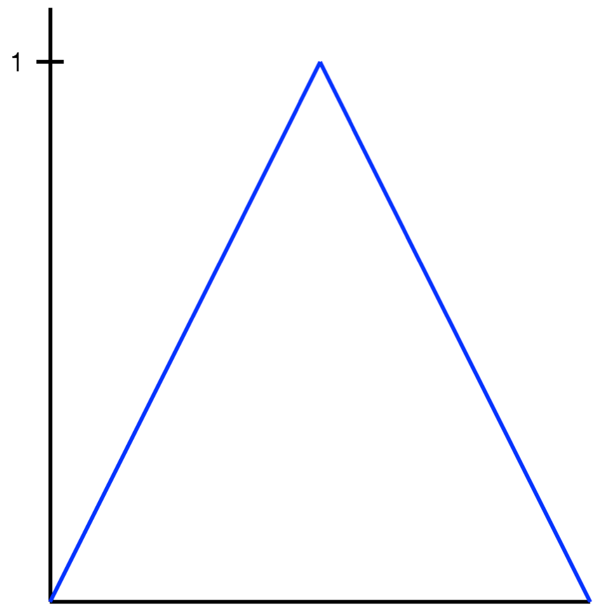
*Insect contamination*

# Description of uncertainty

**Interval analysis**

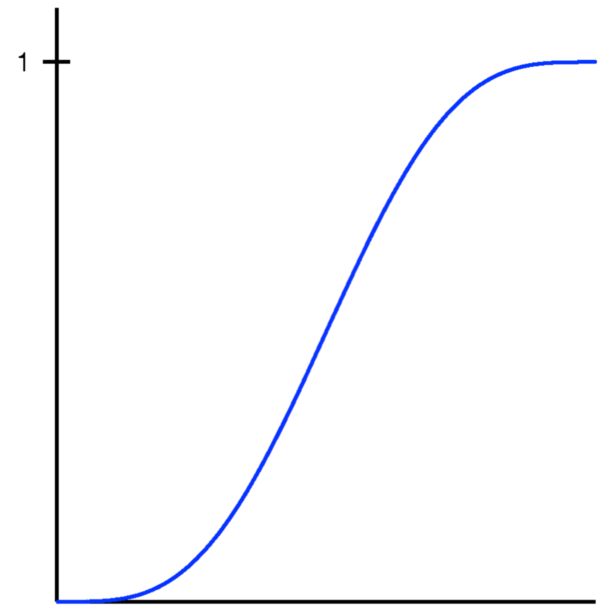
*Interval*

**Fuzzy logic**




*Membership function*

**Probability theory**




*Cumulative distribution*

Increasingly quantitative

# Uncertainty quantification: performing multiple simulations

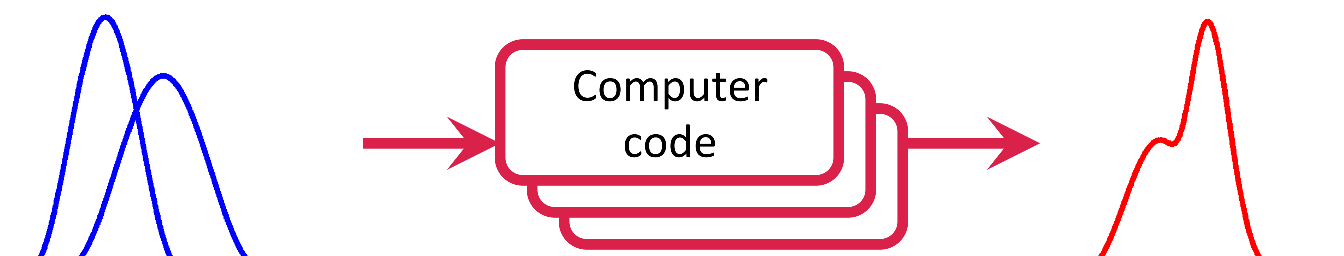
## Conventional CFD simulations:

- Model parameters
- Initial conditions
- Boundary conditions


## Uncertainty quantification in CFD:

*Pre-processing*

*Post-processing*

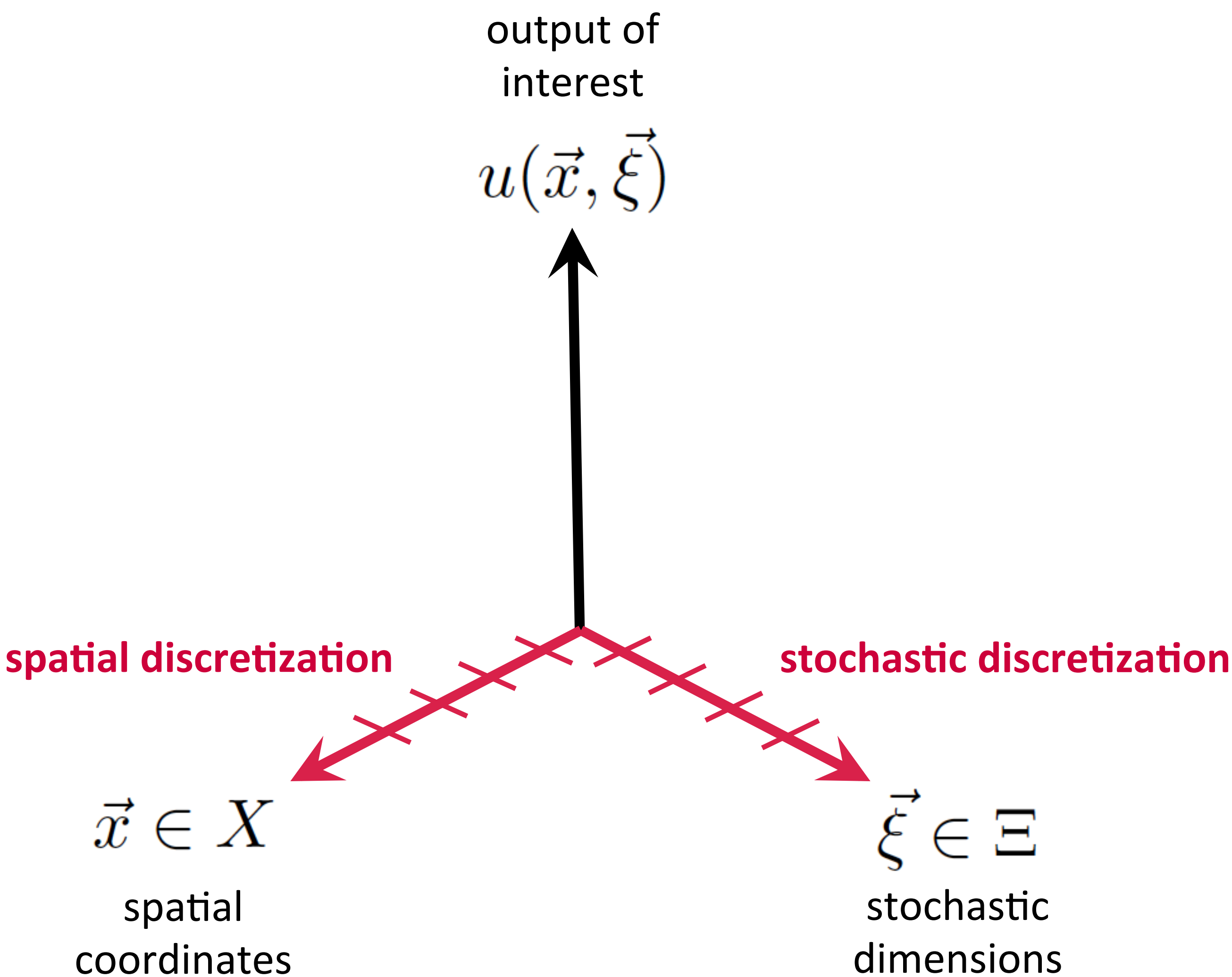



Input probability distributions

Existing black-box CFD simulation software

Output quantity of interest

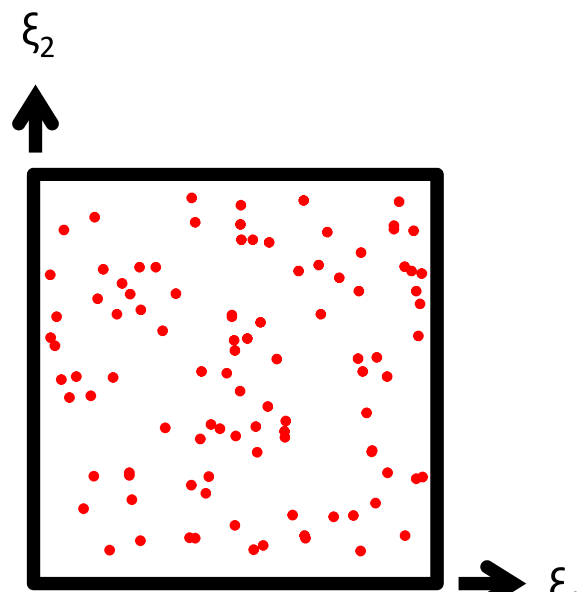
# Discretization of spatial and stochastic dimensions



# Monte Carlo simulation too expensive for CFD

**Monte Carlo simulation:**

- Pre-processing: Random sampling points
- Uncertainty propagation: Many CFD simulations
- Post-processing: Ensemble statistics

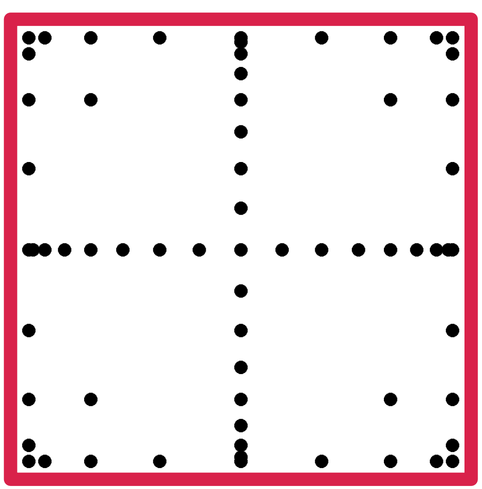



*Random sampling*

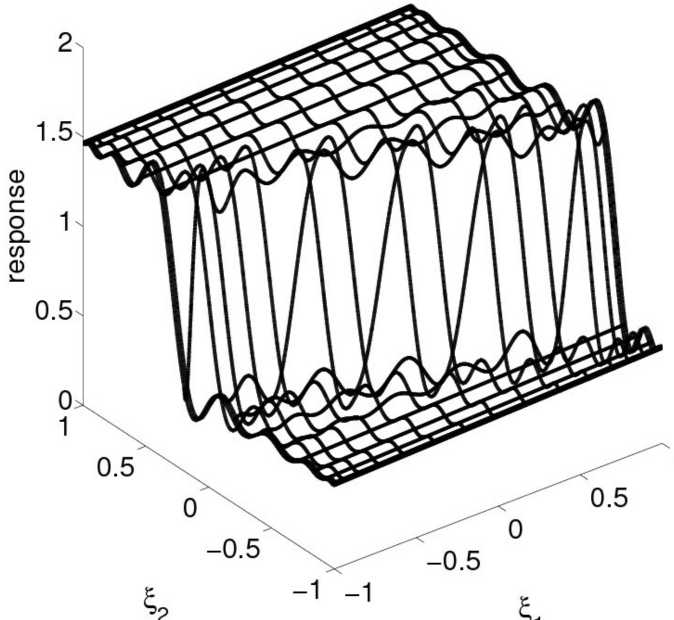
# Reduce number of expensive CFD simulations

**Stochastic Collocation with sparse grids:**

- Pre-processing: Choice of quadrature points
- Uncertainty propagation: Small number of CFD simulations
- Post-processing: Global polynomial interpolation



*Deterministic sampling*




*Interpolation overshoots at discontinuities*

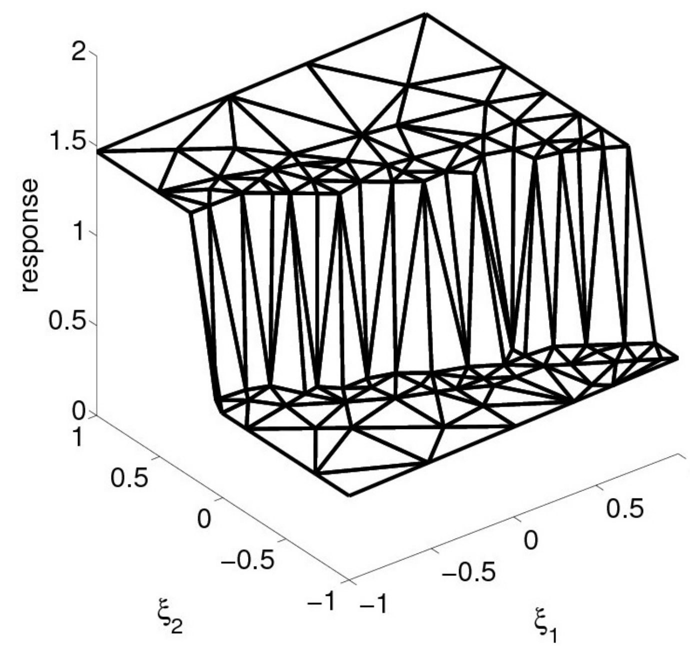
# Same robustness as in spatial discretization

**Simplex Stochastic Collocation:**

- Pre-processing: Minimal number of sampling points
- Uncertainty propagation: Solution-adaptive refinement
- Post-processing: Robust piecewise interpolation

**Robustness concepts:**

- Local Extremum Diminishing (LED)
- Total Variation Diminishing (TVD)
- Monotonicity Preserving (MP)
- Essentially Non-Oscillatory (ENO)
- Subcell Resolution (SR)



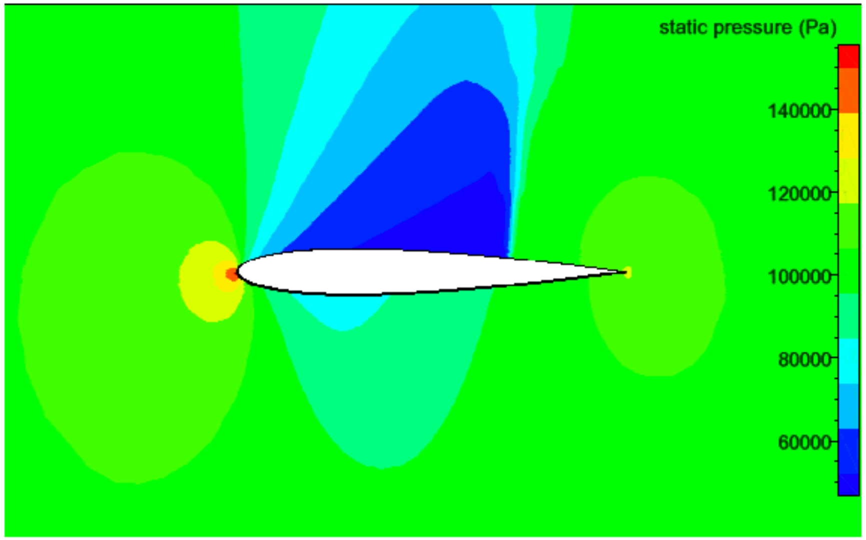

*Robust adaptive approximation*

# Computational fluid dynamics example

**Transonic flow over wing cross-section:**

- Mach number 0.8
- Random flow angle
- Beta distribution on [1º,3º]

*Static fluid pressure for 2º*

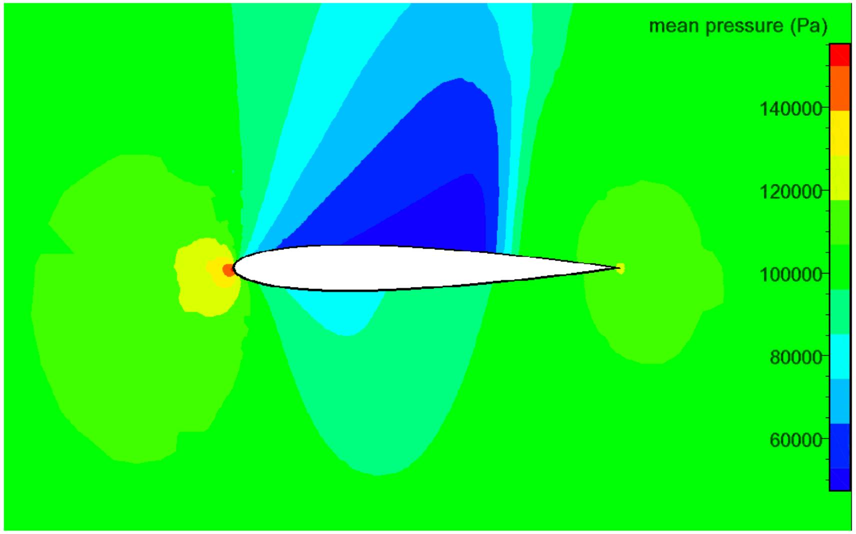


# Shock wave location sensitive for uncertain flow angle

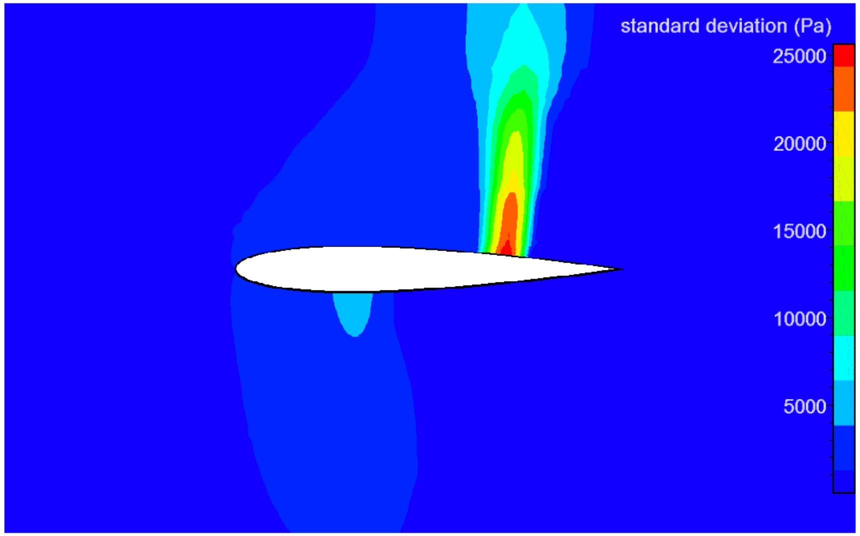
**Uncertainty quantification findings:**

- Mean shock wave smeared
- Local maximum in standard deviation

*Mean pressure*




*Standard deviation*

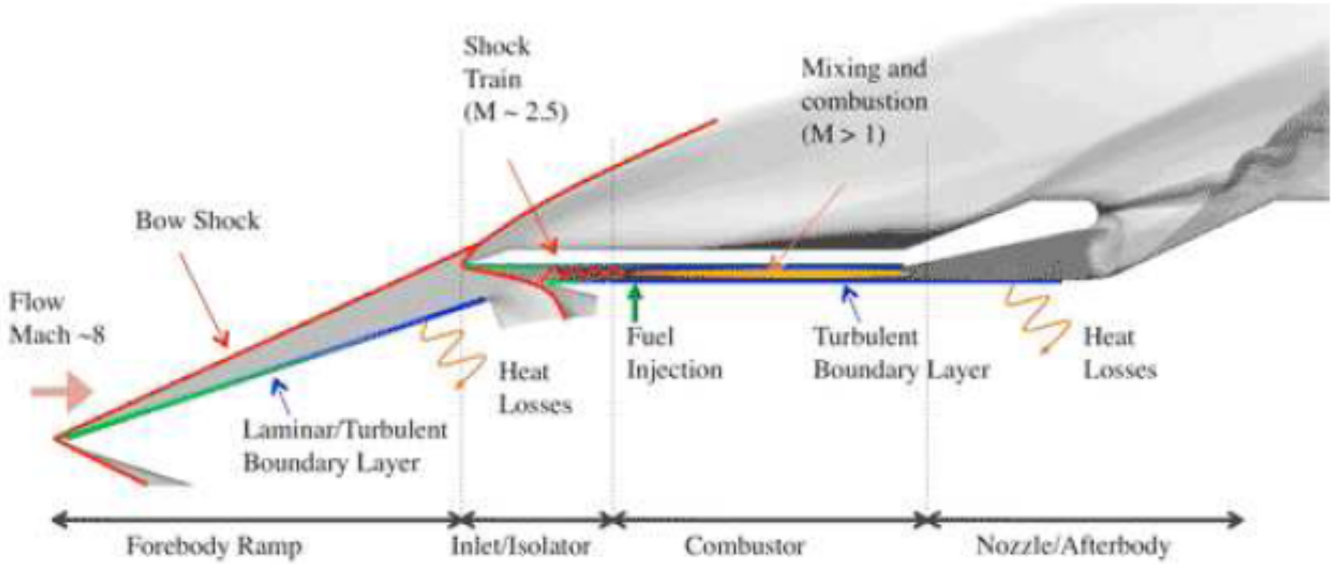


# Combustion simulations with parameter and model uncertainties

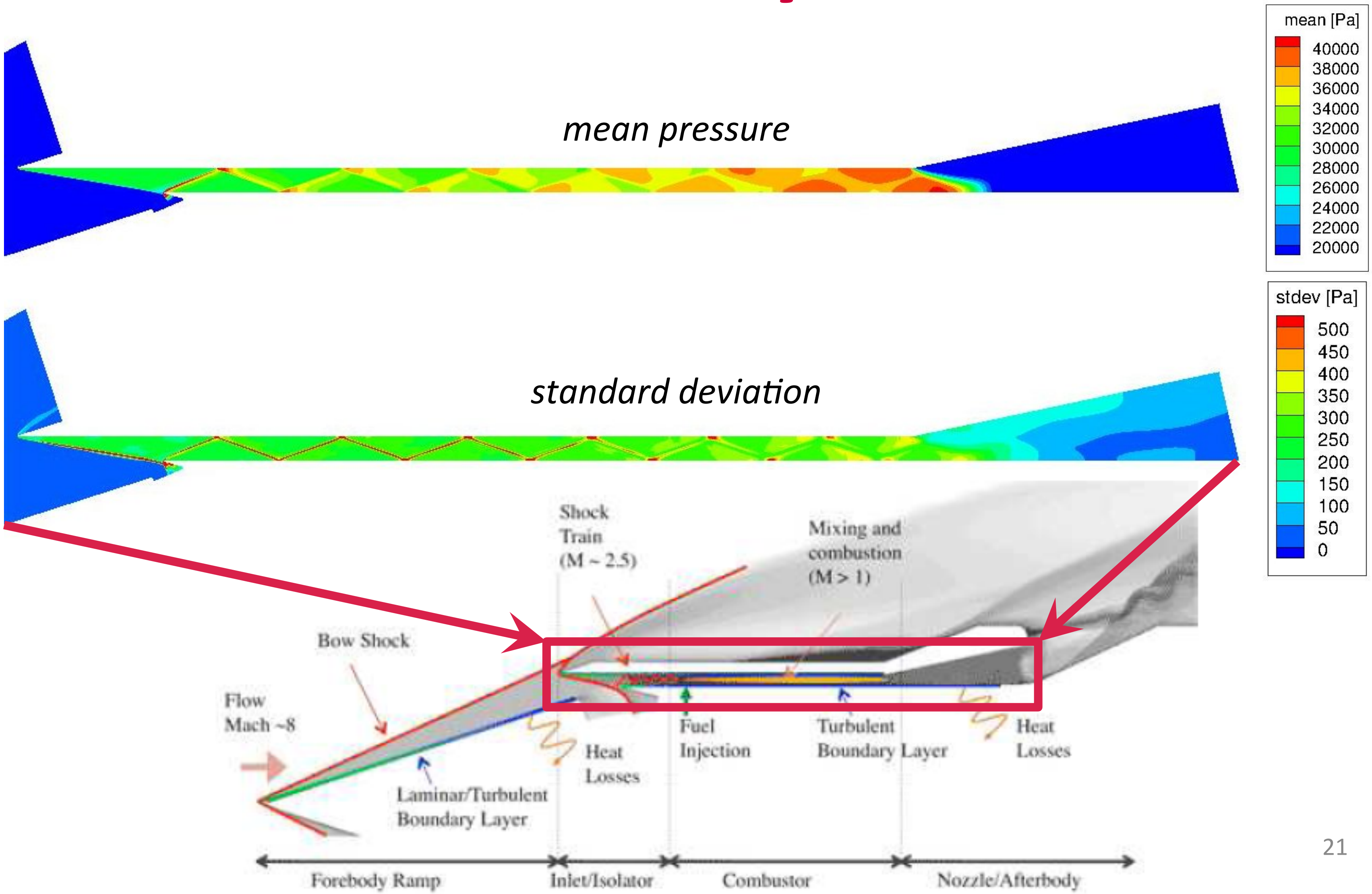
**Supersonic combustion engine for hypersonic flight:**

- Parameter uncertainty: Uncertainty boundary conditions
- Model uncertainty: Turbulence model uncertainty
- Numerical error: Spatial discretization

*Hyshot II scramjet flight experiment*



# Uncertain flow angle and flight altitude boundary conditions

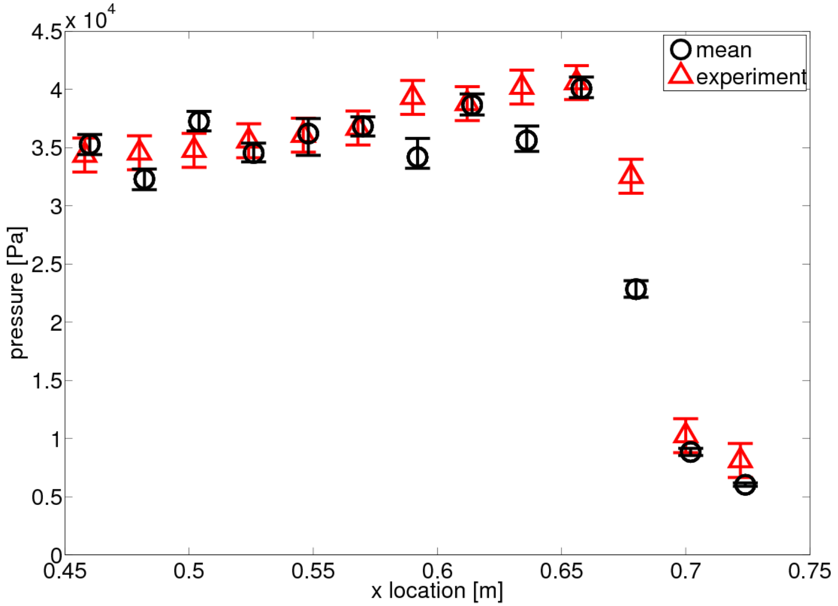


# Model uncertainty dominates turbulent flow simulations

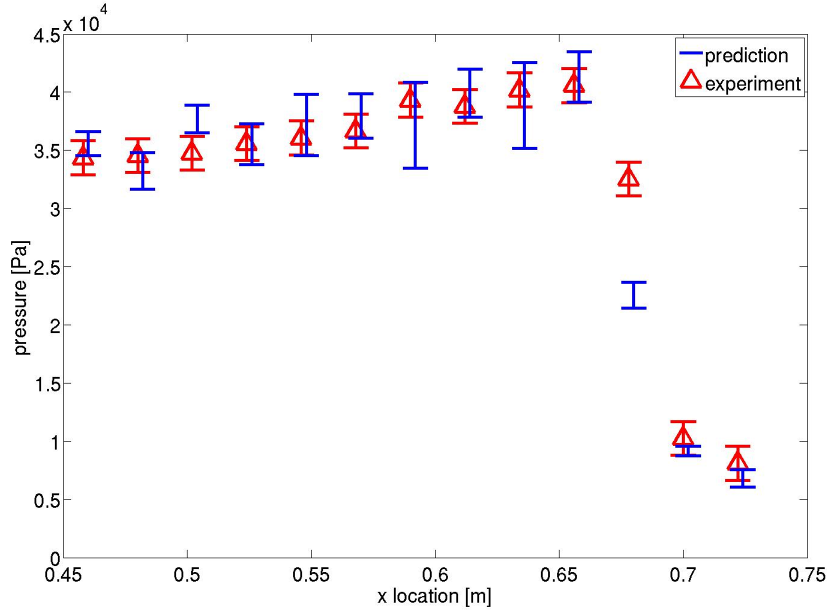
**By an order of magnitude estimated using two different turbulence models**

*Wall pressure simulations, measurements, and uncertainty bars*

*Parameter uncertainty and numerical error*




*Parameter and model uncertainty*



# Wind turbine robust design optimization under uncertainty

50kW AOC 15/50 wind turbine

Acqua Spruzza site in Italy

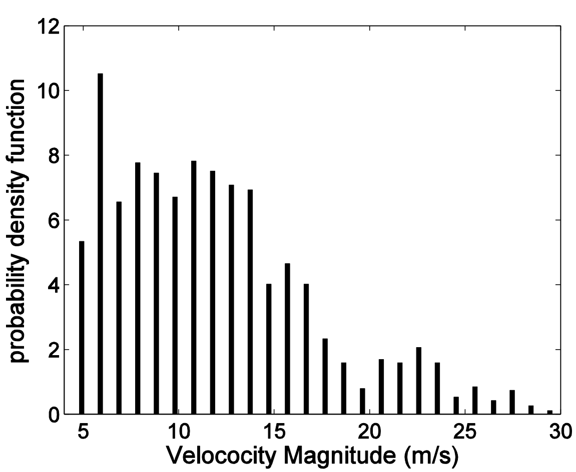
# Uncertain wind conditions given by measured histograms

**Uncertain conditions:**

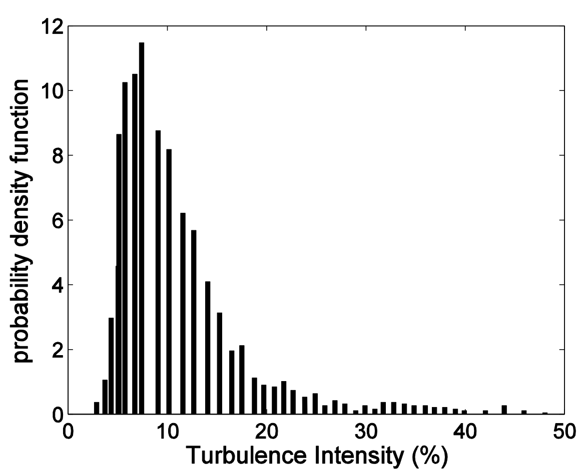
- Wind conditions
- Manufacturing tolerances
- Insect contamination

Input probability density functions for the uncertain wind conditions

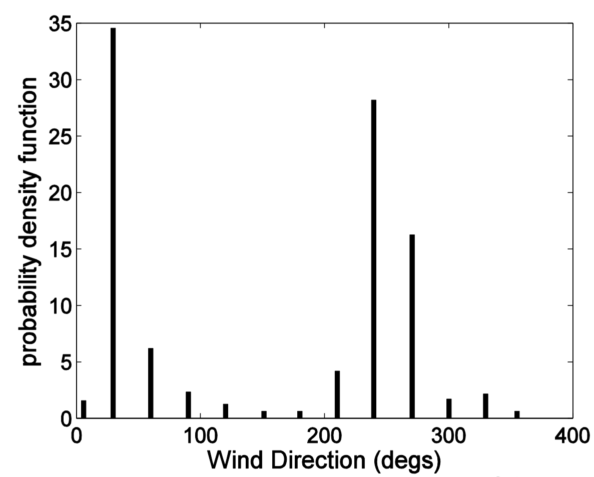
Magnitude




Turbulence intensity

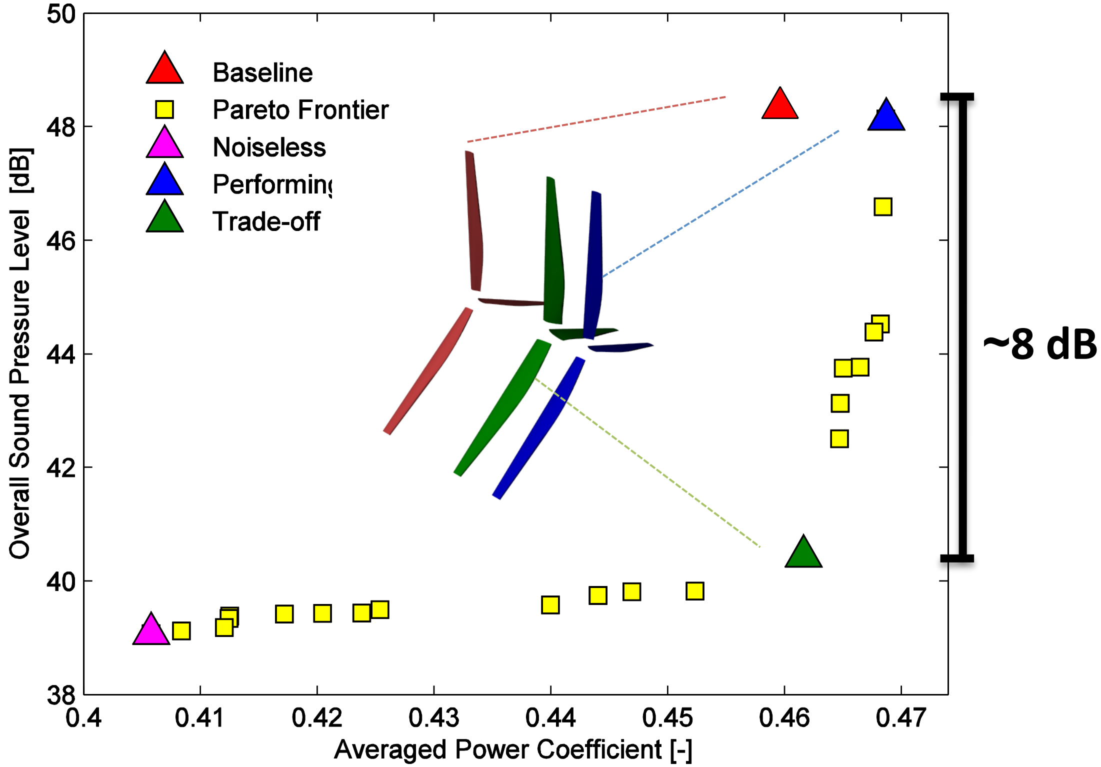



Direction



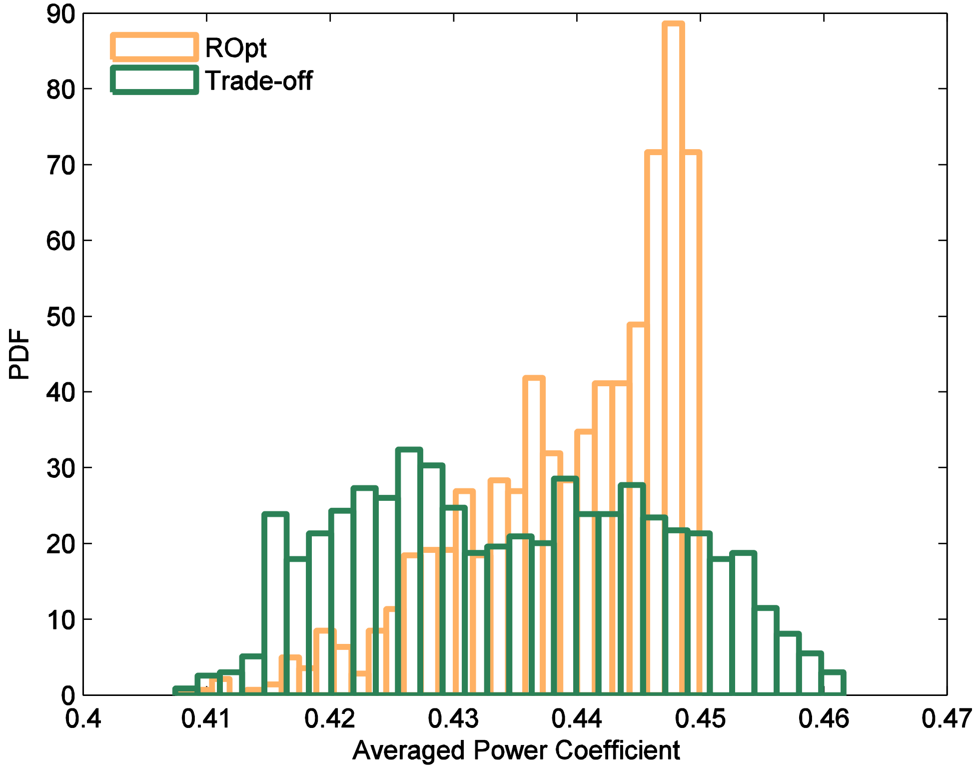
# Deterministic optimization: Significant reduction in noise

Deterministic rank-one Pareto front optimization using genetic algorithm



# Optimization under uncertainty: Peak probability at maximum output

Density of power coefficient for optimum and trade-off



# Conclusions

**Computational Fluid Dynamics:**

- Numerically solving the Navier-Stokes equations
- Robust numerical methods for discontinuities

**Uncertainty Quantification:**

- Robust discretization methods in probability space
- Parameter uncertainty, model uncertainty, numerical error
- Robust performance by optimization under uncertainty

# Questions?

**Thank you**