Achievable Performance of Blind Scheduling Policies

(How to work through your to-do list?)

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To-do list

- Colleagues arrive according to (random) process; rate $\lambda$
- Every colleague gives a task; mean size $\mathbb{E}[T]$
- $\rho := \lambda \mathbb{E}[T] < 1$
- Pre-empting is allowed
- No deadlines
- Minimise average waiting time $\mathbb{E}[W]$ for colleague
To-do list

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To-do list

- Need to make a schedule
- Depends on size (duration) of tasks
  - Omniscient scheduler (SRPT)
  - Blind scheduler (FCFS, LCFS, RMLF)
- How is performance affected by knowledge?
Two CWI departments: N&O and ST

- N&O: competitive analysis
- ST: stochastic analysis

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<th>Competitive analysis</th>
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Two CWI departments: N&O and ST

\[ \mathbb{E}[W_{FCFS}] = \frac{\rho c}{1-\rho} \]

Arbitrarily worse than optimal
Two CWI departments: N&O and ST

- $\mathbb{E}[W_{\text{FCFS}}] = \frac{\rho c}{1-\rho}$
- Arbitrarily worse than optimal
The oblivious scheduler

- Input: sizes known
- Scheduling policy: Shortest Remaining Processing Time (SRPT)
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Performance: optimal

Red: $\mathbb{E}[W_{\text{FCFS}}] \approx \frac{1}{1-\rho}$
Blue: $\mathbb{E}[W_{\text{SRPT}}] \approx \frac{1}{1-\rho} \log \frac{1}{1-\rho}$

Task sizes: Exponential(1)

$F(x) = 1 - e^{-x}$
The oblivious scheduler

Performance: optimal

Red: $\mathbb{E}[W_{FCFS}] \approx \frac{1}{1-\rho}$

Blue: $\mathbb{E}[W_{SRPT}] \approx \frac{1}{\sqrt{1-\rho}}$

Task sizes: Pareto(3)

$F(x) = 1 - x^{-3}$
The blind scheduler

- SRPT: short tasks first
- Task sizes unknown
- Randomised Multilevel Feedback scheduling policy
The blind scheduler

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- Scheduling policy: Randomised Multilevel Feedback (RMLF)
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- Scheduling policy: **Randomised Multilevel Feedback (RMLF)**
The blind scheduler

- Input: sizes unknown
- Scheduling policy: Randomised Multilevel Feedback (RMLF)
- Our theorem:

\[ \mathbb{E}[W_{RMLF}] \leq c \log \left( \frac{1}{1 - \rho} \right) \mathbb{E}[W_{SRPT}] \]
The blind scheduler

Performance: at most factor $c \log \left( \frac{1}{1-\rho} \right)$ from optimal

Red: $\mathbb{E}[W_{FCFS}] \approx \frac{1}{1-\rho}$

Green: $\mathbb{E}[W_{RMLF}] \approx \frac{1}{1-\rho}$

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Task sizes: Exponential(1)

$F(x) = 1 - e^{-x}$
The blind scheduler

Performance: at most factor $c \log\left(\frac{1}{1-\rho}\right)$ from optimal

Red: $\mathbb{E}[W_{FCFS}] 
\approx \frac{1}{1-\rho}$

Green: $\mathbb{E}[W_{RMLF}] 
\approx \frac{1}{\sqrt{1-\rho}} \log \frac{1}{1-\rho}$

Blue: $\mathbb{E}[W_{SRPT}] 
\approx \frac{1}{\sqrt{1-\rho}}$

Task sizes: Pareto(3)
$F(x) = 1 - x^{-3}$
Takeaway:

- Do not apply FCFS when you’re busy
- N&O: RMLF is close to optimal scheduling policy
- ST: bounds on waiting time under RMLF
- Proof needs techniques from both

To do:

- Extend concept to other models
Thank you!

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Achievable Performance of Blind Policies in Heavy Traffic

http://arxiv.org/abs/1512.07771

