

Communication using quantum entanglement: benefits and limitations



Teresa Piovesan

CWI Scientific Meeting
Amsterdam, 13 May 2016



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JAN 06, 2014



24°C SAO PAULO



E-MAIL



CONTACT



MUSIC



VIDEO



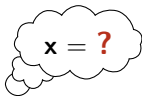
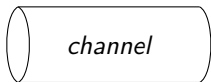
CHAT



SEARCH

Channel coding

$$\mathbf{x} \in \{0, 1\}^m$$

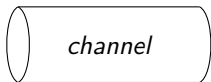


Channel coding

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Alice

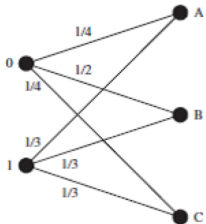
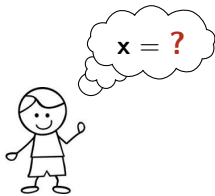
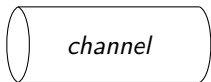


Bob

$\mathbf{x} = ?$

Channel coding

$$\mathbf{x} \in \{0, 1\}^m$$



Channel coding

$$\mathbf{x} \in \{0, 1\}^m$$



S

encoding

$$\{0, 1\}^m \rightarrow S$$



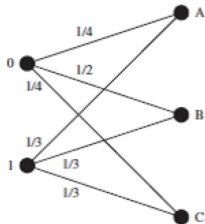
V

decoding $\rightarrow \mathbf{y}$

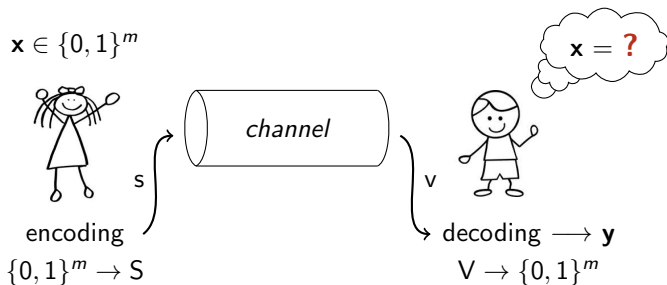


$$\mathbf{x} = ?$$

$$V \rightarrow \{0, 1\}^m$$



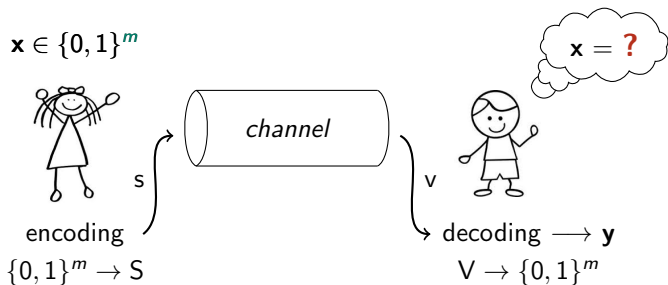
Channel coding



GOAL:

- $y = x$ with high probability

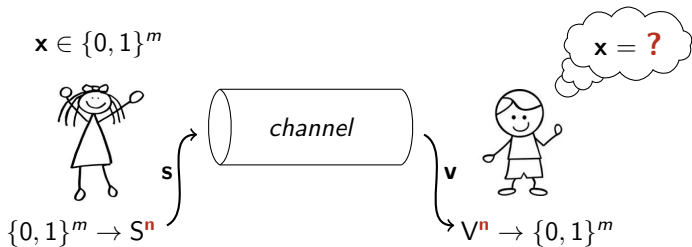
Channel coding



GOAL:

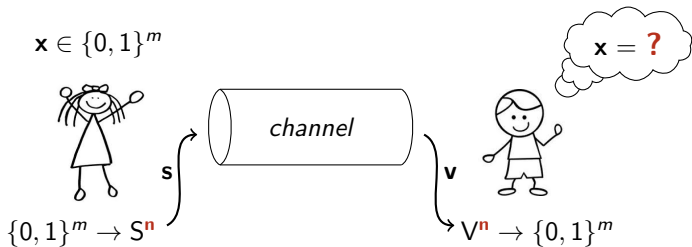
- $\mathbf{y} = \mathbf{x}$ with high probability
- maximize m

Block coding



- Encoding \mathbf{x} into a **sequence** of channel inputs can be more efficient

Block coding

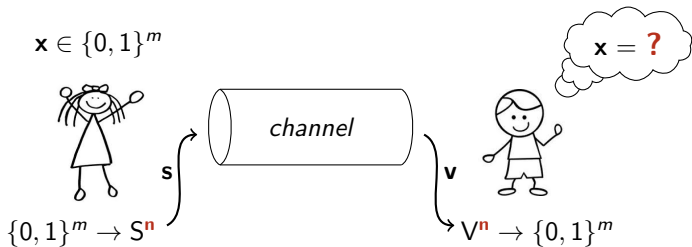


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Shannon (1948):

Number of bits that can be *theoretically* transmitted per channel use

Block coding



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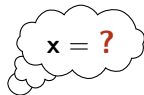
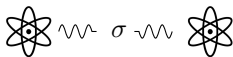
Shannon (1948):

Number of bits that can be *theoretically* transmitted per channel use

- error probability goes to zero as $n \rightarrow \infty$

Channel coding with *entanglement*

$$\mathbf{x} \in \{0, 1\}^m$$



Quantum entanglement

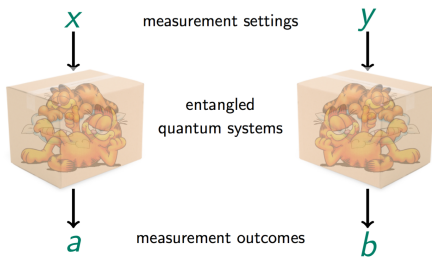
Entanglement is one of the most striking features of quantum mechanics

Measurements on *entangled* quantum systems can give outcomes that are **non-classically correlated**

Quantum entanglement

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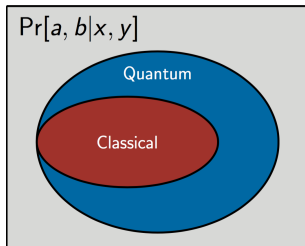
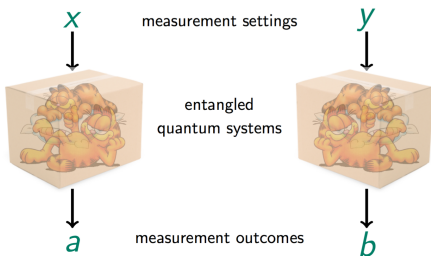
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Quantum entanglement

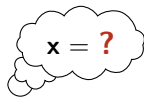
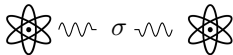
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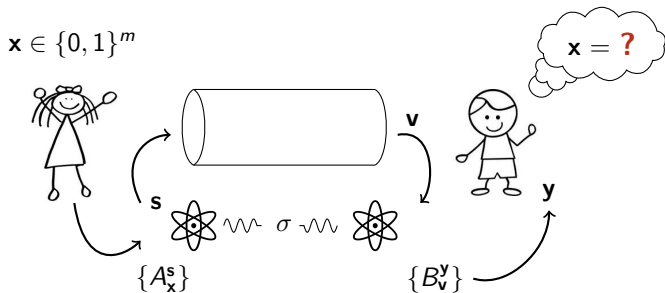


Channel coding with *entanglement*

$$\mathbf{x} \in \{0, 1\}^m$$

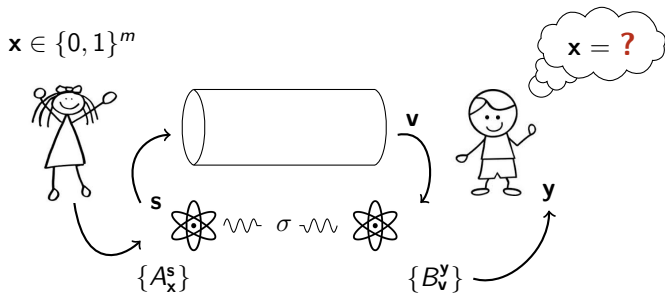


Channel coding with *entanglement*



Can entanglement be beneficial?

Channel coding with *entanglement*



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Depends

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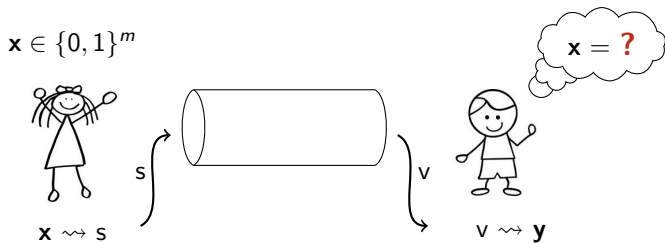


- Probability of error **equal to zero**

Positive results both for *finite* and *asymptotic* number of uses of the channel

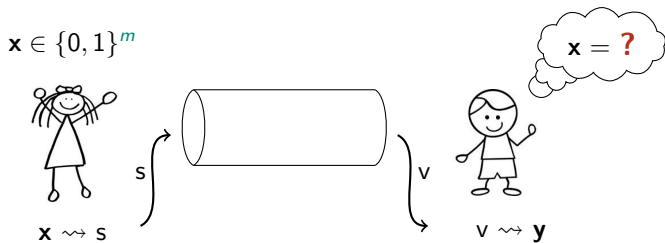


Zero-error channel coding

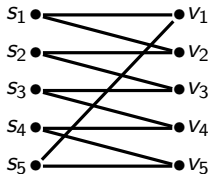


GOAL: $\mathbf{y} = \mathbf{x}$ with **zero** probability of error and **maximize** m

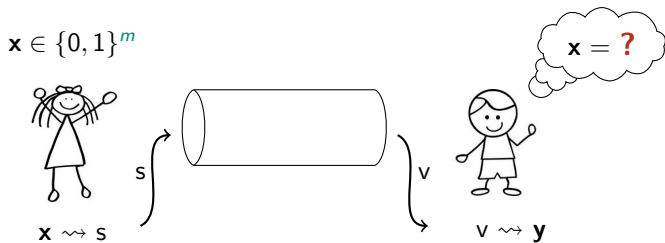
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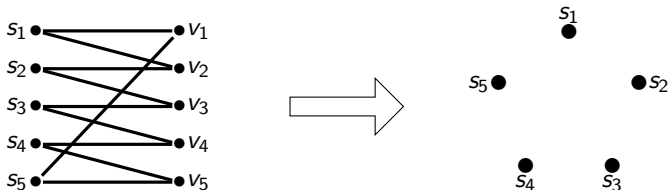
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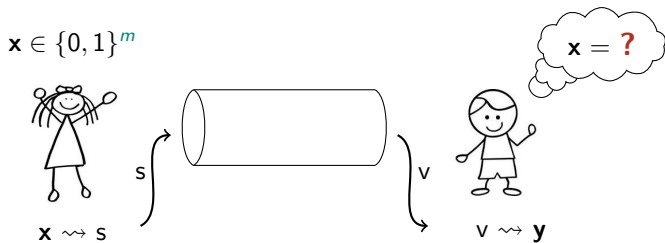
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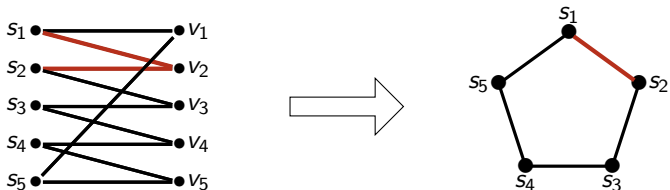
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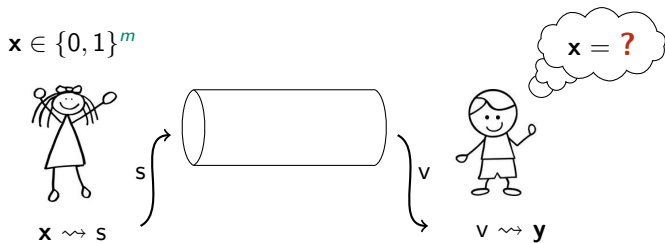
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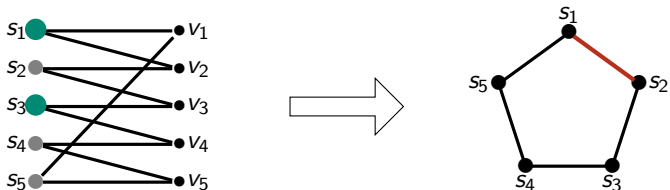
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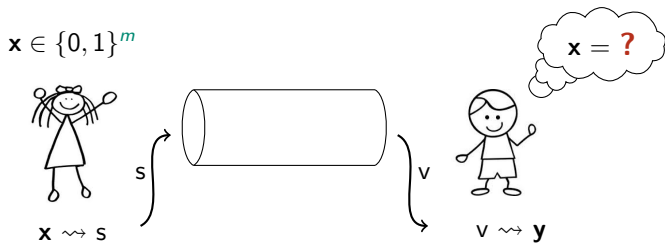
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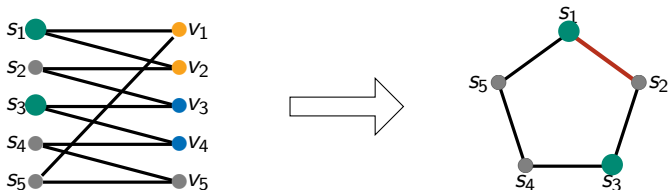
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Zero-error channel coding



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Channel capacities

	One-shot	Asymptotic
Classical		
Quantum		

Channel capacities

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Classical	$\alpha(G)$	$c(G)$
Quantum		

Channel capacities

	One-shot		Asymptotic
Classical	$\alpha(G)$	\leq	$c(G)$
Quantum			

Channel capacities

	One-shot		Asymptotic	
Classical	$\alpha(G)$	$<$	$c(G)$	Shannon 1956
Quantum				

- \exists graph $G : \alpha(G) < c(G)$

Channel capacities

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Classical	$\alpha(G)$	\leq	$c(G)$
	\wedge		
Quantum	$\alpha^*(G)$		

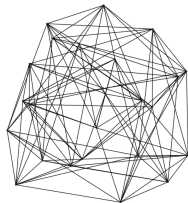
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Cubitt et al. 2010

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Channel capacities

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	\wedge	$\not\leq$	
Quantum	$\alpha^*(G)$	\leq	$c^*(G)$

Leung et al. 2012

Briët et al. 2012

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Thank you!