

**Booking.com**

# **Subpopulation-aware experimentation platform**

Christina Katsimerou

Principal Machine Learning Scientist

# A

Booking.com SS Refer Friends & Earn List Your Property Alexander Vorobyev  
Frequent Traveler

Accommodations Flights Car Rentals Airport Taxis

Where to next, Alexander?  
Find exclusive Genius rewards in every corner of the world.  
Destination, property name or address:  
New York City  
Check-in: Friday, May 25, 2018 Check-out: Saturday, May 26, 2018  
1-night stay  
2 adults No children 1 room  
 I'm traveling for work [Search](#)

Welcome back, Alexander!  
How was Pension Elisabeth? [Rate your stay](#)

**London**   
Top reasons to visit: sightseeing, shopping, museums  
Still interested in your previous searches for London?  
May 24 - May 26 4 rooms, 2 adults, 3 children [X](#)

**New York City**   
Top reasons to visit: shopping, sightseeing, museums  
Still interested in your previous searches for New York City?  
May 25 - May 26 1 room, 2 adults [X](#)

**Penza**   
Top reasons to visit: city walks, friendly locals, sightseeing  
Still interested in your previous searches for Penza?  
Aug 16 - Aug 24 1 room, 2 adults, 3 children [X](#)  
Average price **SS 420**

**Amsterdam**   
2,136 properties  
Average price **SS 420**

**Paris**   
3,965 properties  
Average price **SS 304**

Register. Earn. Play for Your Passions  
List your place on Booking.com  
[Learn more](#)

Learn more about Refer a Friend for employees  
[Start earning!](#)  
Refer a friend to Booking.com. Your friend gets a 10% refund, you get € 0!

# B

Booking.com SS Refer Friends & Earn List Your Property Alexander Vorobyev  
Frequent Traveler

Accommodations Flights Car Rentals Airport Taxis

Where to next, Alexander?  
Find exclusive Genius rewards in every corner of the world.

Amsterdam, Noord-Holland, Netherlands Fri, May 25 - Sat, May 26 2 adults · 0 Children [Search](#)

I'm traveling for work

How was Pension Elisabeth? [Rate your stay](#)

**London**   
Top reasons to visit: sightseeing, shopping, museums  
Still interested in your previous searches for London?  
May 24 - May 26 4 rooms, 2 adults, 3 children [X](#)

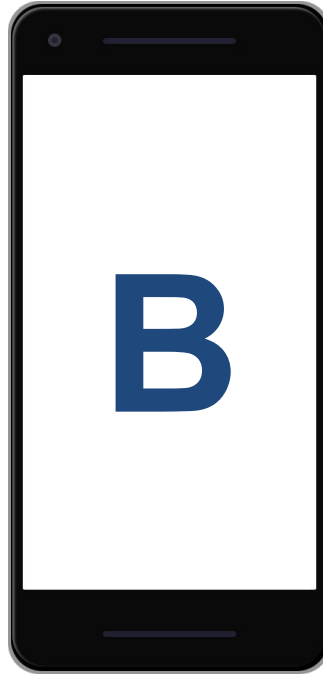
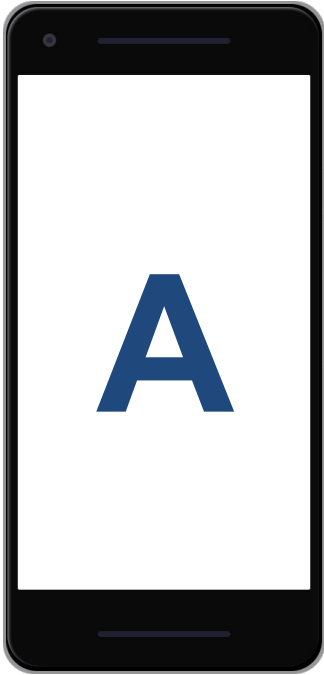
**New York City**   
Top reasons to visit: shopping, sightseeing, museums  
Still interested in your previous searches for New York City?  
May 25 - May 26 1 room, 2 adults [X](#)

**Penza**   
Top reasons to visit: city walks, friendly locals, sightseeing  
Still interested in your previous searches for Penza?  
Aug 16 - Aug 24 1 room, 2 adults, 3 children [X](#)  
Average price **SS 420**

**Amsterdam**   
2,136 properties  
Deal alert **SS 224**  
Average price **SS 420**

**Paris**   
3,965 properties  
Deal alert **SS 121**  
Average price **SS 304**

Learn more about Refer a Friend for employees



...

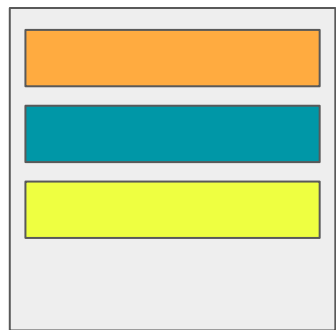


## A/B/n testing

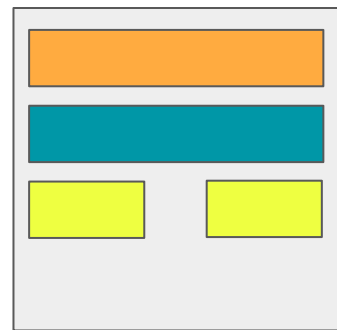
- ❖ Decision making
- ❖ Learning & Inspiration

# Currently

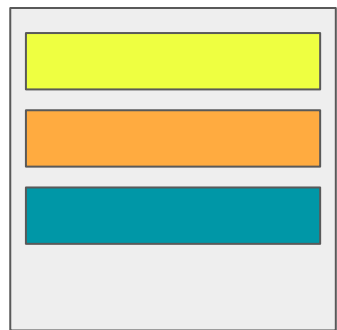
Control



Variant 1

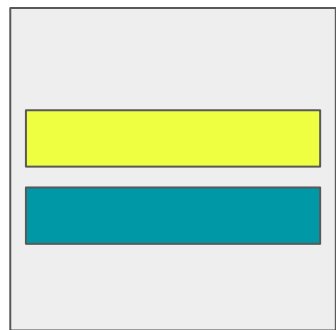


Variant 2

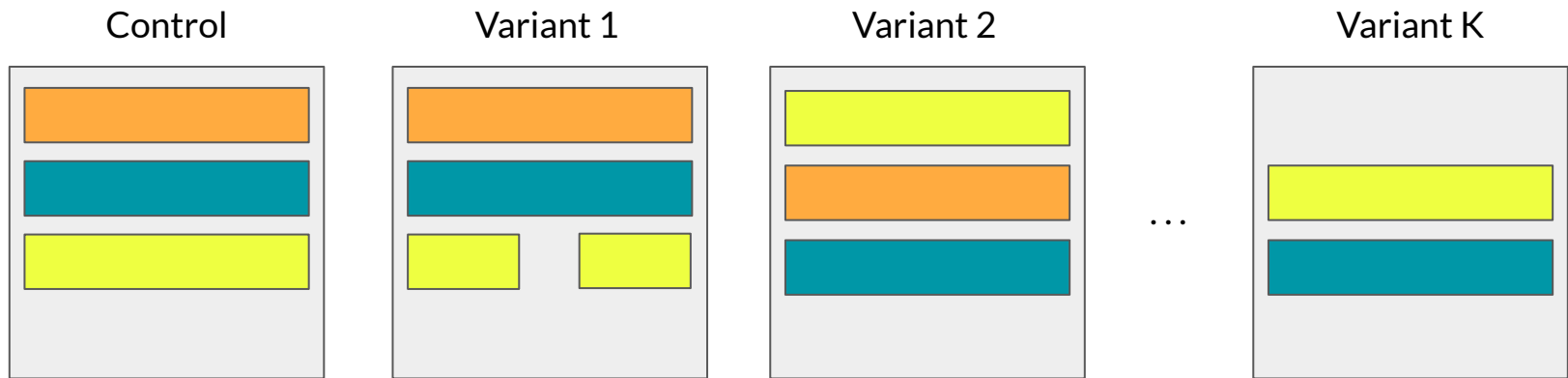


...

Variant K

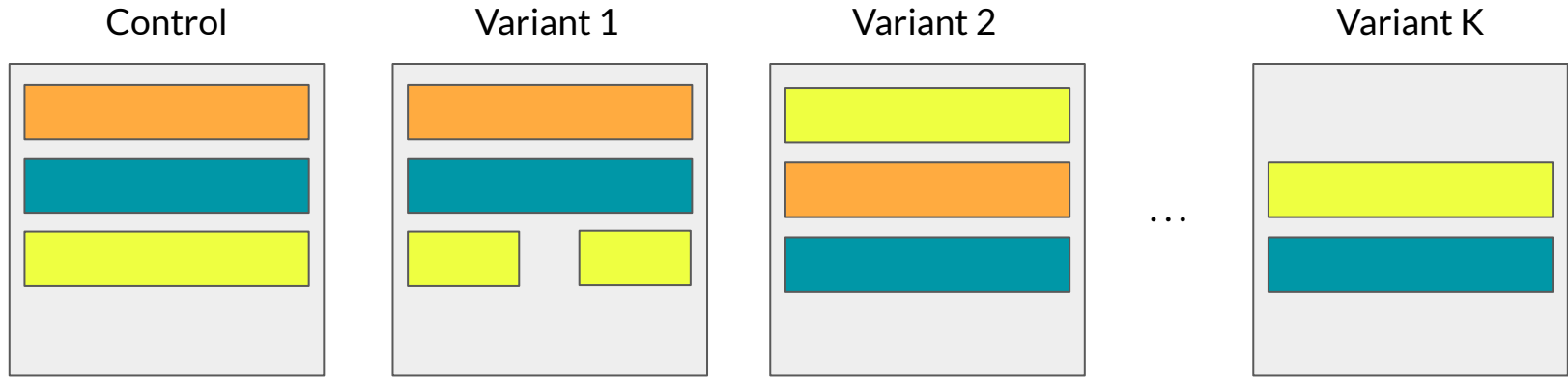


# Currently



$$H_{0,k} : \mu_0 \leq \mu_k, k \in \{1, \dots, K\}$$

# Currently

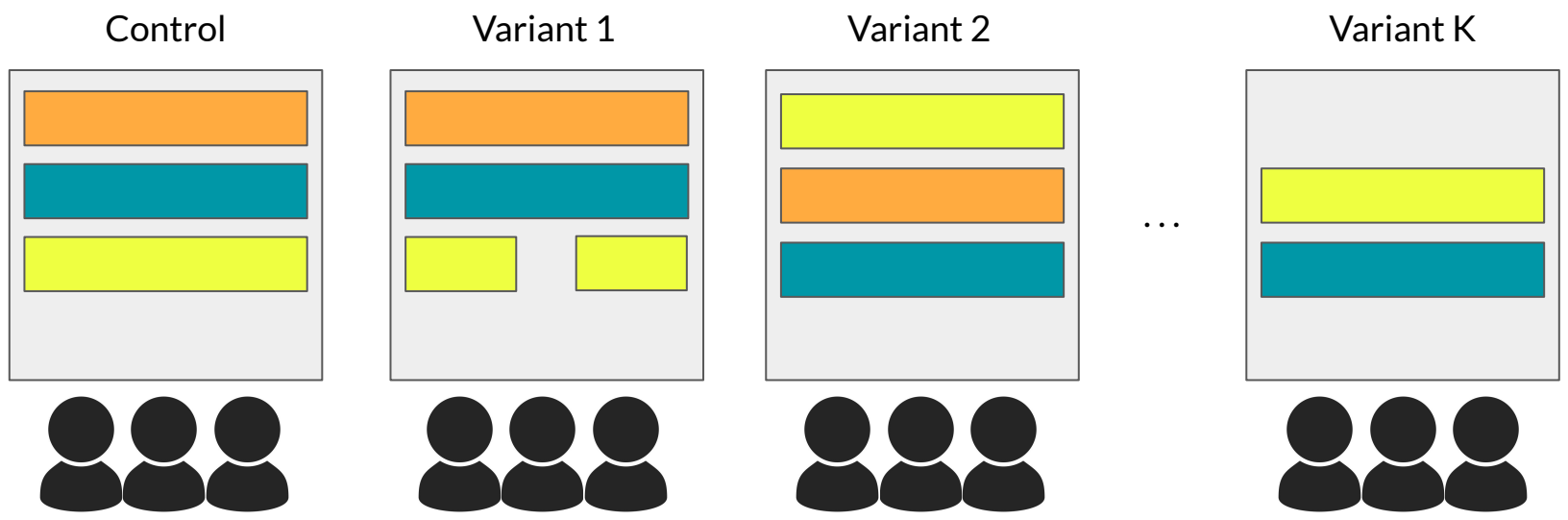


$$H_{0,k} : \mu_0 \leq \mu_k, k \in \{1, \dots, K\}$$

For a fixed risk  $\delta$ :

- Determine minimum desired detectable effect
- Compute sample size  $N$

# Currently

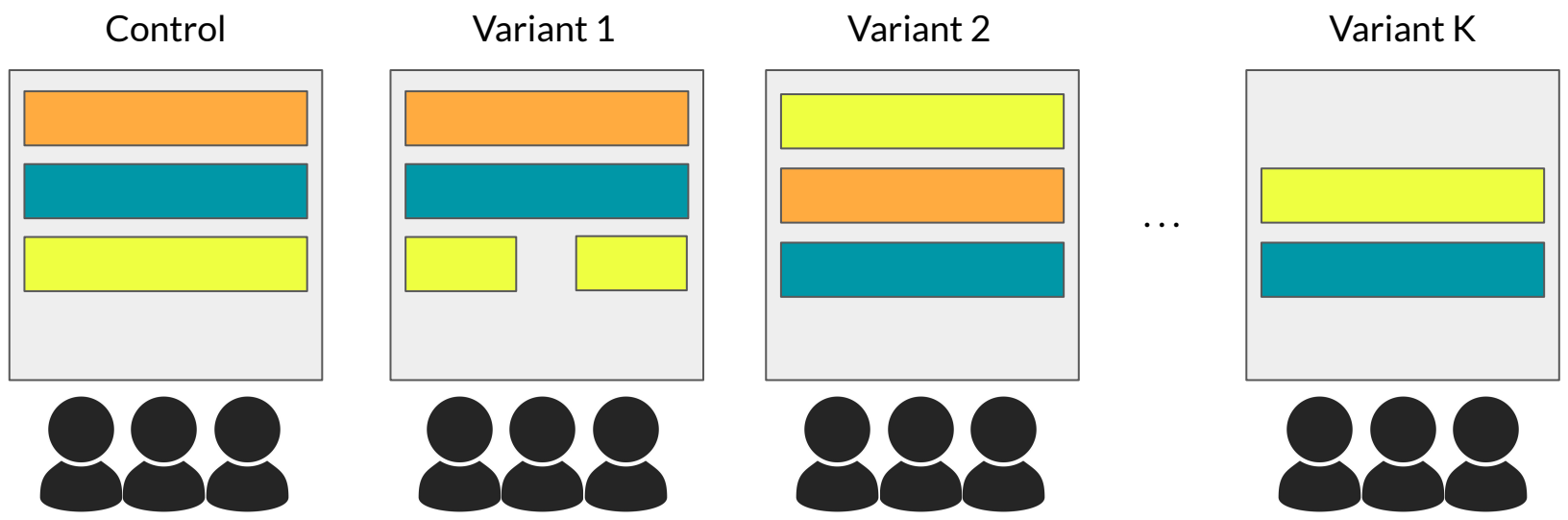


$$H_{0,k} : \mu_0 \leq \mu_k, k \in \{1, \dots, K\}$$

For a fixed risk  $\delta$ :

- Determine minimum desired detectable effect
- Compute sample size  $N$
- Distribute traffic uniformly among variants

# Currently



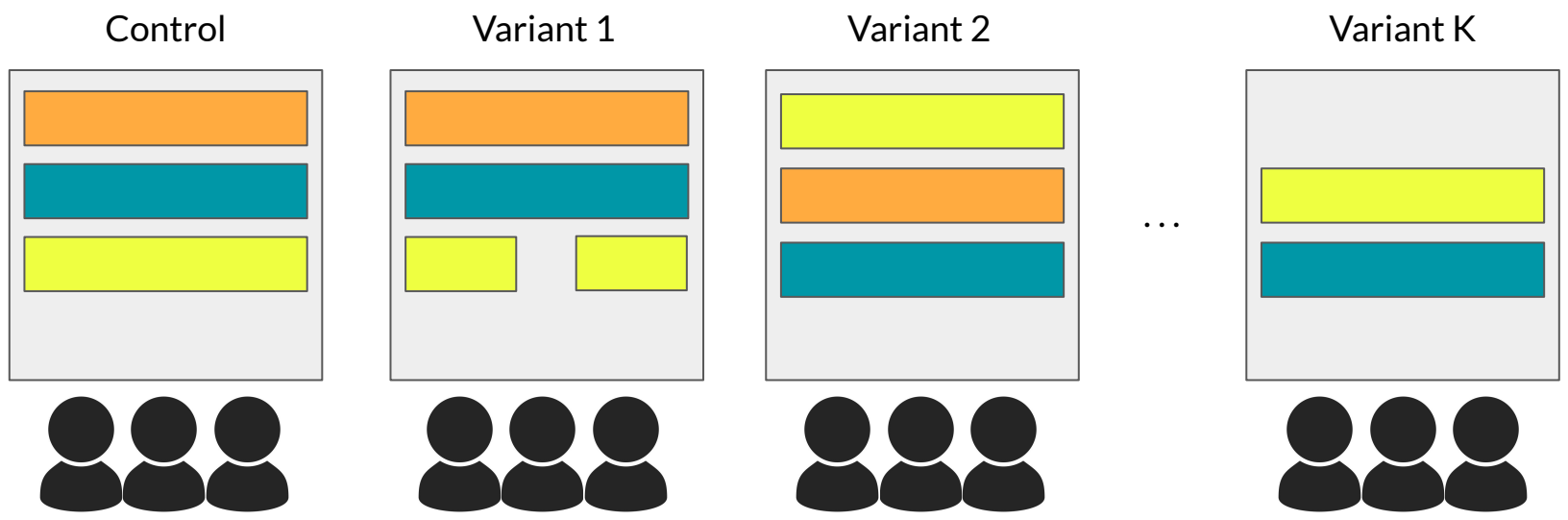
$$H_{0,k} : \mu_0 \leq \mu_k, k \in \{1, \dots, K\}$$

For a fixed risk  $\delta$ :

- Determine minimum desired detectable effect
- Compute sample size  $N$
- Distribute traffic uniformly among variants
- At sample size  $N$  compute p-values  $p_k$  per hypothesis



# Currently



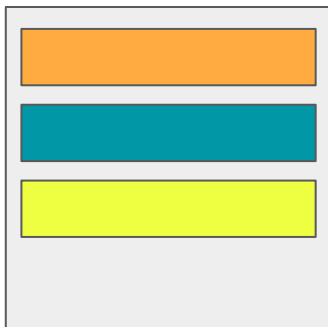
$$H_{0,k} : \mu_0 \leq \mu_k, k \in \{1, \dots, K\}$$

For a fixed risk  $\delta$ :

- Determine minimum desired detectable effect
- Compute sample size  $N$
- Distribute traffic uniformly among variants
- At sample size  $N$  compute p-values  $p_k$  per hypothesis
- Reject  $H_{0,k}$  if  $p_k \leq \frac{\delta}{K}$

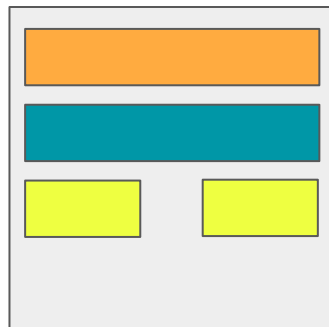
# Problem #1: Uniform allocation

Control



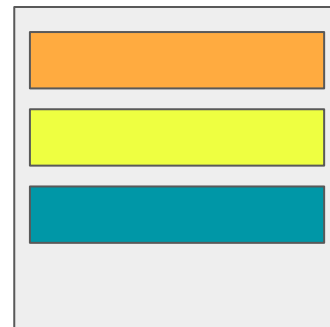
$$\mu_0 = 0.1$$

Variant 1



$$\mu_1 = 0.1001$$

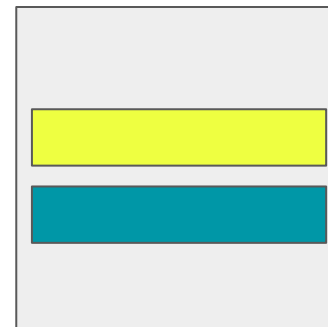
Variant 2



$$\mu_2 = 0.11$$

...

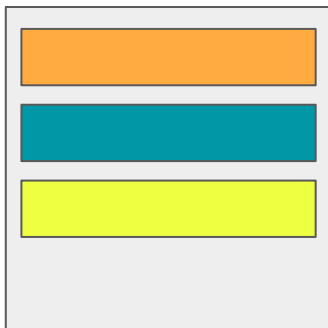
Variant K



$$\mu_K = 0.08$$

# Problem #1: Uniform allocation

Control



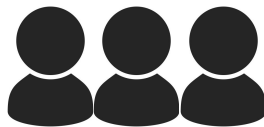
$$\mu_0 = 0.1$$



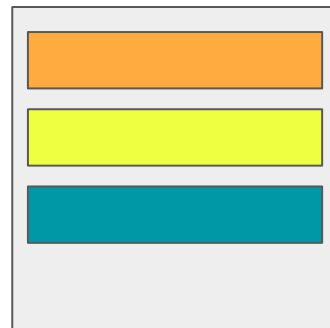
Variant 1



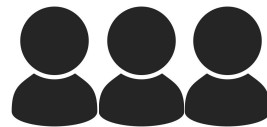
$$\mu_1 = 0.1001$$



Variant 2

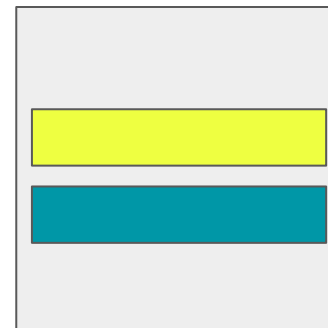


$$\mu_2 = 0.11$$



...

Variant K

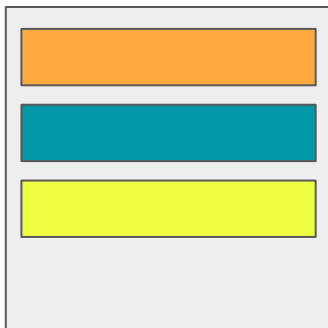


$$\mu_K = 0.08$$



# Problem #1: Uniform allocation

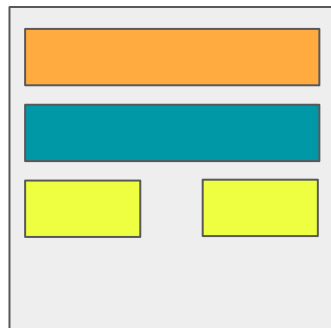
Control



$$\mu_0 = 0.1$$



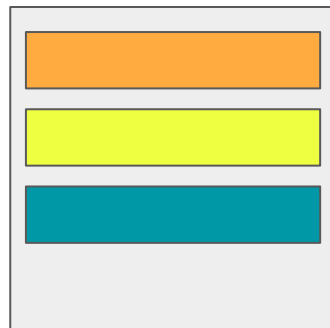
Variant 1



$$\mu_1 = 0.1001$$



Variant 2

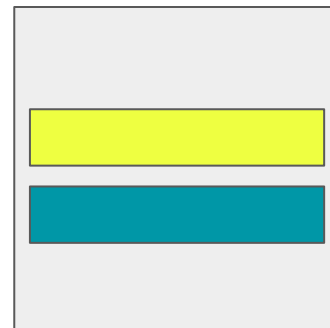


$$\mu_2 = 0.11$$



...

Variant K



$$\mu_K = 0.08$$



## Problem #2: No monitoring

Experimenters want to **monitor continuously** their experiments to:

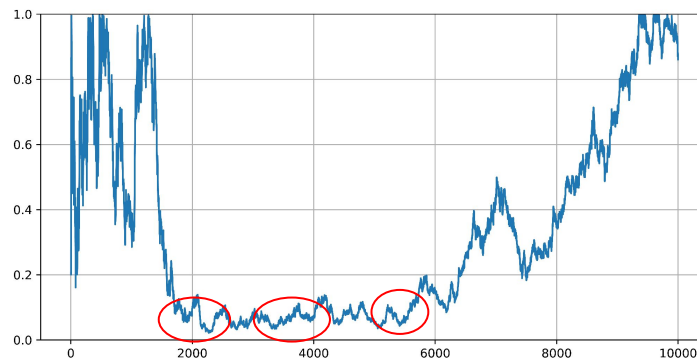
- Detect effects as quickly as possible
- Give up soon when there is no effect

# Problem #2: No monitoring

Experimenters want to monitor continuously their experiments to:

- Detect effects as quickly as possible
- Give up soon when there is no effect

The platform does not allow **data dependent stopping times**



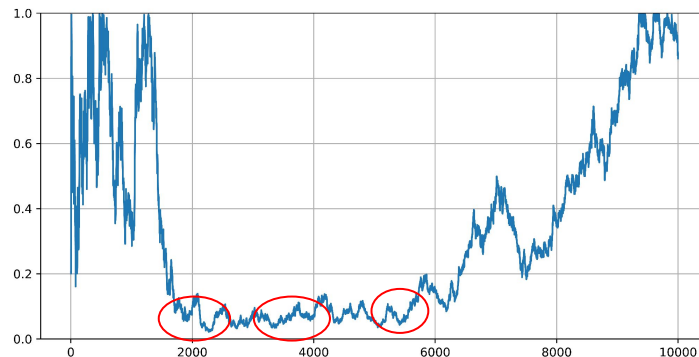
$p(t)$  under  $H_0 : \mu_1 = \mu_2$

# Problem #2: No monitoring

Experimenters want to monitor continuously their experiments to:

- Detect effects as quickly as possible
- Give up soon when there is no effect

The platform does not allow **data dependent stopping times**



$p(t)$  under  $H_0 : \mu_1 = \mu_2$

Monitoring without properly correcting for it inflates the false positive rate.

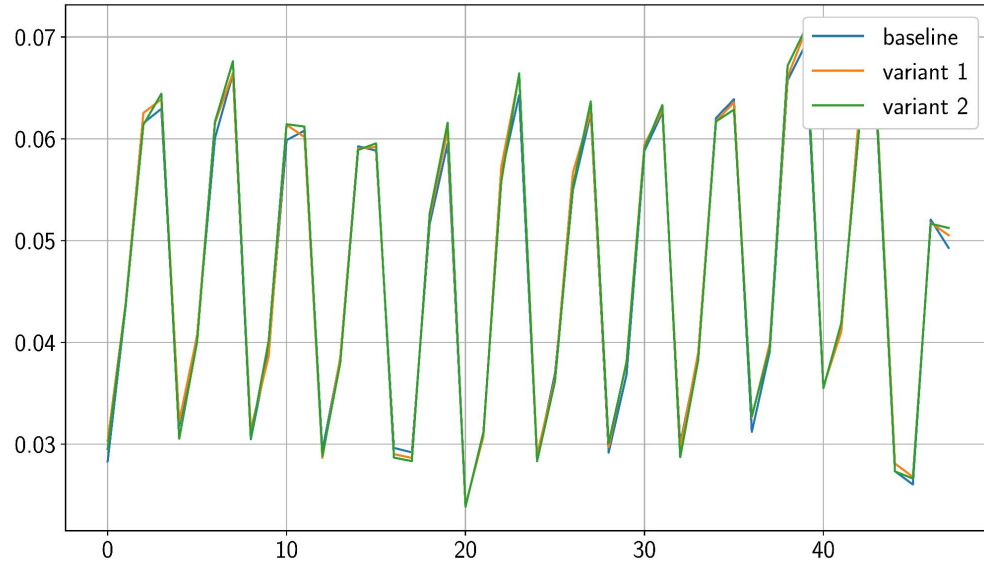
# New formulation

We aim to identify the subset of **all variants that are better than the control**, while:

- optimizing adaptively the stopping time of the A/B/n experiment
- being  **$\delta$ -PAC**: the probability of stopping and returning a wrong answer must be  $\leq \delta$
- optimizing adaptively the allocation of variants to users

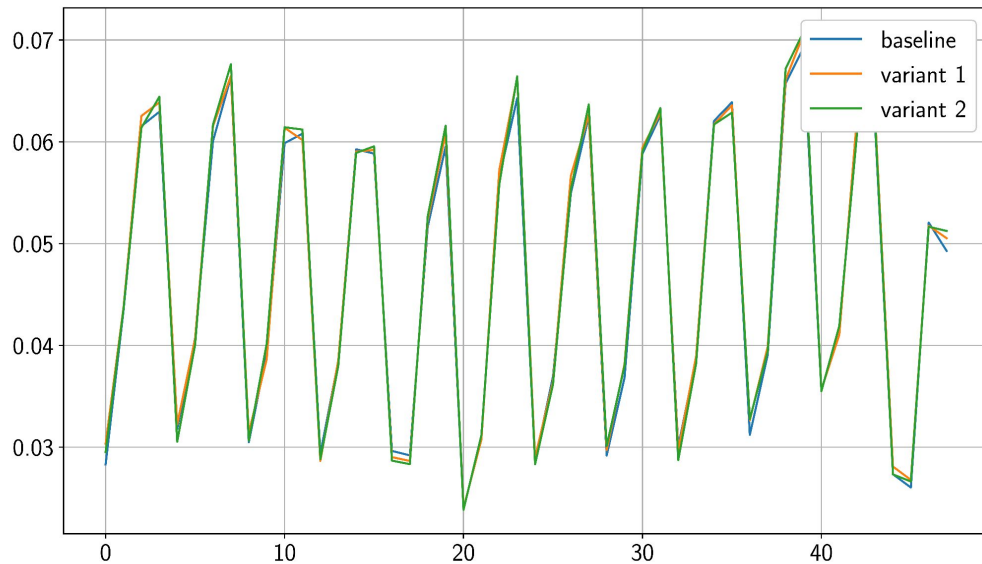


# Seasonality



Click-through rate per 6 hours for 12 days

# Seasonality



Click-through rate per 6 hours for 12 days

Seasons → subpopulations

With  $J$  subpopulations, each variant is a mixture of  $J$  distributions.

The value of the variant  $a$  is:

$$\mu_a = \sum_{i=1}^J \alpha_i \mu_{a,i}$$

# Interaction with subpopulations

- Pick variant
- Pick subpopulation

(a) Active

- See subpopulation
- Pick variant

(b) Proportional

- Pick variant
- Don't see subpopulation

(c) Oblivious

# Interaction with subpopulations

- ❑ Pick variant
- ❑ Pick subpopulation

(a) Active

- ❑ See subpopulation
- ❑ Pick variant

(b) Proportional

- ❑ Pick variant
- ❑ Don't see subpopulation

(c) Oblivious



A bandit with  $K + 1$  arms and  $J$  subpopulations is

- a  $(K + 1) \times J$  matrix  $\mu$  of reward distributions
- distribution  $\alpha$  of  $J$  subpopulations

# Model

A bandit with  $K + 1$  arms and  $J$  subpopulations is

- a  $(K + 1) \times J$  matrix  $\mu$  of reward distributions
- distribution  $\alpha$  of  $J$  subpopulations

The correct answer for  $(\mu, \alpha)$  is

$$\mathcal{S}(\mu) = \left\{ k \in \{1, \dots, K\} \text{ s.t. } \sum_{j=1}^J \alpha_j \mu_{k,j} > \sum_{j=1}^J \alpha_j \mu_{0,j} \right\} .$$

For any sampling strategy, the expected number of rounds  $\mathbb{E}_{\mu}[\tau_{\delta}]$  is bounded from below.

$\mathbb{E}_{\mu}[\tau_{\delta}]$ :

- depends on  $\mu$
- gets minimized when we sample with the optimal sampling proportions  $w^*(\mu)$ , which depend on the mode of interaction with the subpopulations

# Example

$$\boldsymbol{\mu} = \begin{array}{c} \text{subpopulations} \\ \left[ \begin{array}{cc} 0.15 & 0.05 \\ 0.16 & 0.06 \\ 0.17 & 0.07 \end{array} \right] \begin{array}{c} \text{arms} \end{array} \end{array} \quad \boldsymbol{\alpha} = [0.5, 0.5]$$



# Example

$$\boldsymbol{\mu} = \begin{matrix} & \begin{matrix} \text{subpopulations} \end{matrix} \\ \begin{matrix} \left[ \begin{array}{cc} 0.15 & 0.05 \\ 0.16 & 0.06 \\ 0.17 & 0.07 \end{array} \right] & \begin{matrix} \text{arms} \end{matrix} \end{matrix} \quad \boldsymbol{\alpha} = [0.5, 0.5]$$

	i=1	i=2
0	0.288	0.182
1	0.283	0.178
2	0.042	0.027
	<b>0.613</b>	<b>0.387</b>

$$\mathbb{E}_{\boldsymbol{\mu}}[\tau_{\delta}]_{\text{active}} \geq 22370$$

# Example

$$\mu = \begin{matrix} & \text{subpopulations} \\ \begin{matrix} \left[ \begin{array}{cc} 0.15 & 0.05 \\ 0.16 & 0.06 \\ 0.17 & 0.07 \end{array} \right] & \text{arms} \end{matrix} & \alpha = [0.5, 0.5] \end{matrix}$$

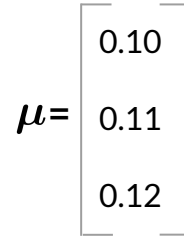
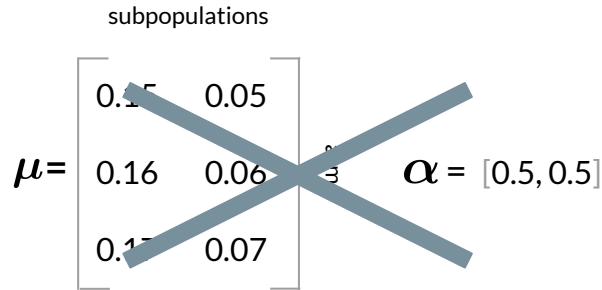
	i=1	i=2
0	0.288	0.182
1	0.283	0.178
2	0.042	0.027
	<b>0.613</b>	<b>0.387</b>

$$\mathbb{E}_{\mu}[\tau_{\delta}]_{\text{active}} \geq 22370$$

	i=1	i=2
0	0.236	0.231
1	0.230	0.234
2	0.034	0.035
	<b>0.5</b>	<b>0.5</b>

$$\mathbb{E}_{\mu}[\tau_{\delta}]_{\text{proportional}} \geq 23516$$

# Example



	i=1	i=2
0	0.288	0.182
1	0.283	0.178
2	0.042	0.027
	<b>0.613</b>	<b>0.387</b>

$$\mathbb{E}_{\mu}[\tau_{\delta}]_{\text{active}} \geq 22370$$

	i=1	i=2
0	0.236	0.231
1	0.230	0.234
2	0.034	0.035
	<b>0.5</b>	<b>0.5</b>

$$\mathbb{E}_{\mu}[\tau_{\delta}]_{\text{proportional}} \geq 23516$$

0	0.47
1	0.46
2	0.07
	<b>1</b>

$$\mathbb{E}_{\mu}[\tau_{\delta}]_{\text{oblivious}} \geq 24158$$

# Track & Stop Algorithm

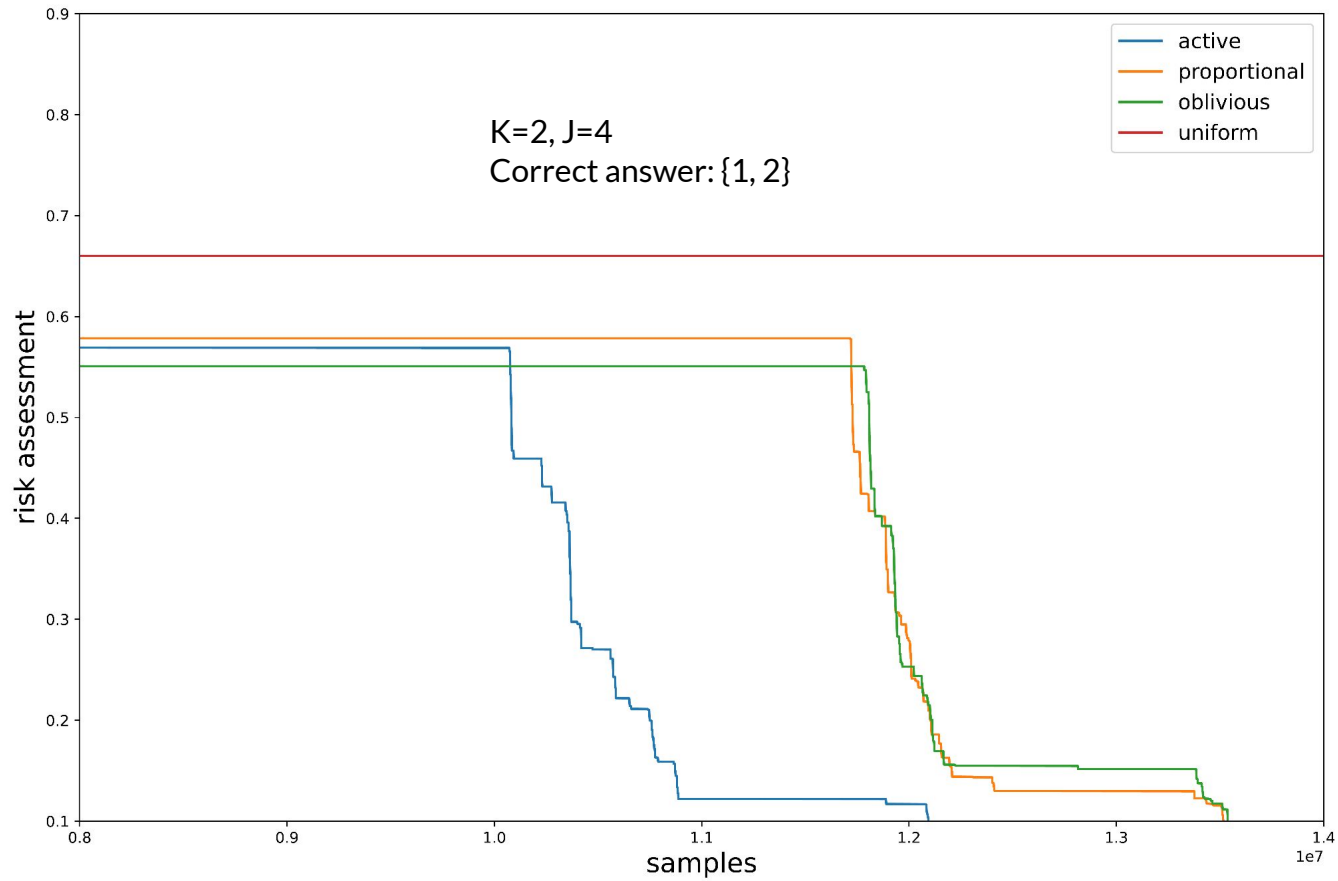
For  $t \geq 1$ :

- Sampling rule: given the current estimates  $\hat{\boldsymbol{\mu}}(t)$ 
  - ① estimate the target weights  $\mathbf{w}_t(\hat{\boldsymbol{\mu}})$
  - ② pick arm s.t.  $\frac{N_{a,t}}{t} \rightarrow \mathbf{w}_t(\hat{\boldsymbol{\mu}})$

# Track & Stop Algorithm

For  $t \geq 1$ :

- Sampling rule: given the current estimates  $\hat{\boldsymbol{\mu}}(t)$ 
  - ① estimate the target weights  $\mathbf{w}_t(\hat{\boldsymbol{\mu}})$
  - ② pick arm s.t.  $\frac{N_{a,t}}{t} \rightarrow \mathbf{w}_t(\hat{\boldsymbol{\mu}})$
- Recommendation:  
 $\mathcal{S}(\hat{\boldsymbol{\mu}}_t) = \{a \in \{1, \dots, K\} : \hat{\mu}_a(t) > \hat{\mu}_0(t)\}$  with risk assessment  $\hat{\delta}_t = \min\{\delta \in (0, 1) | \Lambda(t) \geq \beta(t, \delta)\}$



# Conclusion

The sub-population aware experiment platform:

- Is more efficient allowing adaptive sampling and stopping in the presence of seasonality
- Experimenters can monitor continuously without errors

# Conclusion

The sub-population aware experiment platform:

- Is more efficient allowing adaptive sampling and stopping in the presence of seasonality
- Experimenters can monitor continuously without errors

We have already run successfully A/B/n experiments, which we validated on the non adaptive platform



# Conclusion

The sub-population aware experiment platform:

- Is more efficient allowing adaptive sampling and stopping in the presence of seasonality
- Experimenters can monitor continuously without errors

We have already run successfully A/B/n experiments which we validated on the non adaptive platform

Still work to be done on:

- Delayed rewards
- epsilon-better
- ...

Thank you!

[christina.katsimerou@booking.com](mailto:christina.katsimerou@booking.com)

[paper](#)

[booking.ai](#)

<https://jobs.booking.com/careers>



# References

- Garivier, A. and E. Kaufmann (2016). “Optimal best arm identification with fixed confidence”. In: Conference on Learning Theory. PMLR, pp. 998–1027.
- Russac, Y., C. Katsimerou, D. Bohle, O. Cappé, A. Garivier, and W. M. Koolen (Dec. 2021). “A/B/n Testing with Control in the Presence of Subpopulations”. In: Advances in Neural Information Processing Systems (NeurIPS) 34.

# Appendix

# Complexity

## Theorem 1

For any strategy, the expected number of rounds for the ABC-S problem satisfies

$$\liminf_{\delta \rightarrow 0} \frac{\mathbb{E}_{\boldsymbol{\mu}}[\tau_{\delta}]}{\ln(1/\delta)} \geq T^*(\boldsymbol{\mu}) \quad (1)$$

where

$$T^*(\boldsymbol{\mu})^{-1} = \sup_{\mathbf{w} \in \mathcal{C}} \min_{b \neq 0} \inf_{\lambda \in \mathcal{L}: \lambda_0 < \lambda_b} \sum_{a \in \{0, b\}} \sum_{i=1}^J w_{a,i} d(\mu_{a,i}, \lambda_{a,i}) \quad (2)$$

- A min-max optimization problem
- Optimal weights  $\mathbf{w}^*$  = target relative frequencies of draws

For  $t \geq 1$ :

- Sampling rule: given the current estimates
  - ① estimate the target weights  $\mathbf{w}_t(\hat{\boldsymbol{\mu}})$
  - ② pick arm s.t.  $\frac{N_{a,t}}{t} \rightarrow \mathbf{w}_t(\hat{\boldsymbol{\mu}})$
- Recommendation:
  - ① Stop at  $\tau_\delta = \inf\{t \in \mathbb{N} \mid \text{test statistic} \geq \text{threshold}\}$
  - ② Recommend  $\mathcal{S}(\hat{\boldsymbol{\mu}}_t) = \{a \in \{1, \dots, K\} : \hat{\mu}_a(\tau_\delta) > \hat{\mu}_0(\tau_\delta)\}$

Generalised Likelihood Ratio Statistic for exponential family bandit models:

$$\Lambda(t) = \min_{b \neq 0} \inf_{\lambda \in \mathcal{L}: \lambda_0 = \lambda_b} \sum_{a \in \{0, b\}} \sum_{i=1}^J N_{a,i}(t) d(\hat{\mu}_{a,i}(t), \lambda_{a,i})$$

# Example

$$\boldsymbol{\mu} = \begin{bmatrix} 0.15 & 0.05 \\ 0.16 & 0.06 \end{bmatrix}, \boldsymbol{\alpha} = [0.5 \quad 0.5]$$

$T_{\text{active}} = 16013$

	i=1	i=2
0	0.309	0.194
1	0.305	0.192
	0.614	0.386

$T_{\text{proportional}} = 16840$

	i=1	i=2
0	0.252	0.246
1	0.248	0.254
	0.5	0.5

$T_{\text{agnostic}} = 16841$

	i=1	i=2
0	0.251	0.251
1	0.249	0.249
	0.5	0.5