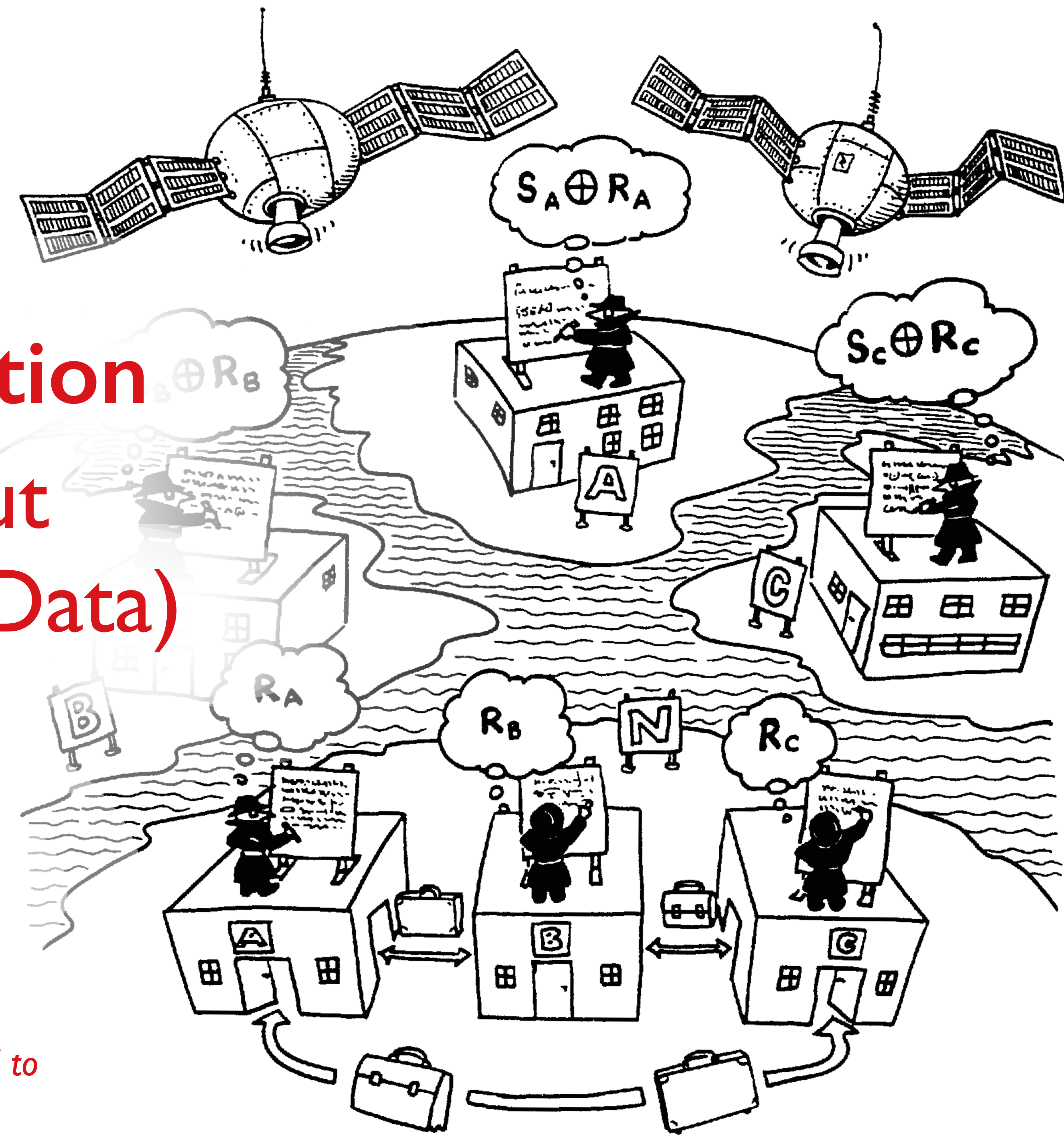
Centrum Wiskunde & Informatica

Universiteit Leiden

CWI Lectures
November 21 & 22, 2019

Multiparty Computation

Collaborate Without Compromise(ing Your Data)



Serge Fehr

Centrum Wiskunde & Informatica (CWI)

Mathematical Institute, Leiden University

On the occasion of the Dijkstra Fellowship being awarded to David Chaum

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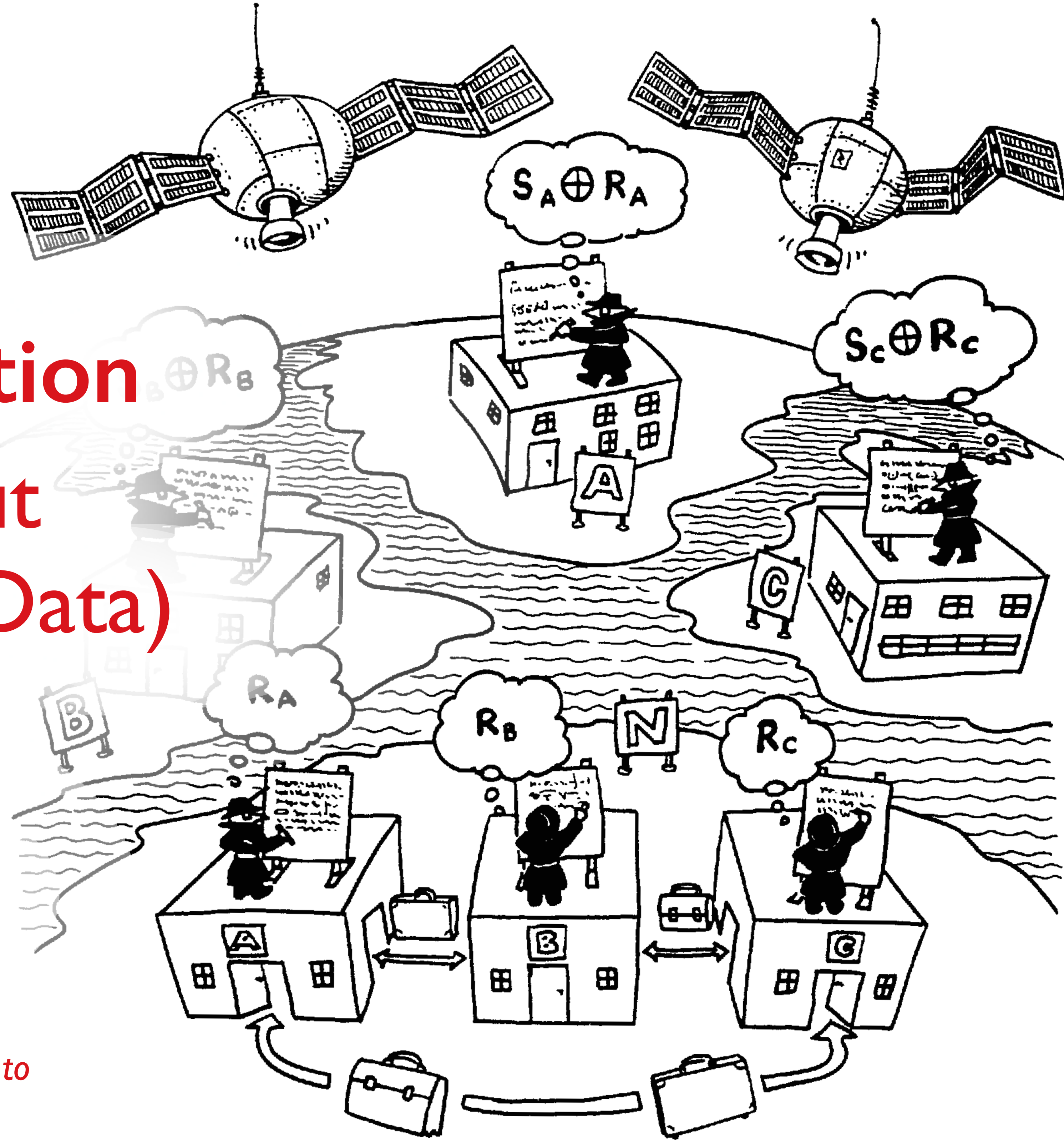

Road Map

- WHAT is multiparty computation?
- HOW does multiparty computation work?
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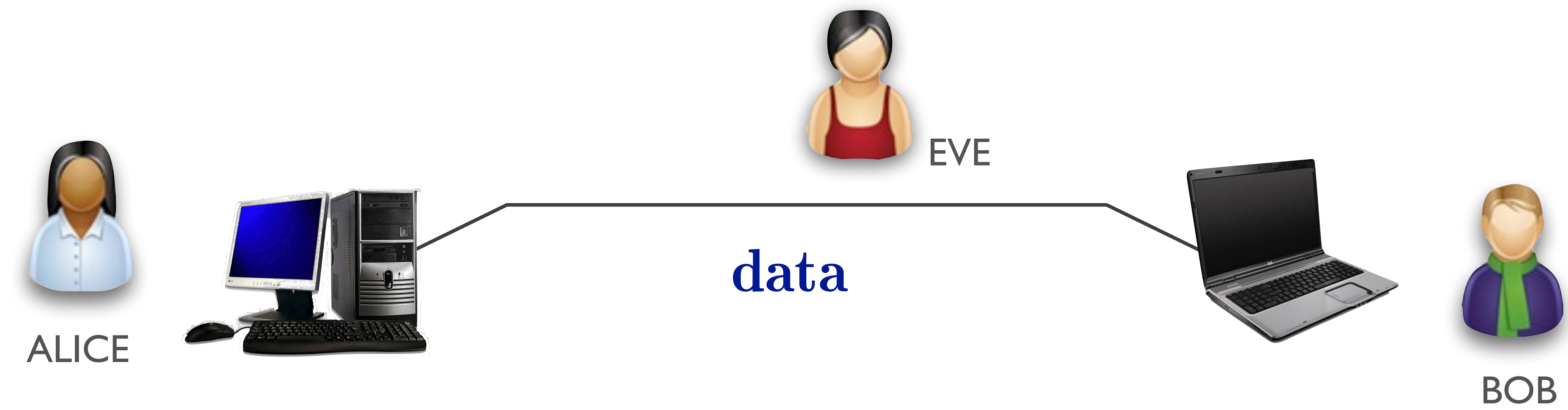

Original goal of cryptography:

Protect data from an eavesdropper/hacker/etc.

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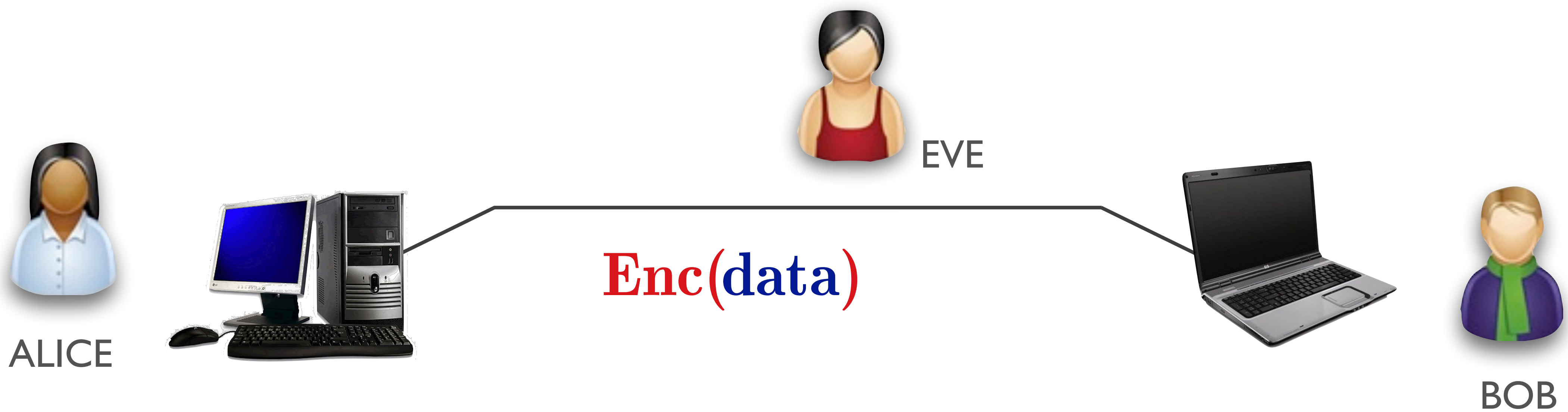

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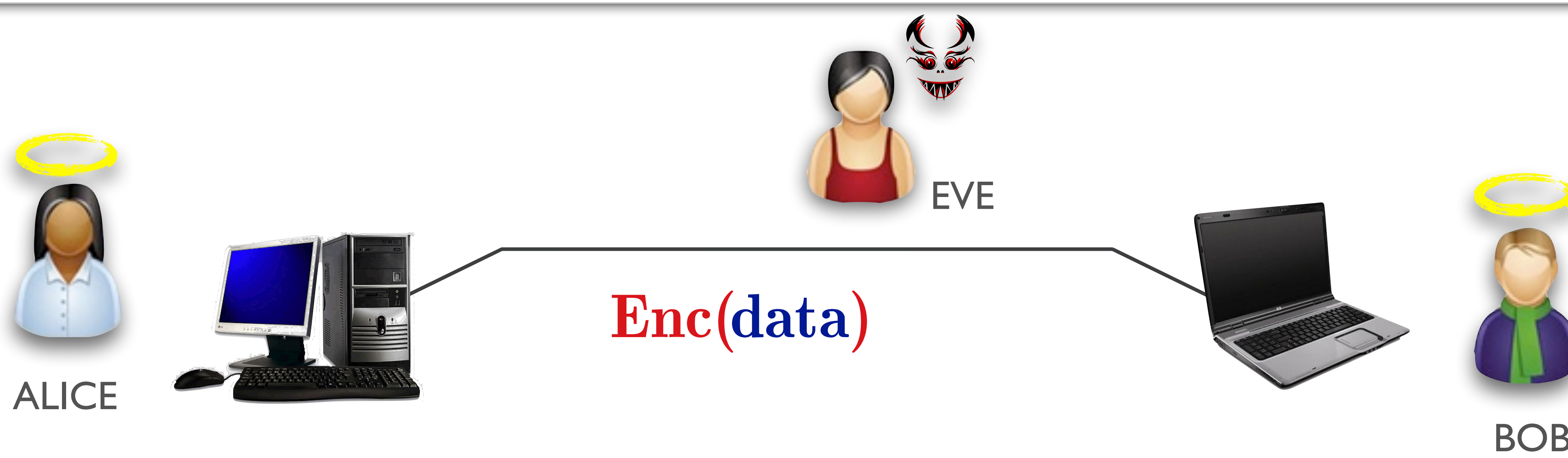

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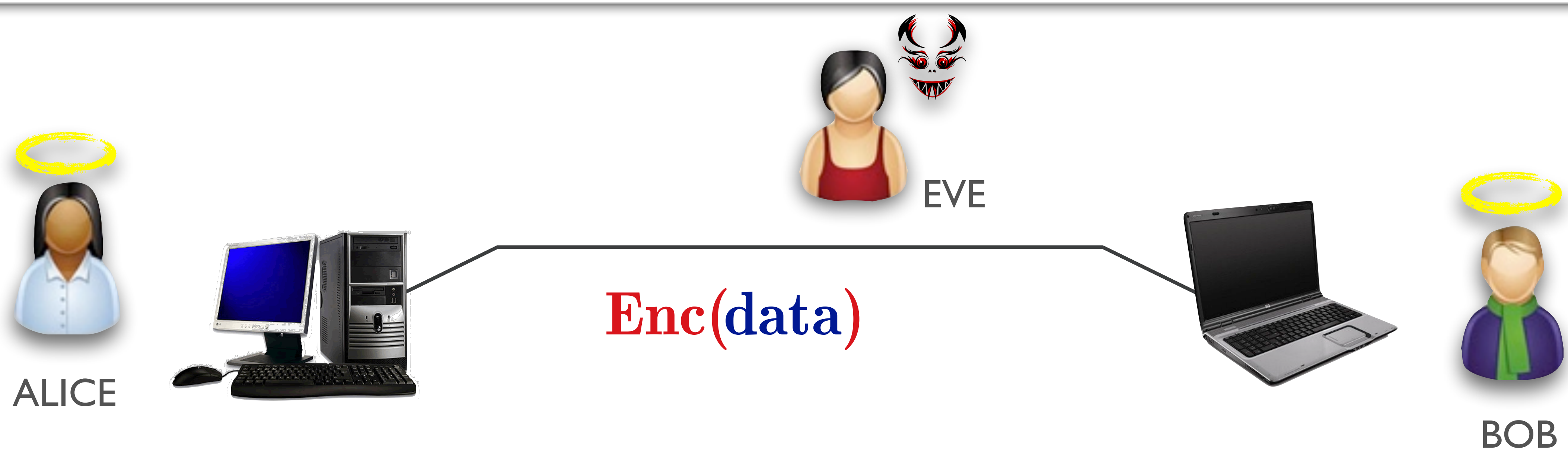

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Situation may not always be so clear cut…

Cryptography

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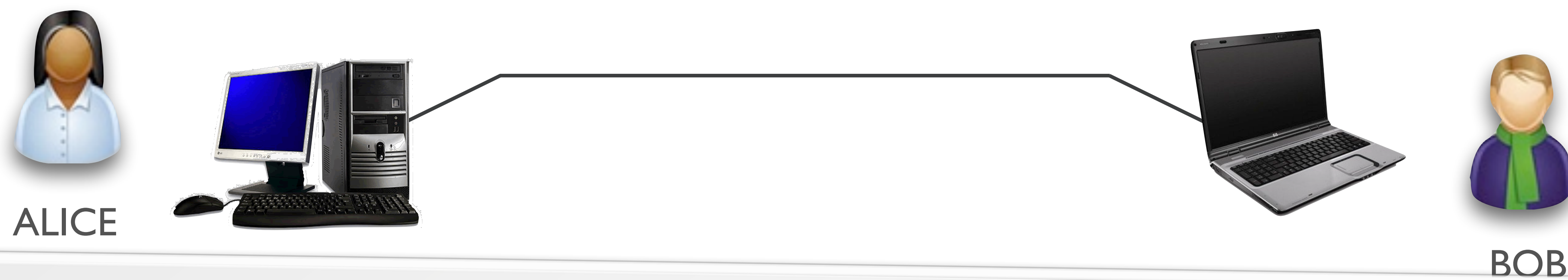
Goal: Collaborate **without** the need to trust each other, and so that **nothing gets revealed** beyond what is necessary.

Cryptography



Multiparty Computation (MPC)

An advanced cryptographic concept

- for **protecting individual data** of different parties
- while **using the data** in collaboration with other parties

Goal: Collaborate **without** the need to **trust each other**, and so that **nothing gets revealed** beyond what is necessary.

Example: Yao's "Millionairs' Problem"

Two millionaires want to find out who is richer,

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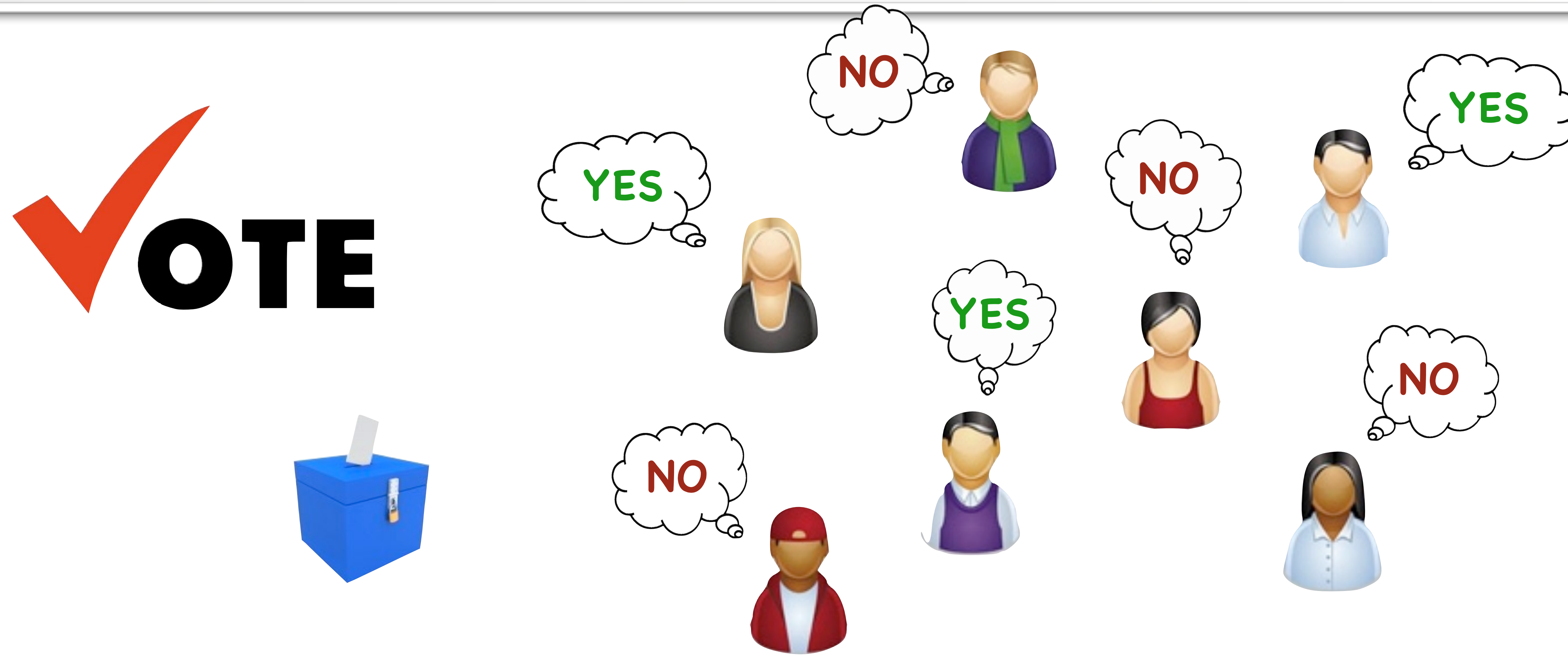
Two millionaires want to find out who is richer,
but **without** telling each other how much they own:
both should learn **nothing beyond**

$$y \in \{\text{“Richard is richer”}, \text{“Elon is richer (or equally rich)”}\}$$

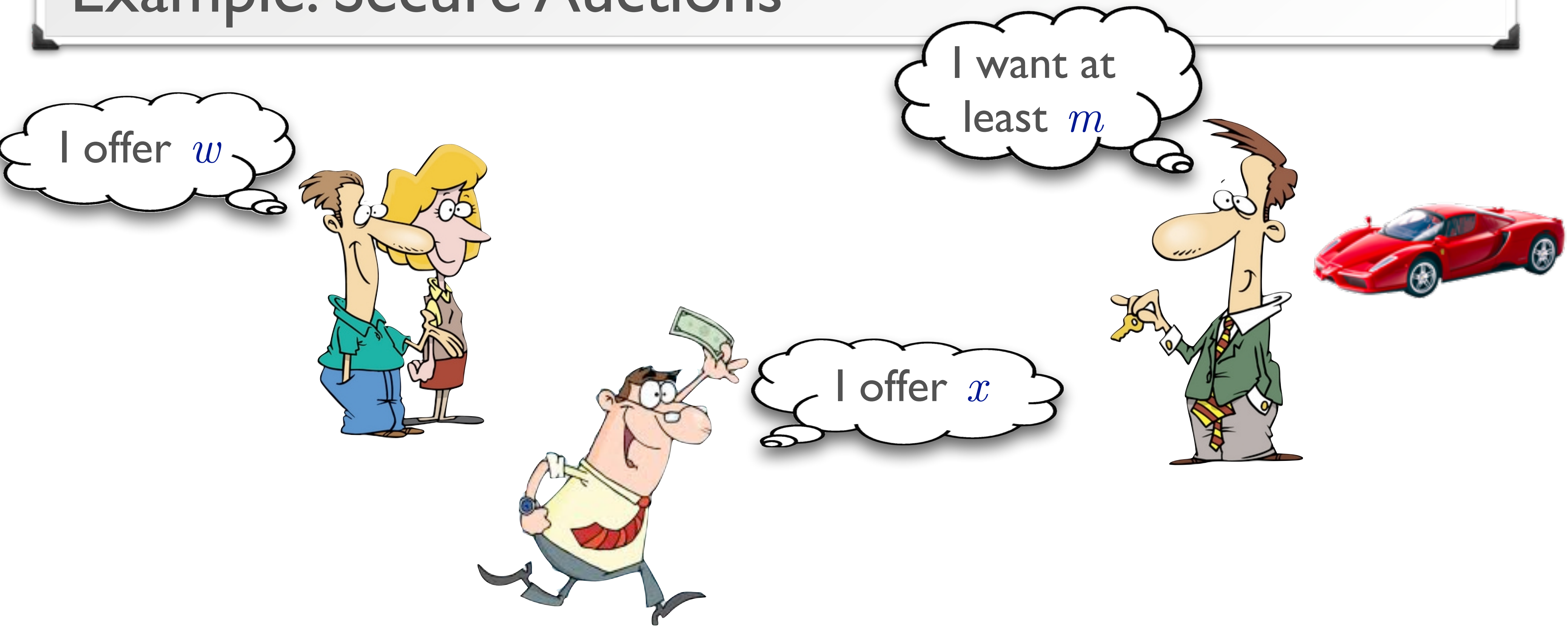
Example: Secure Voting



Find out what the majority wants, i.e., tally the votes,
without revealing individual opinions/votes:
everyone should learn **nothing beyond**, say,

$$y = (\text{sum of YES votes}, \text{sum of NO votes})$$

Example: Secure Auctions



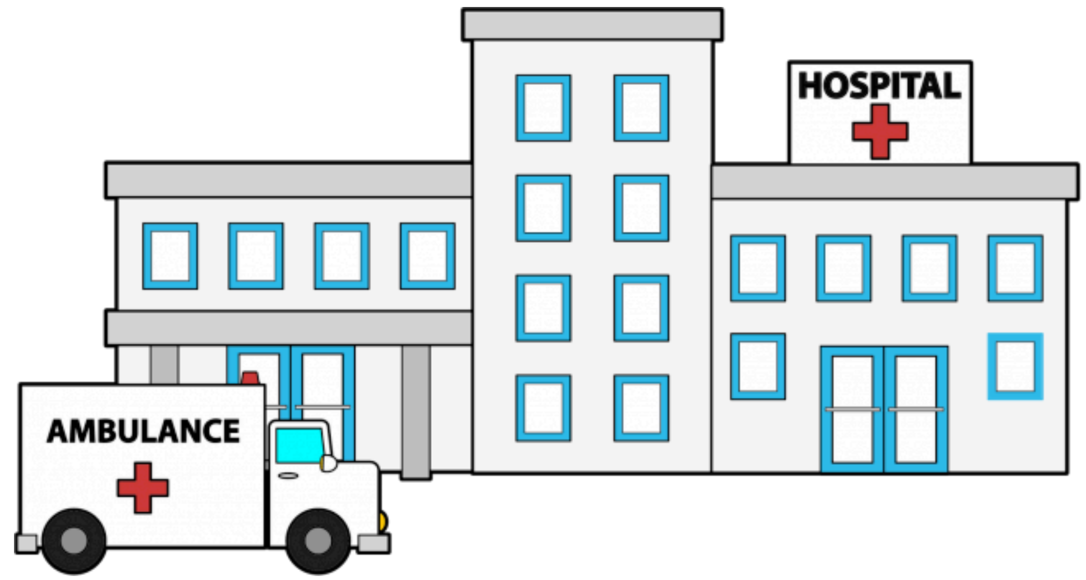
Find the winning bid, while keeping individual bids private. Everyone should learn **nothing beyond**, say,

$$y = \text{"identity of the largest bid} \geq m \text{ if one exists"}$$

i.e., more formally,

$$y = arg\,max\{w,x,m\}.$$

Etc.



Perform a scientific study on patient data,
without the hospital having to reveal such sensitive data.

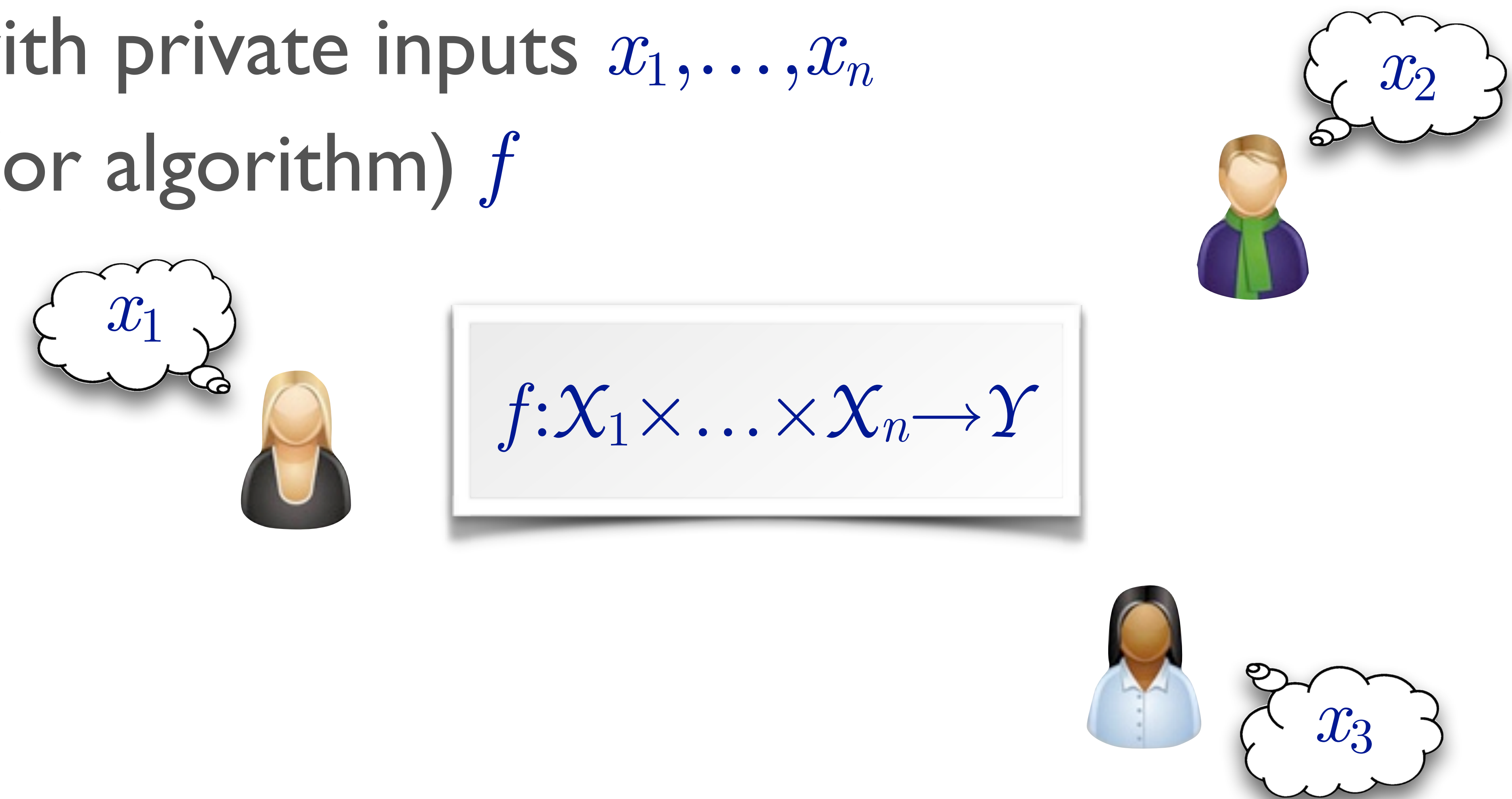
Etc. etc.

Find Facebook friends that are nearby, **without** letting Facebook (or friends not nearby) know where you are.

The General Goal

Given:

- n parties with private inputs $x_1, \ldots, x_n$
- a function (or algorithm) f



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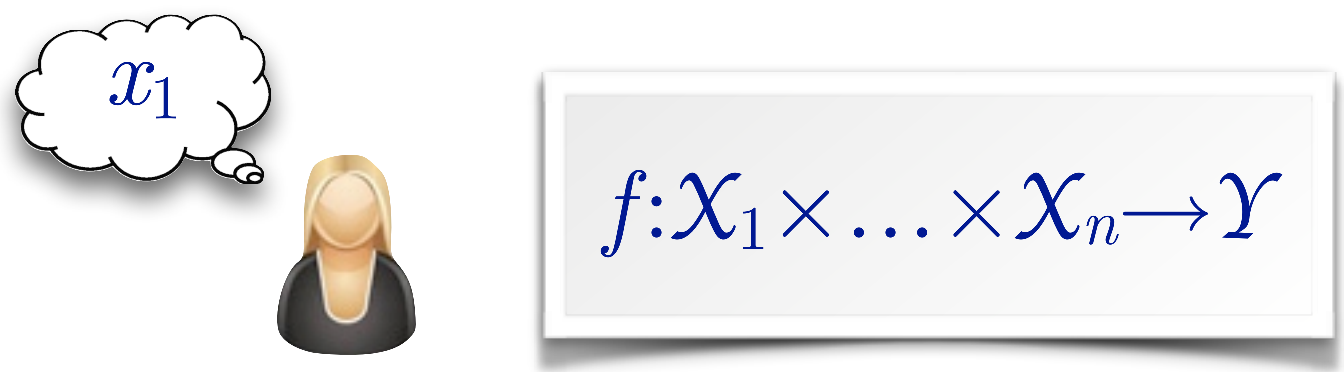
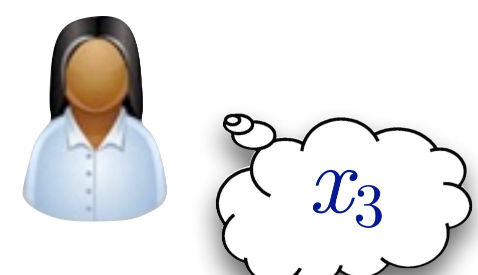




Want: compute $y=f(x_1,\ldots,x_n)$, so that

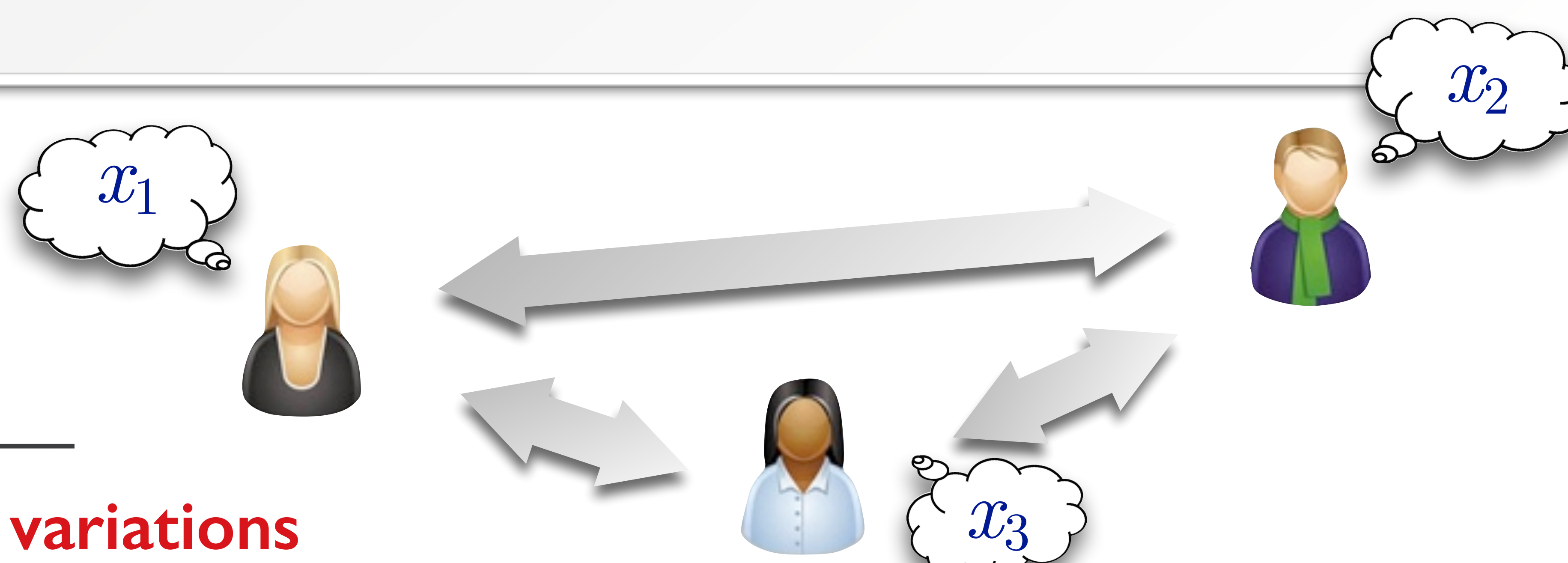
- everyone learns the (correct) result $y=f(x_1,\ldots,x_n)$
- but **nothing more** (in particular, the x_i **remain secret**)

Multiparty Computation (MPC)

Fundamental Theorem of MPC (*)

Originally invented/proven by [Yao 80's, Goldwasser-Micali-Wigderson 87, Chaum-Crépeau-Damgård 88, BenOr-Goldwasser-Wigderson 88]

Any function $f{:}\mathcal{X}_1\times\ldots\times\mathcal{X}_n\rightarrow\mathcal{Y}$ can be jointly computed by means of an **interactive protocol** in a **secure** way, so that:



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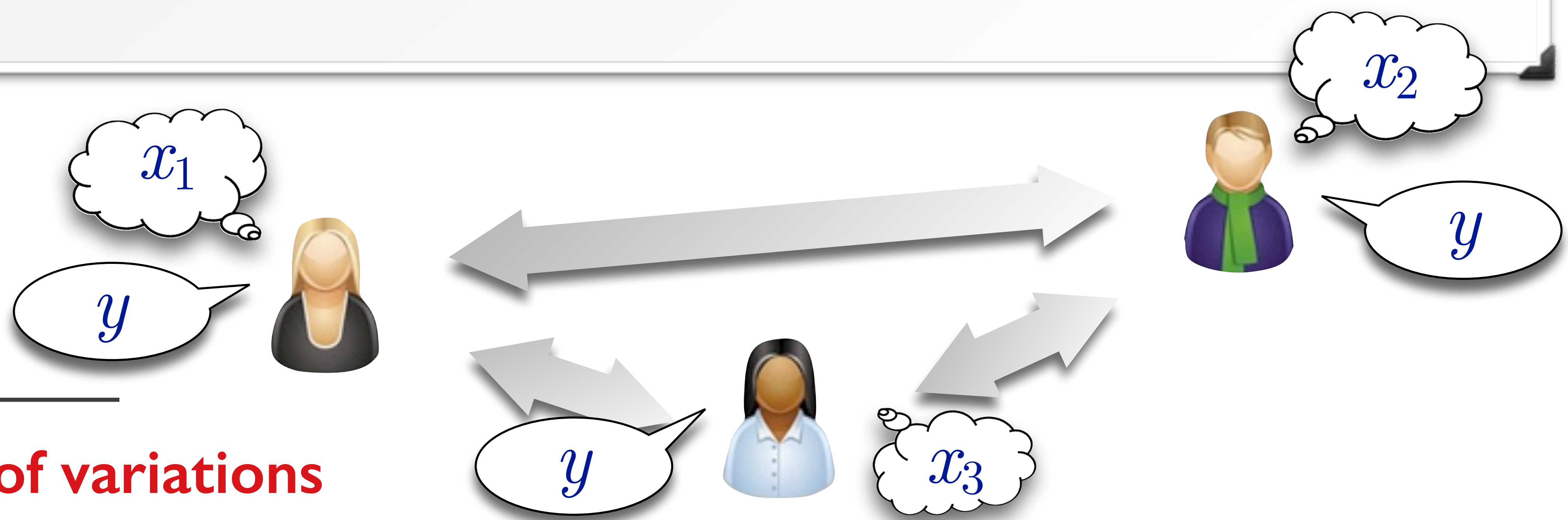

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- everyone learns the **correct result** $y = f(x_1,\ldots,x_n)$,
- yet **nothing more** than than,
- even if some of the parties are **dishonest**.

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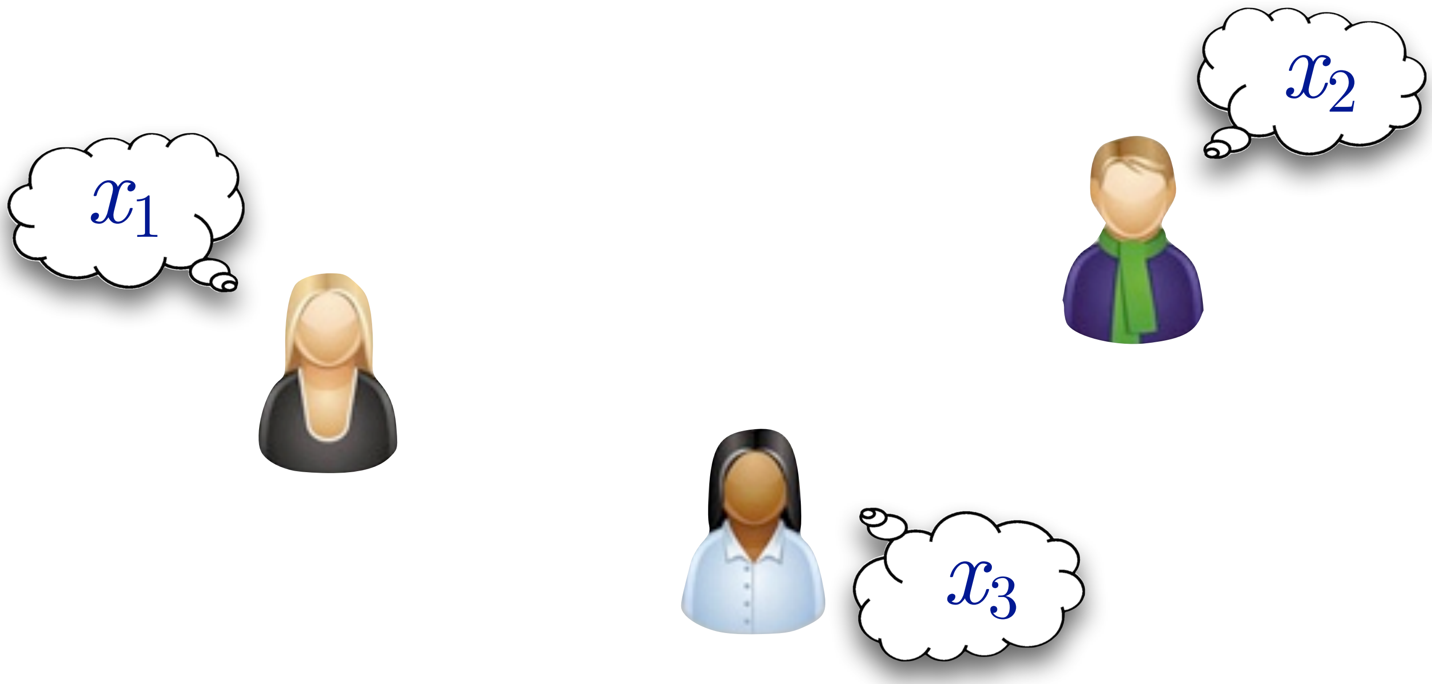
Multiparty Computation (MPC) - In Clip Arts

Fundamental Theorem of MPC (*)



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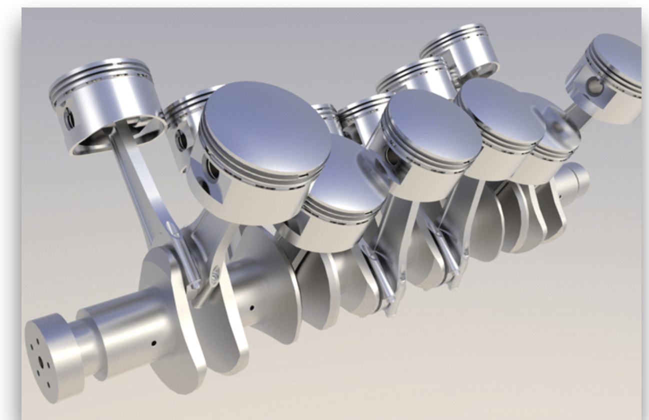

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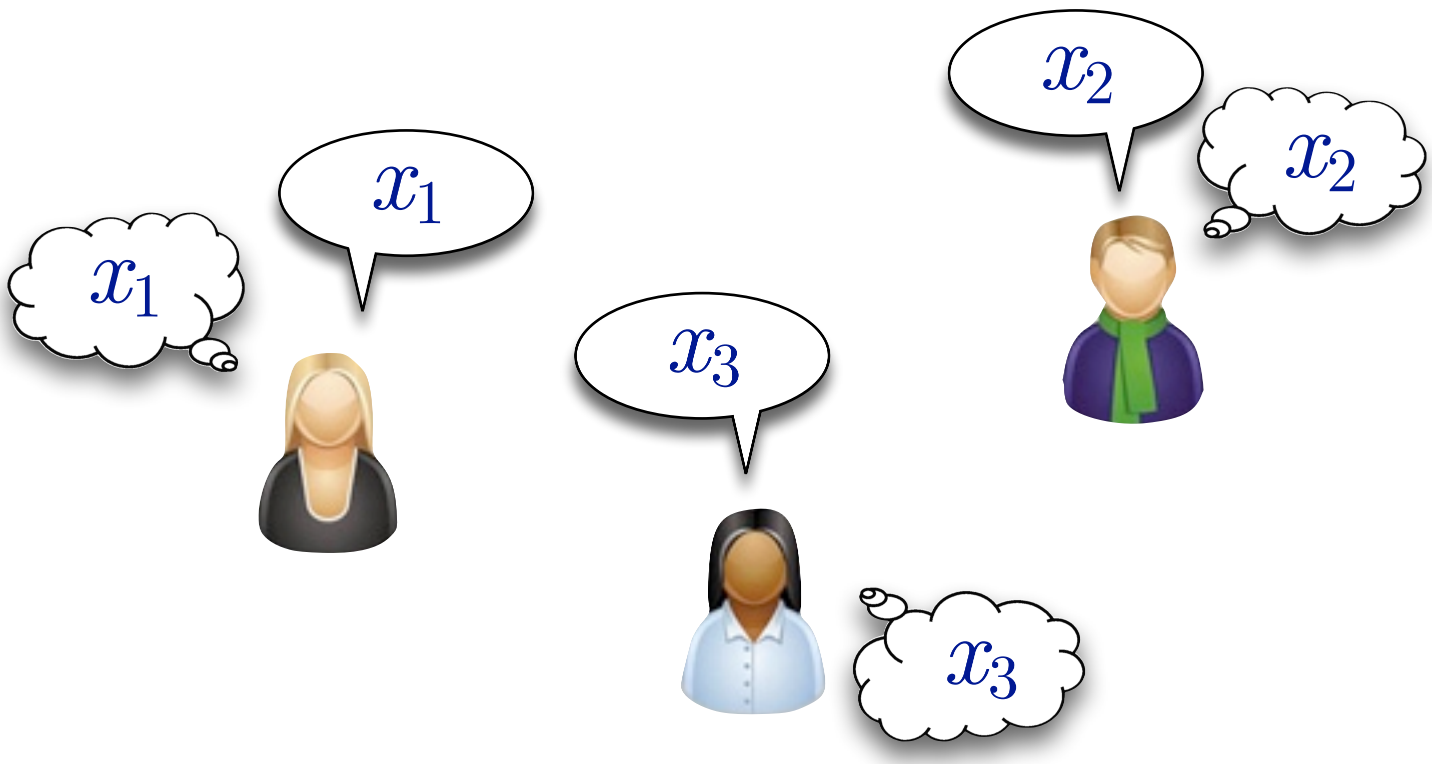

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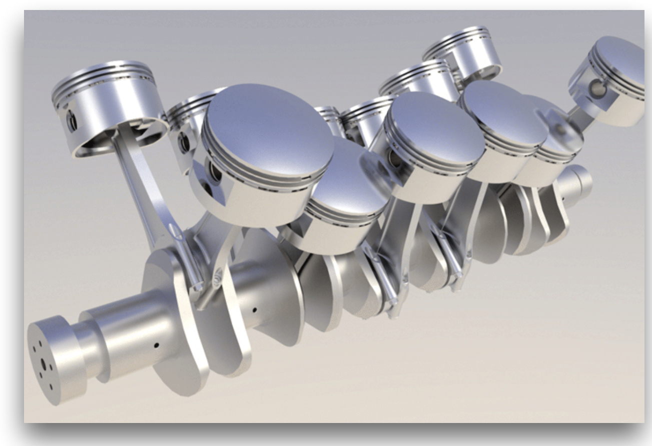

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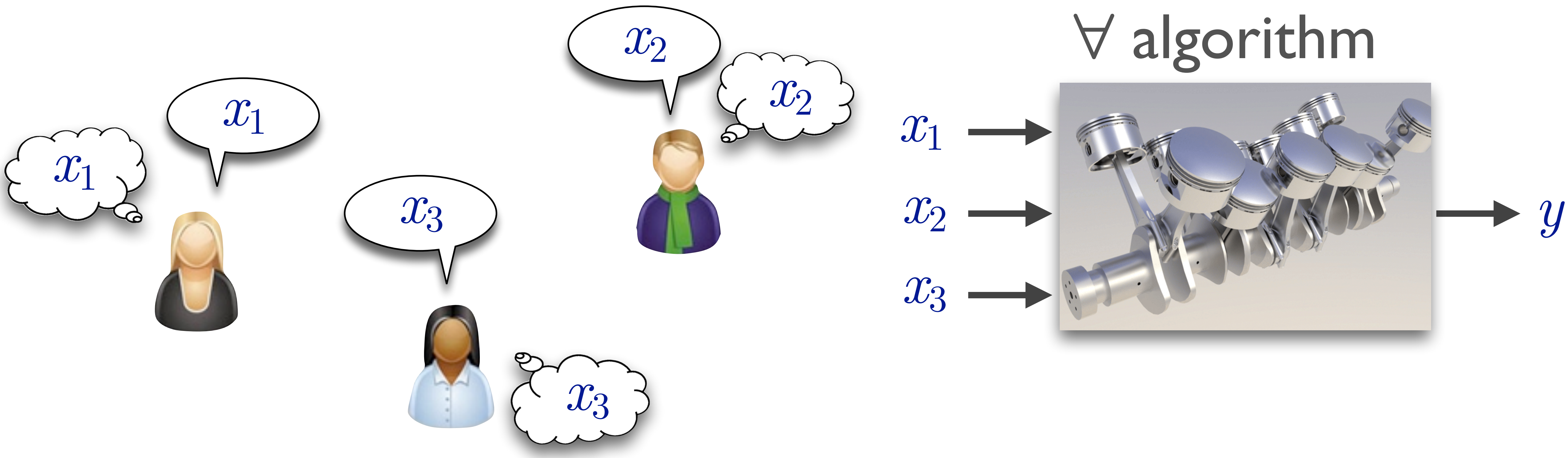

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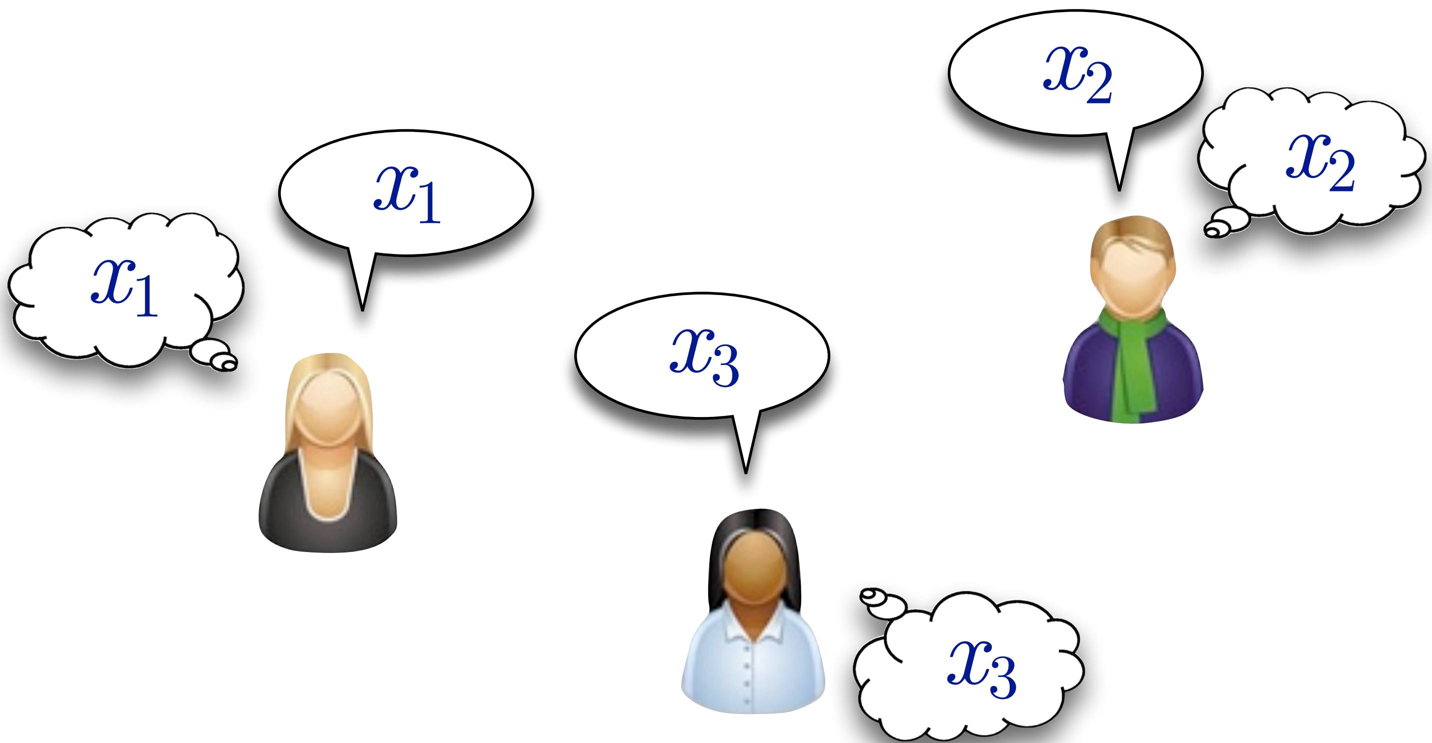

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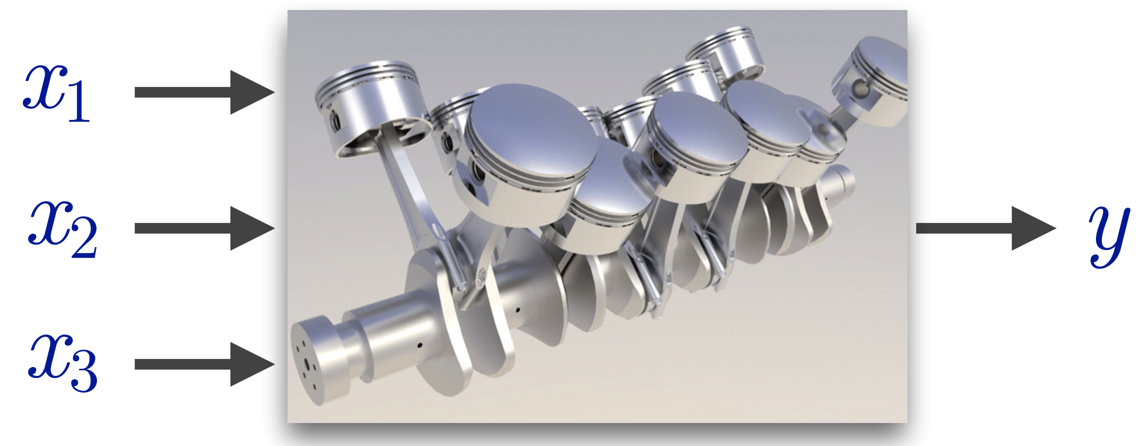
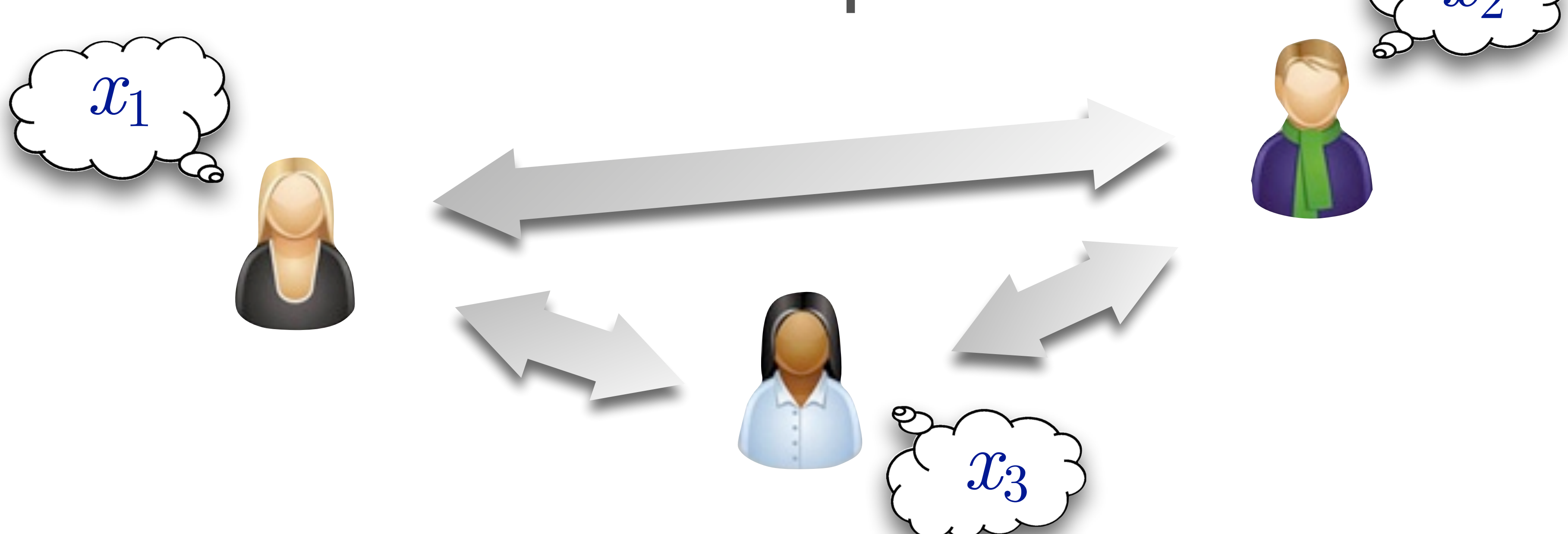


$\exists$ interactive protocol



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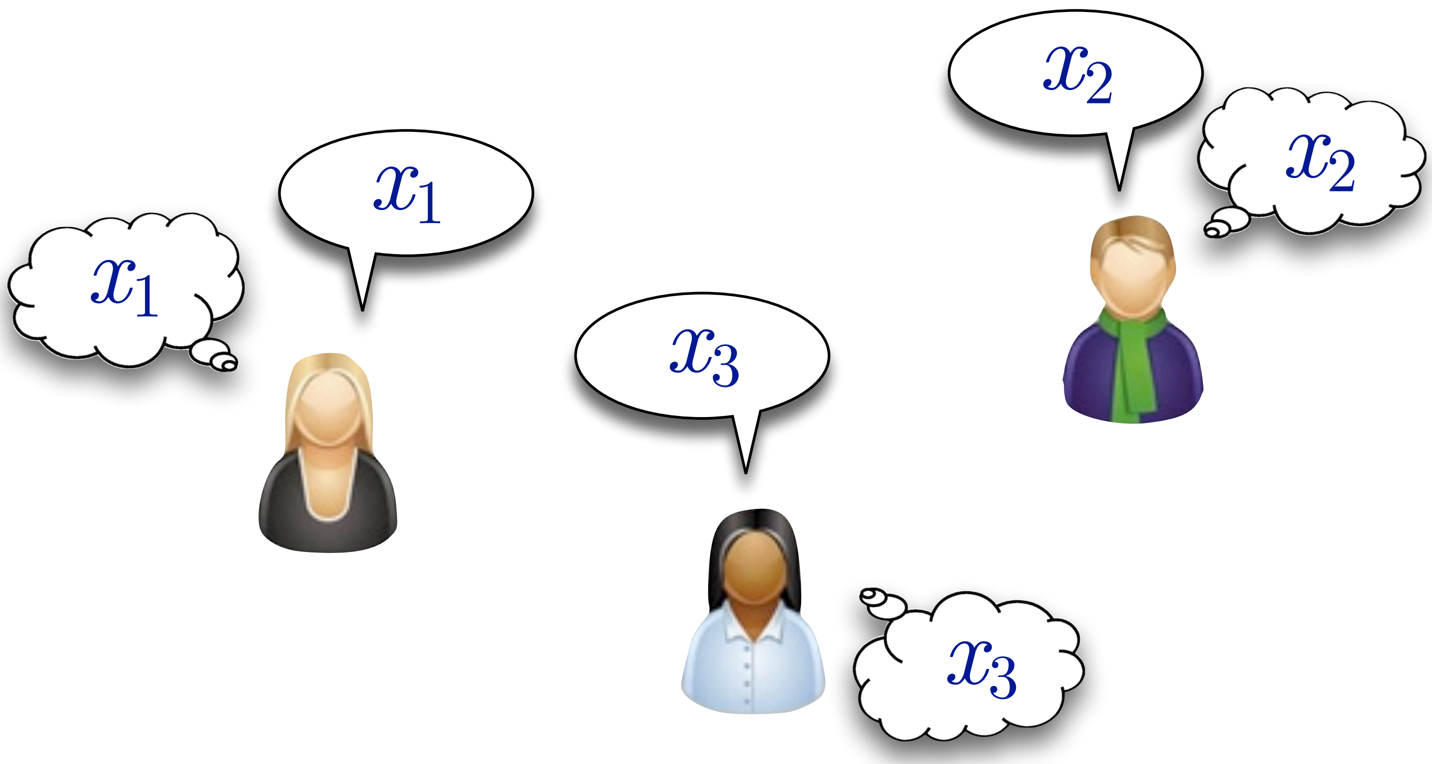

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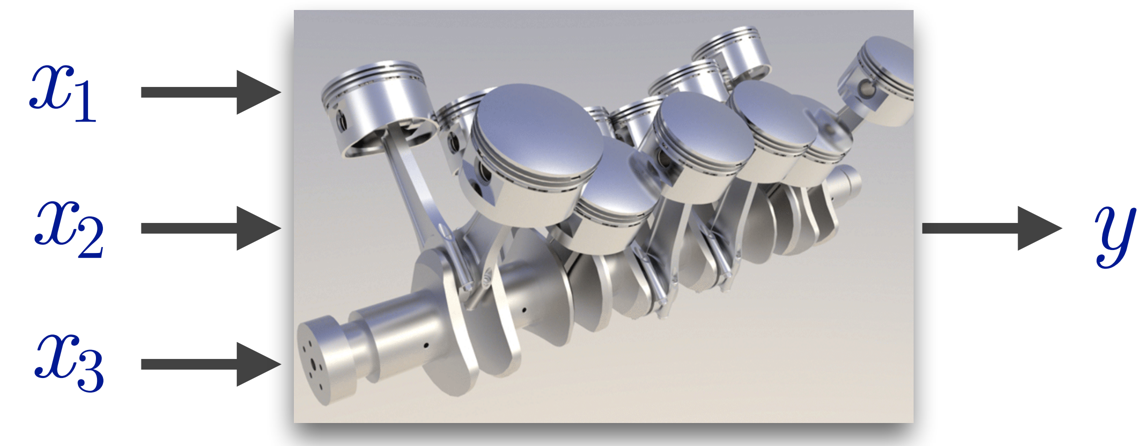

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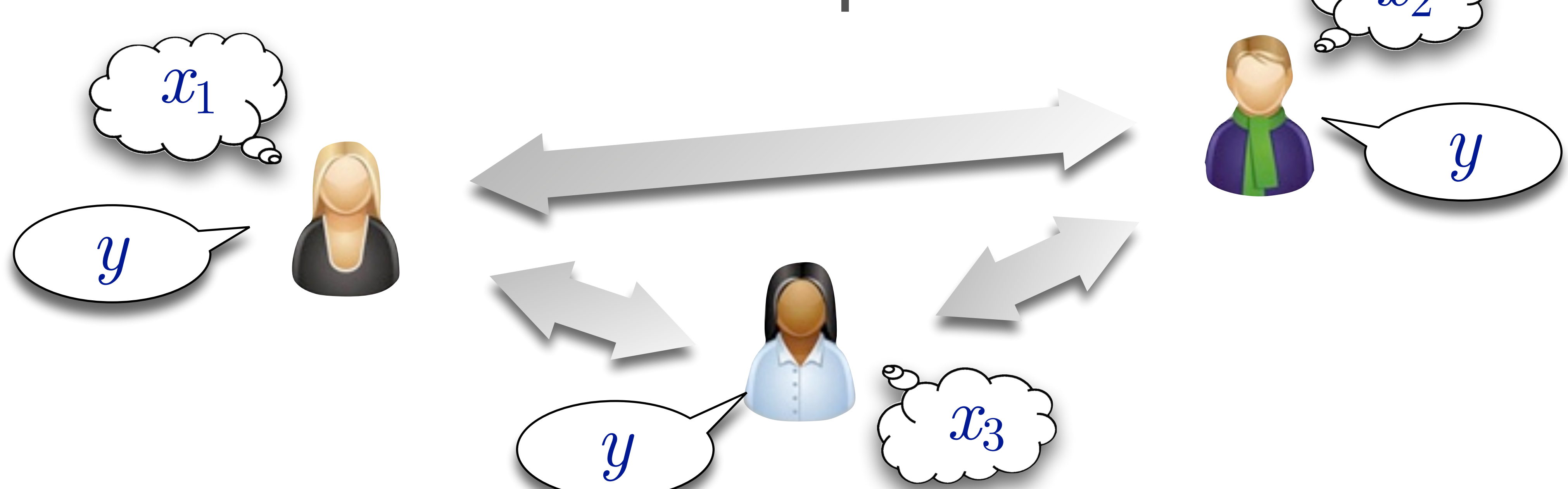

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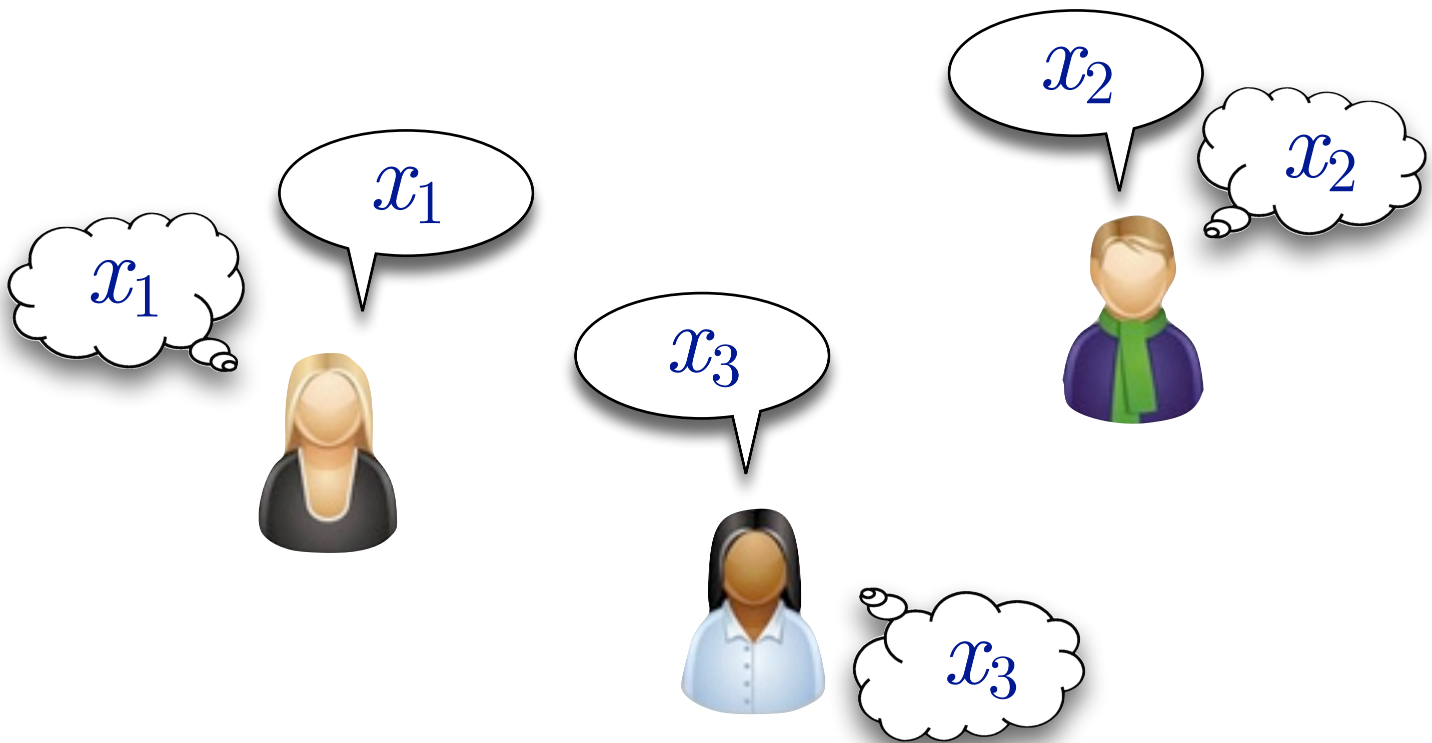

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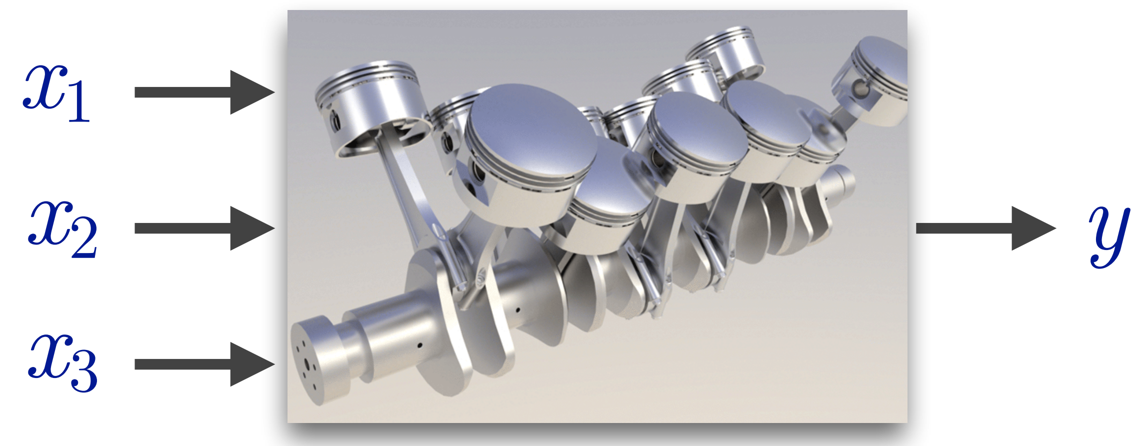
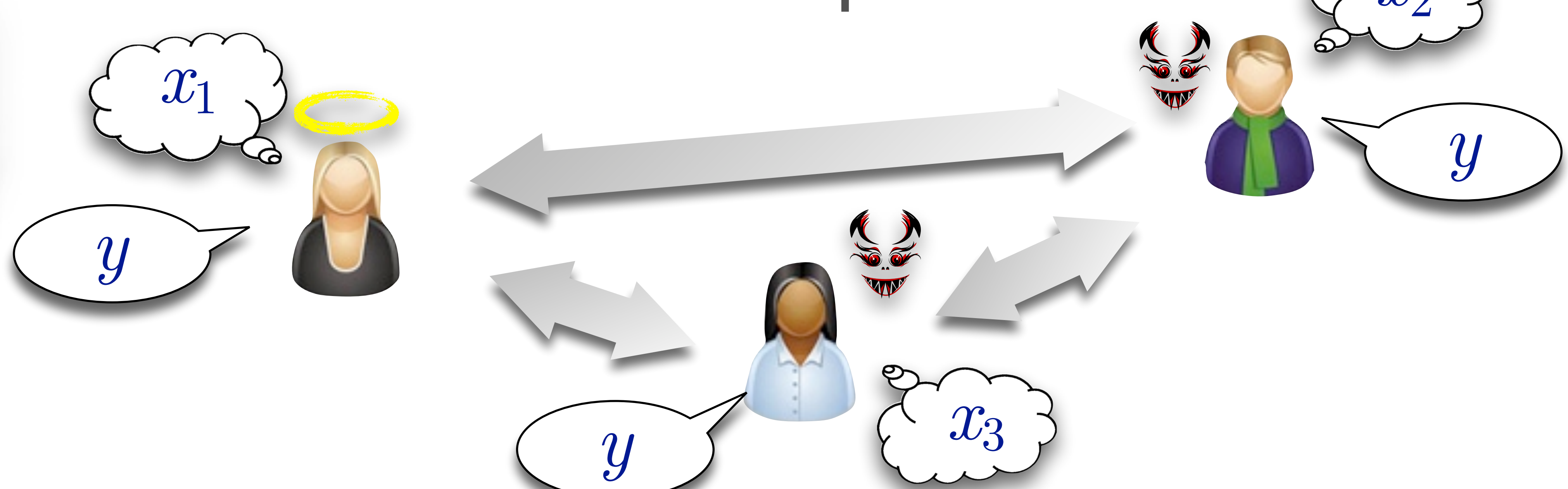


$\exists$ interactive protocol

x_i's remain secret



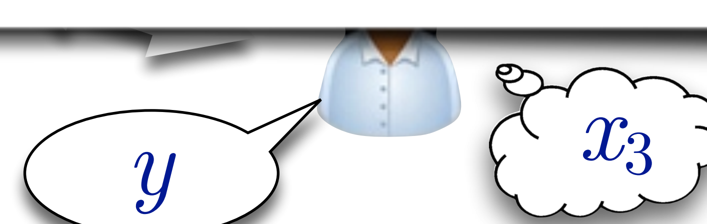
Multiparty Computation (MPC) - In Clip Arts

Fundamental Theorem of MPC (*)

(*) Comes in lots of **different variations**, in terms of:

- number of conspiring dishonest parties it tolerates
- assumed capabilities of dishonest parties
- considered communication infrastructure
- (dis)allowing the protocol to abort
- (not) requiring fairness and/or cheater detection
- etc.

Also, comes with a (significant) **overhead** in computation and communication.



Road Map

- WHAT is multiparty computation?
- HOW does multiparty computation work?
- WHERE can/is multiparty computation be/ used?

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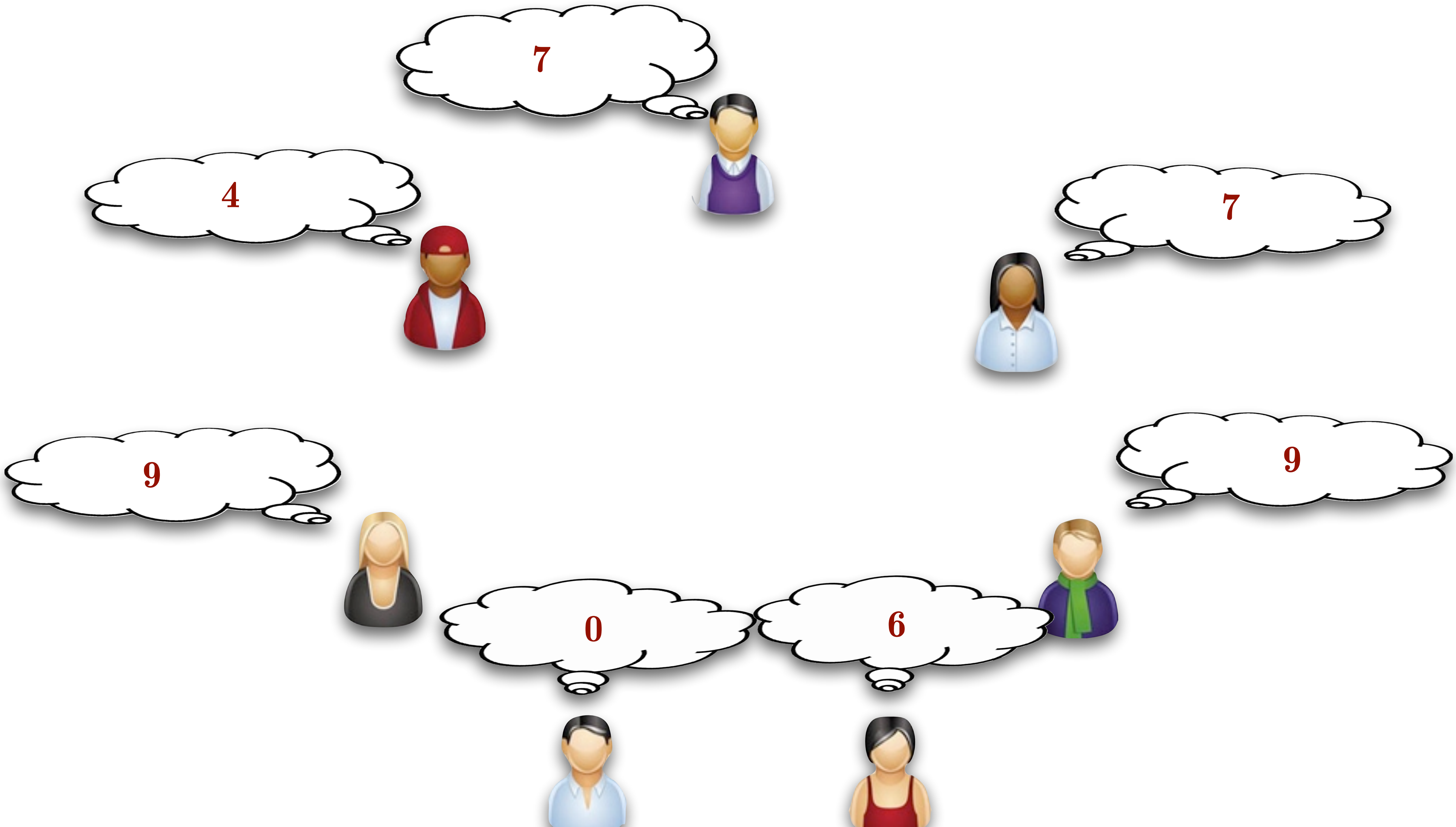
Goal: Computing the sum, i.e., $f(x_1,\ldots,x_n) = \sum x_i$



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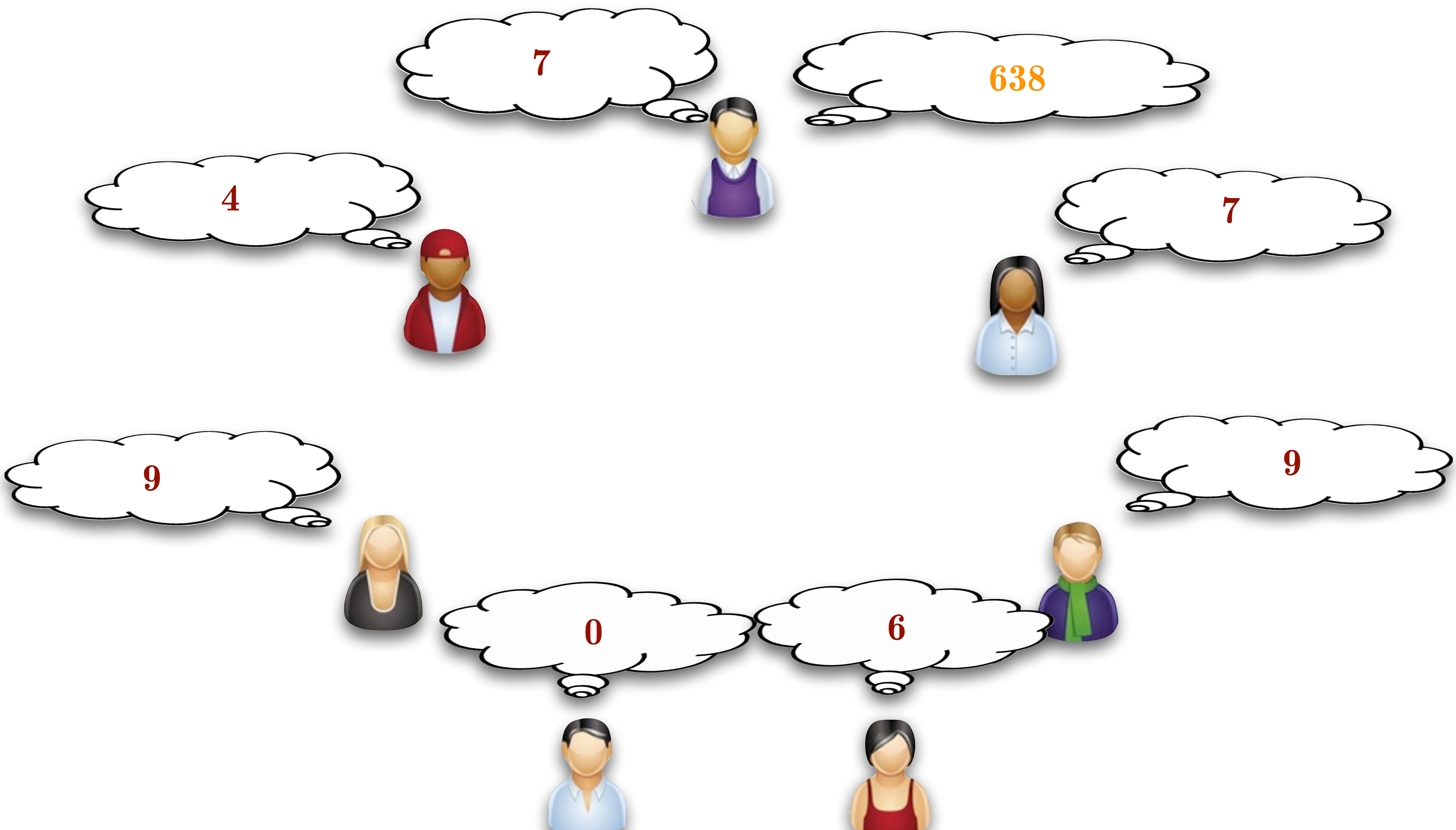

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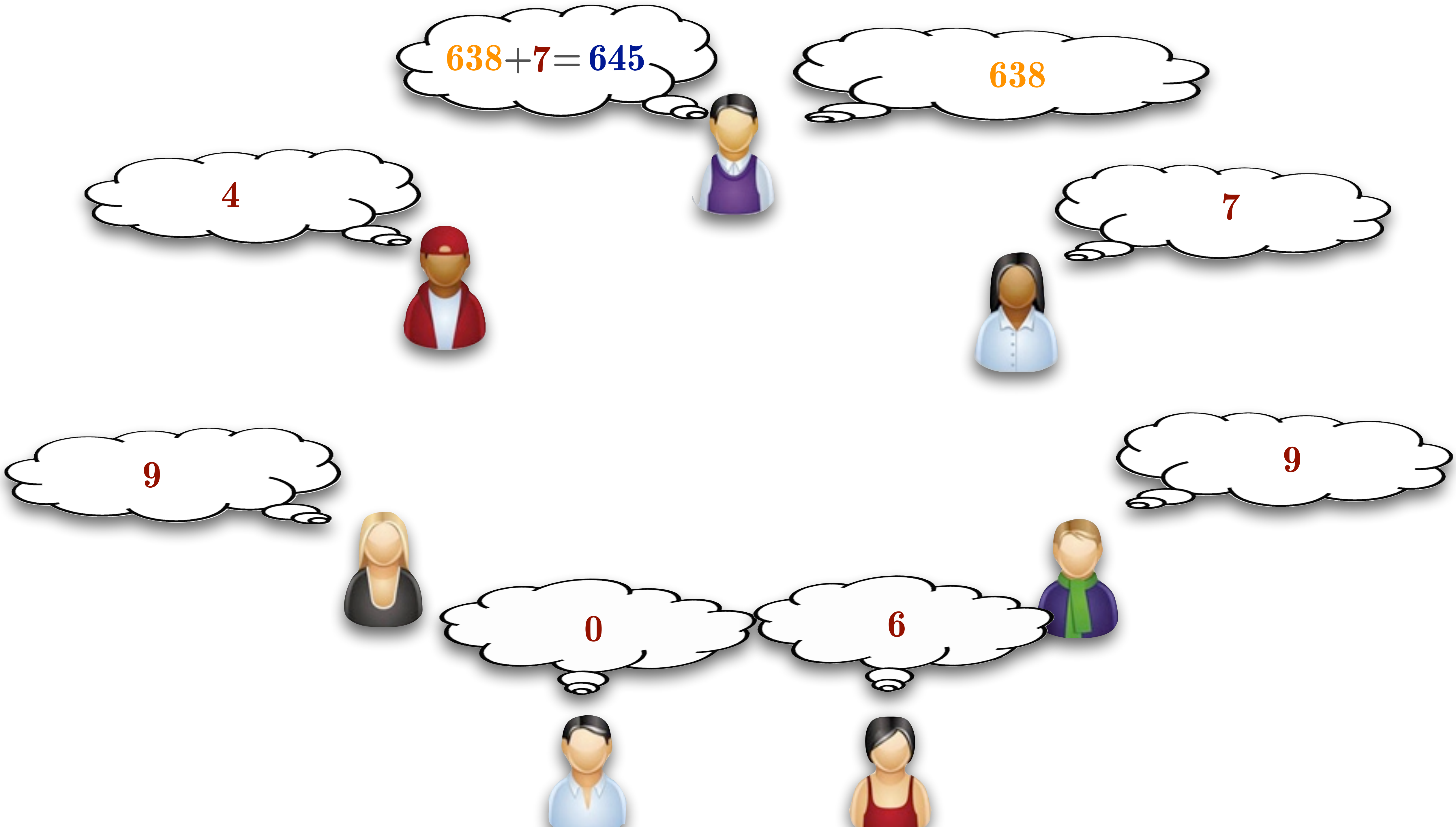

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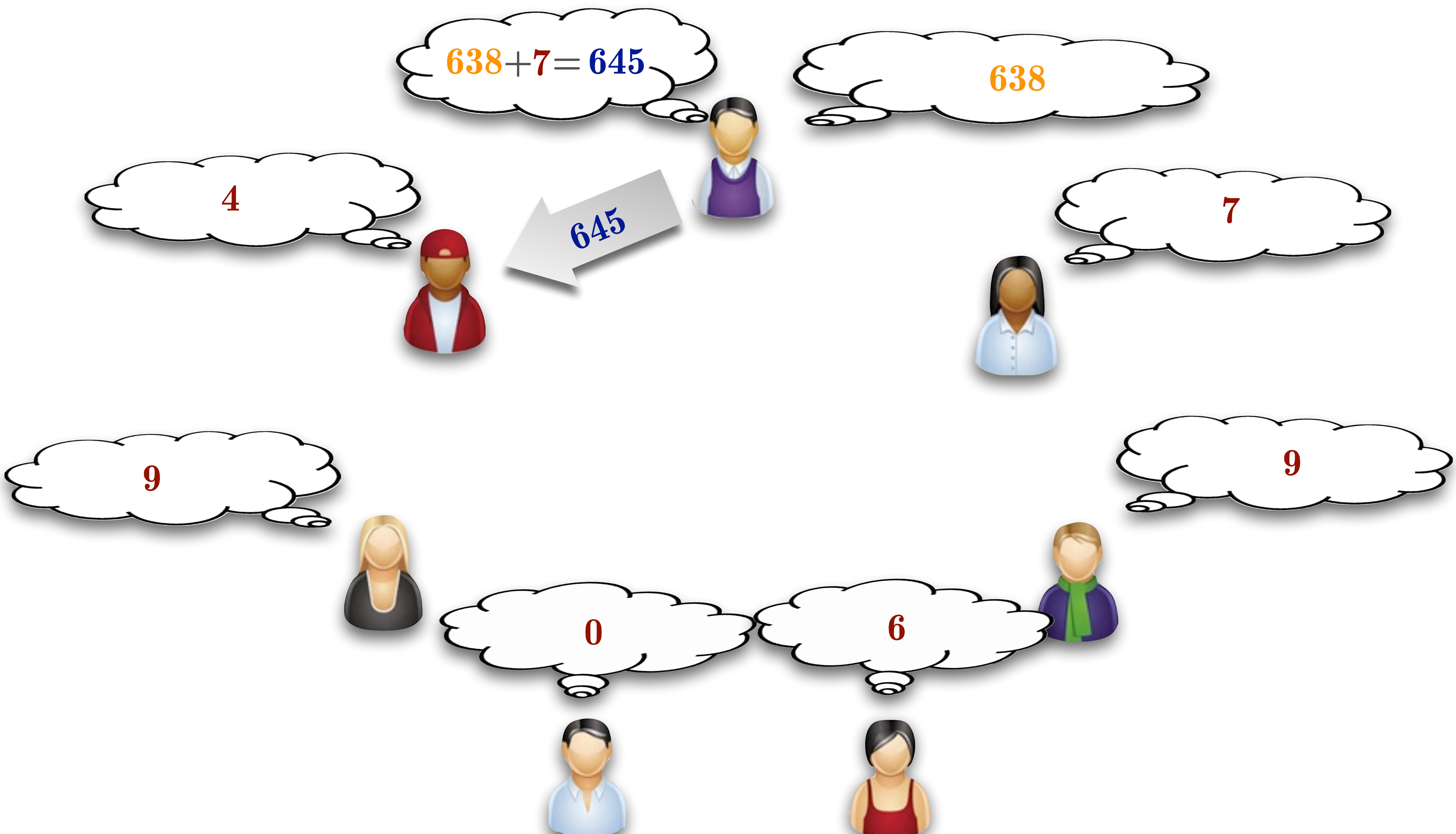

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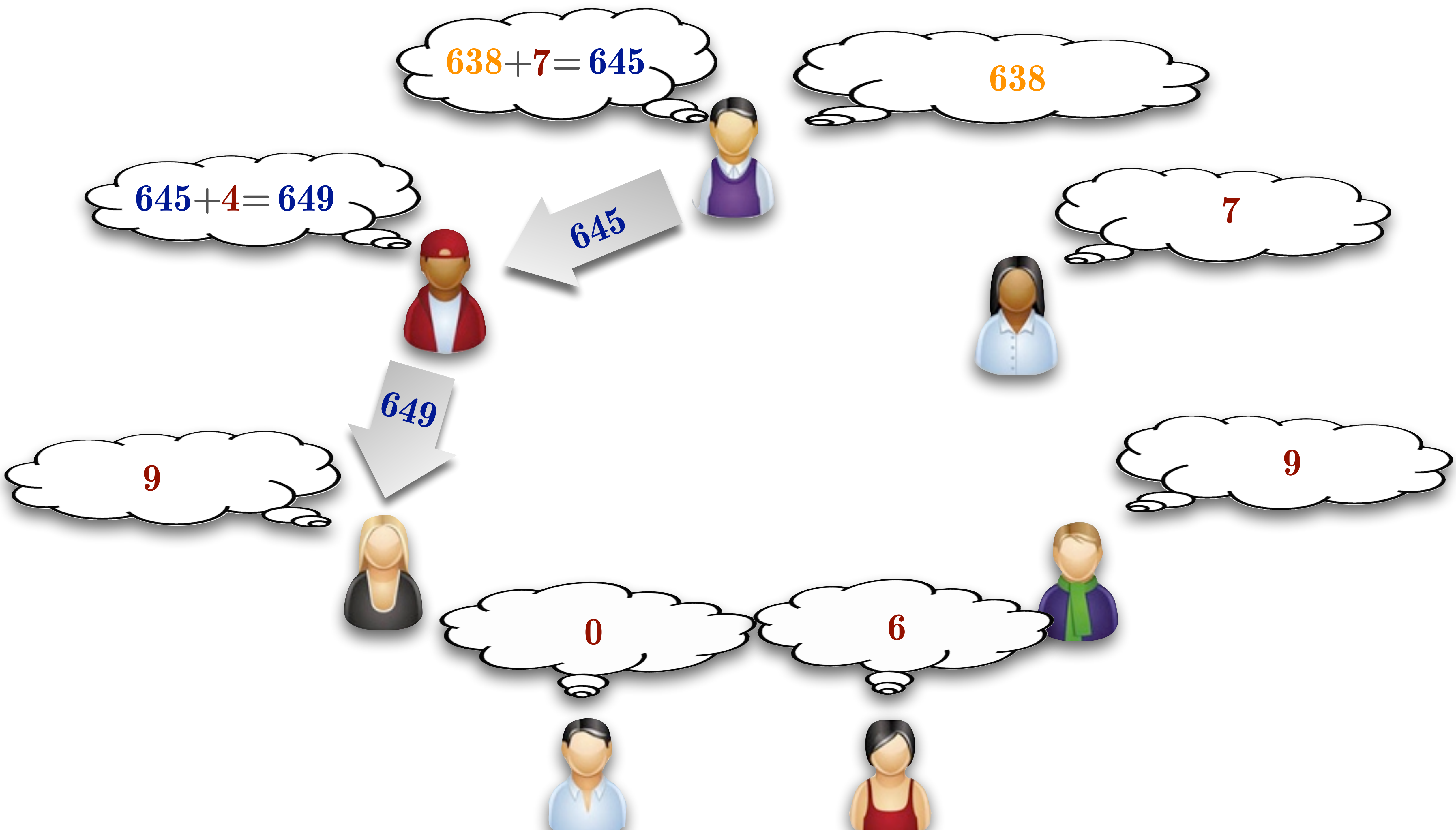

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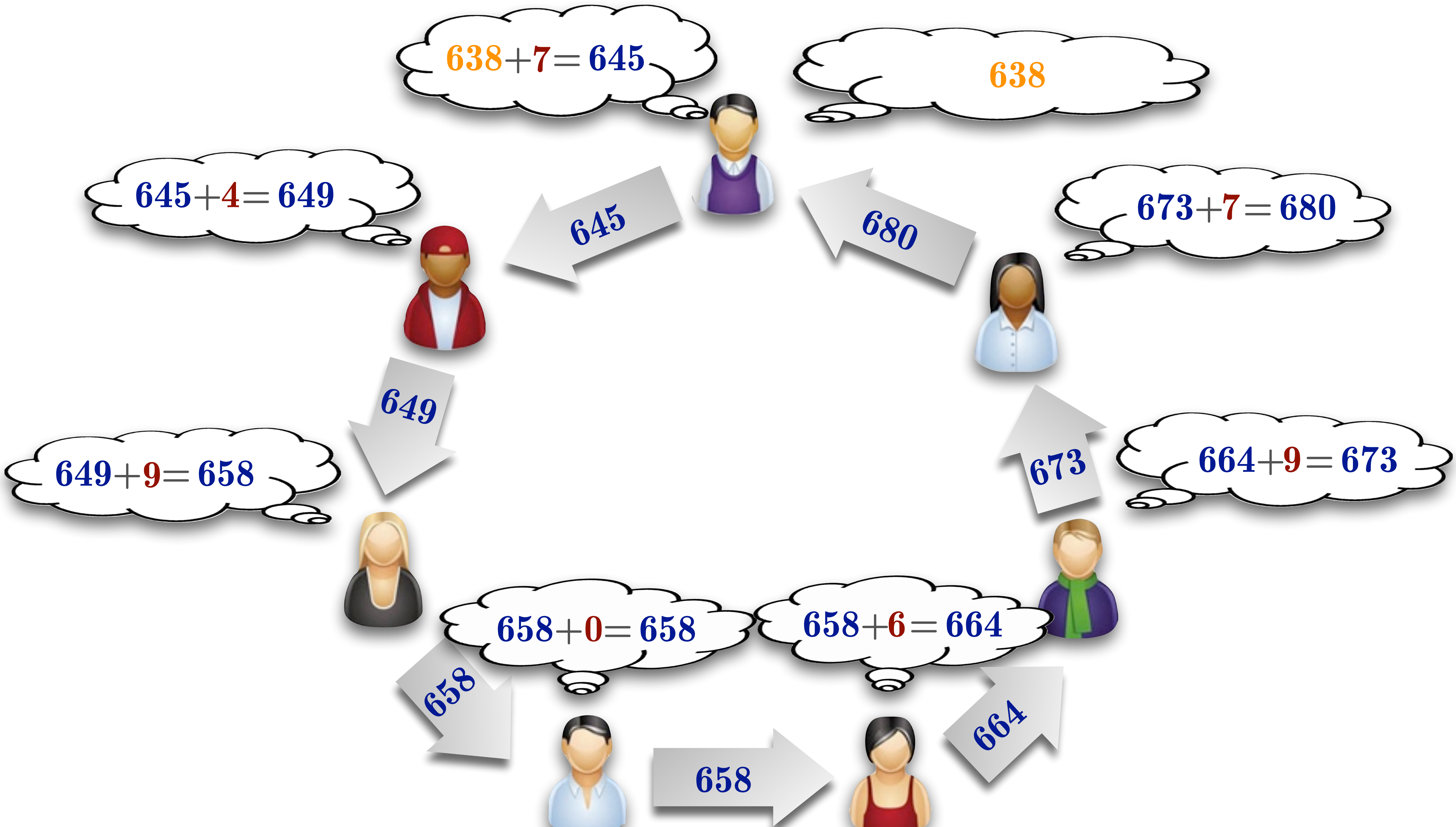

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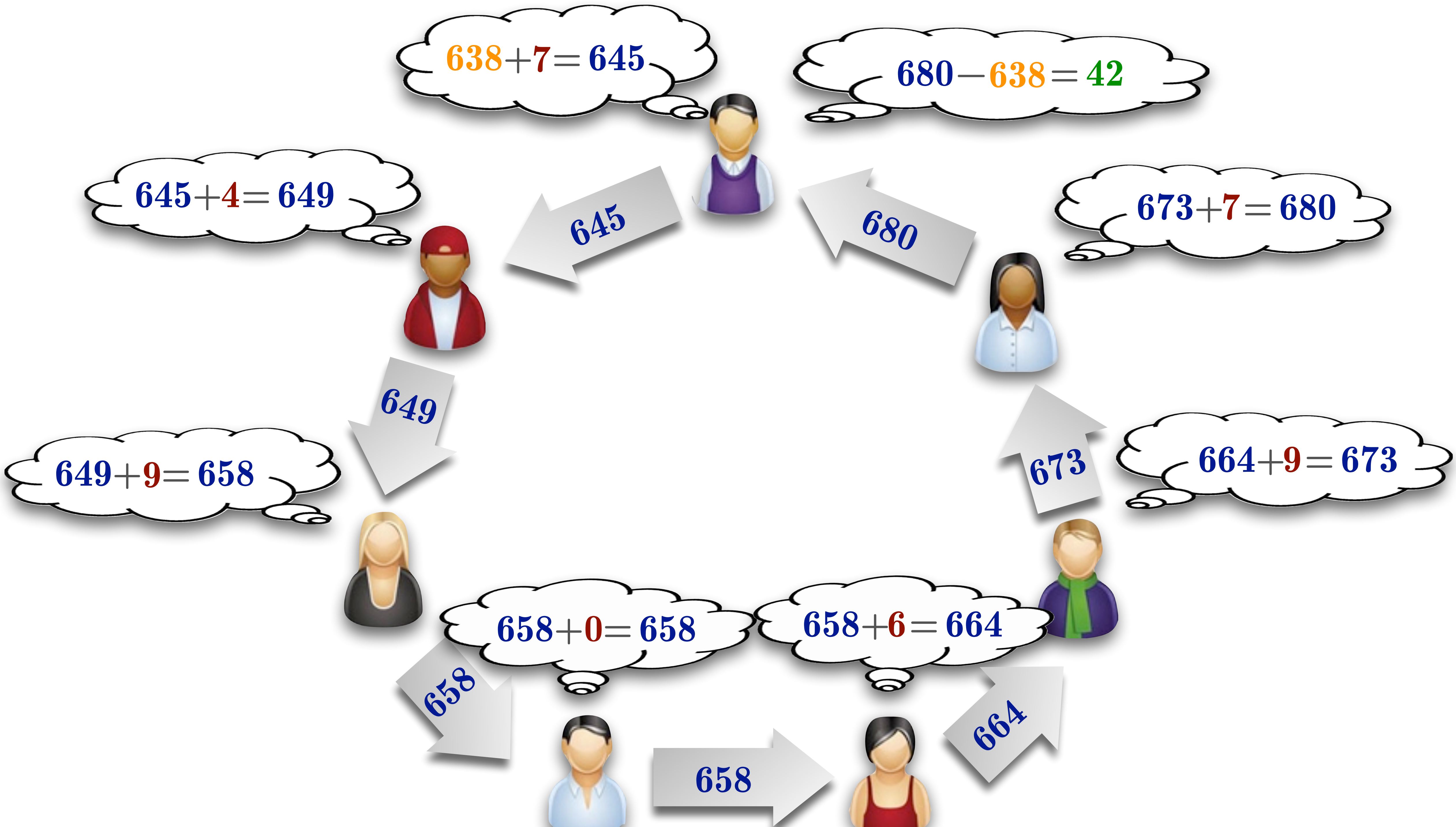

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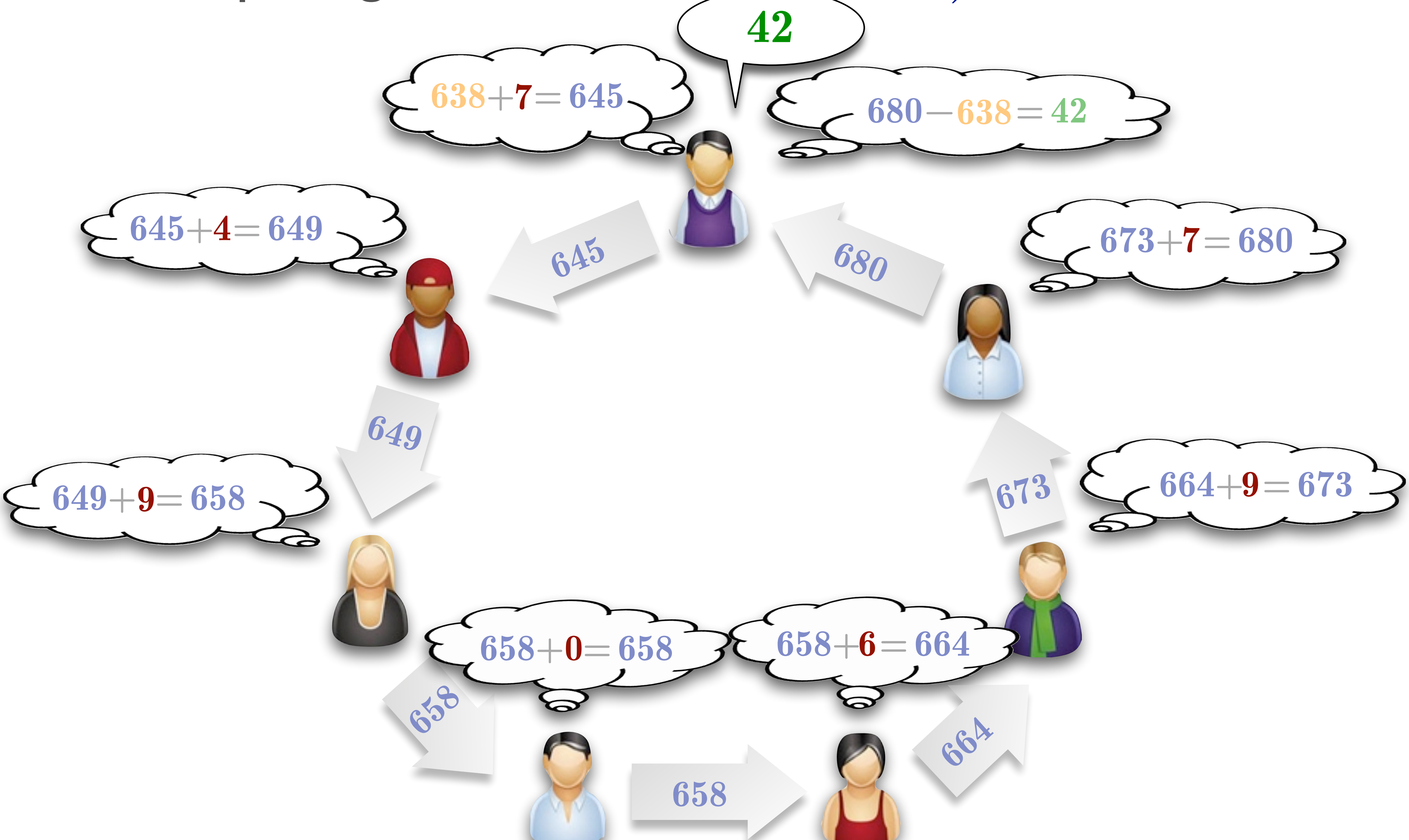

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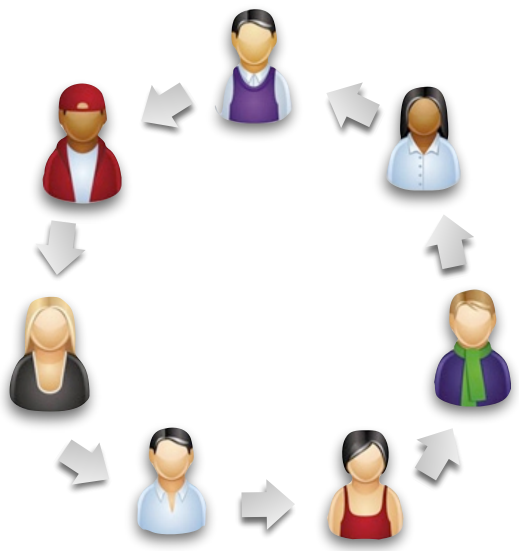

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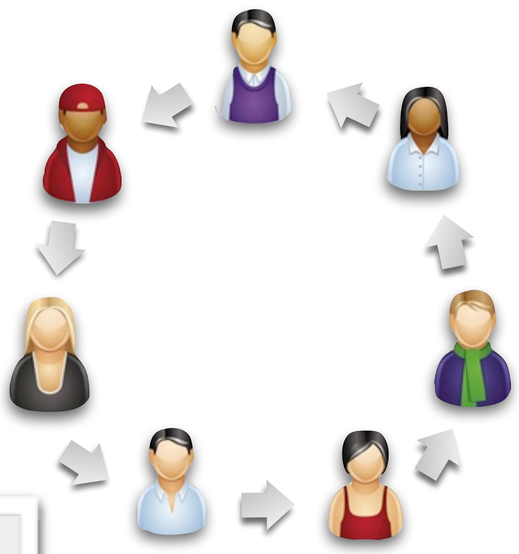

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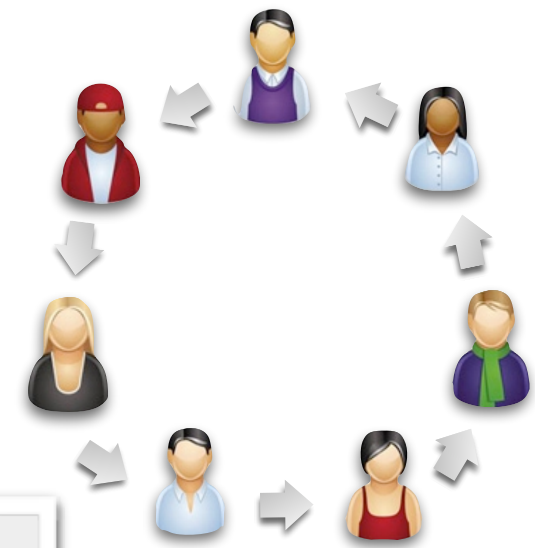

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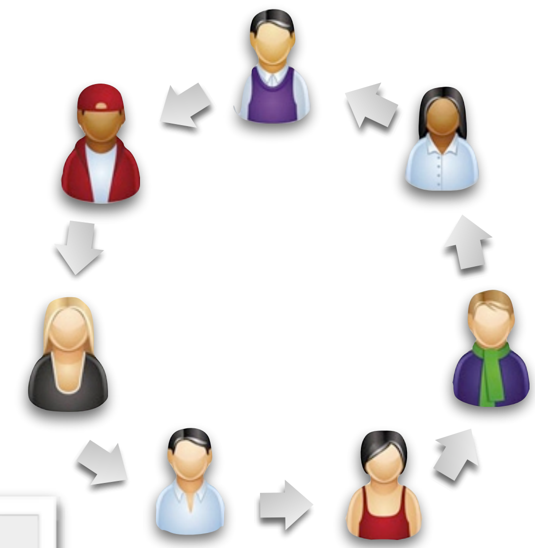

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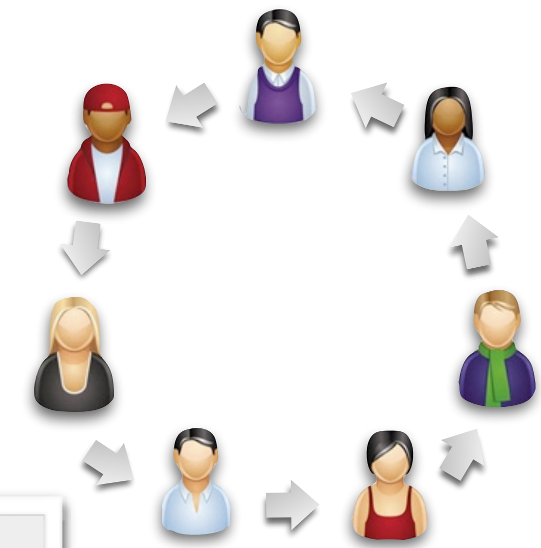

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Approach/solution **limited** to linear functions

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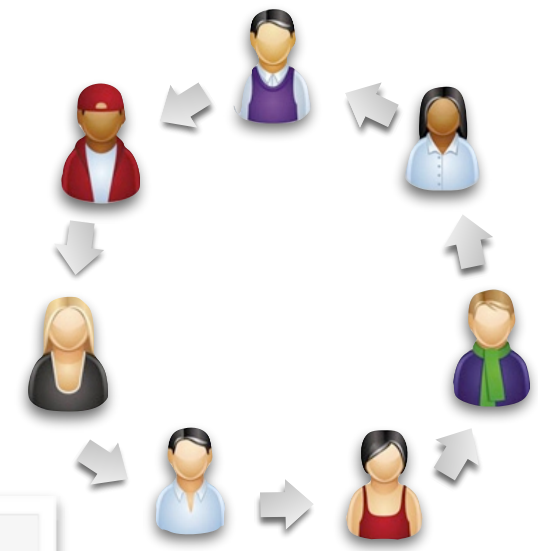

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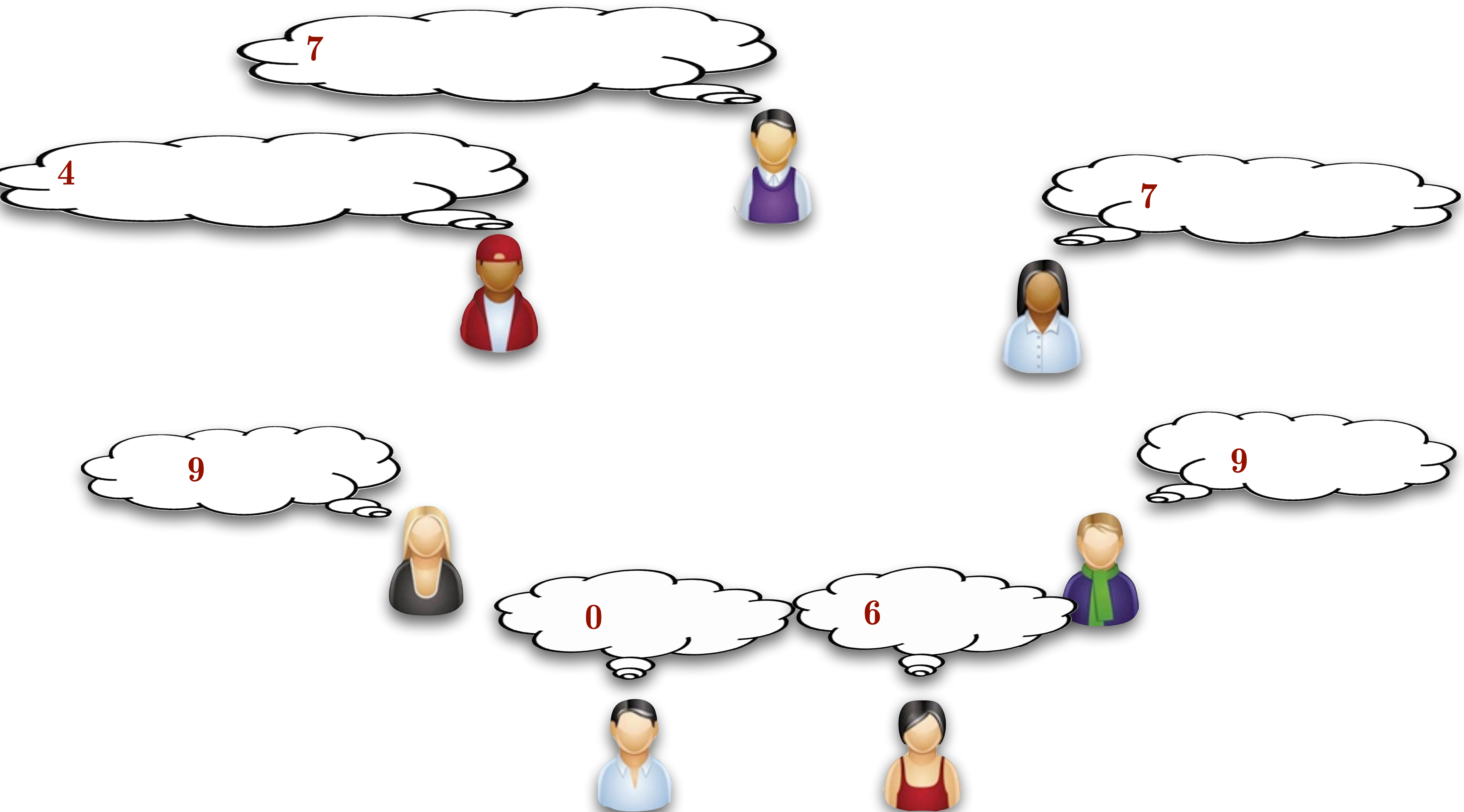

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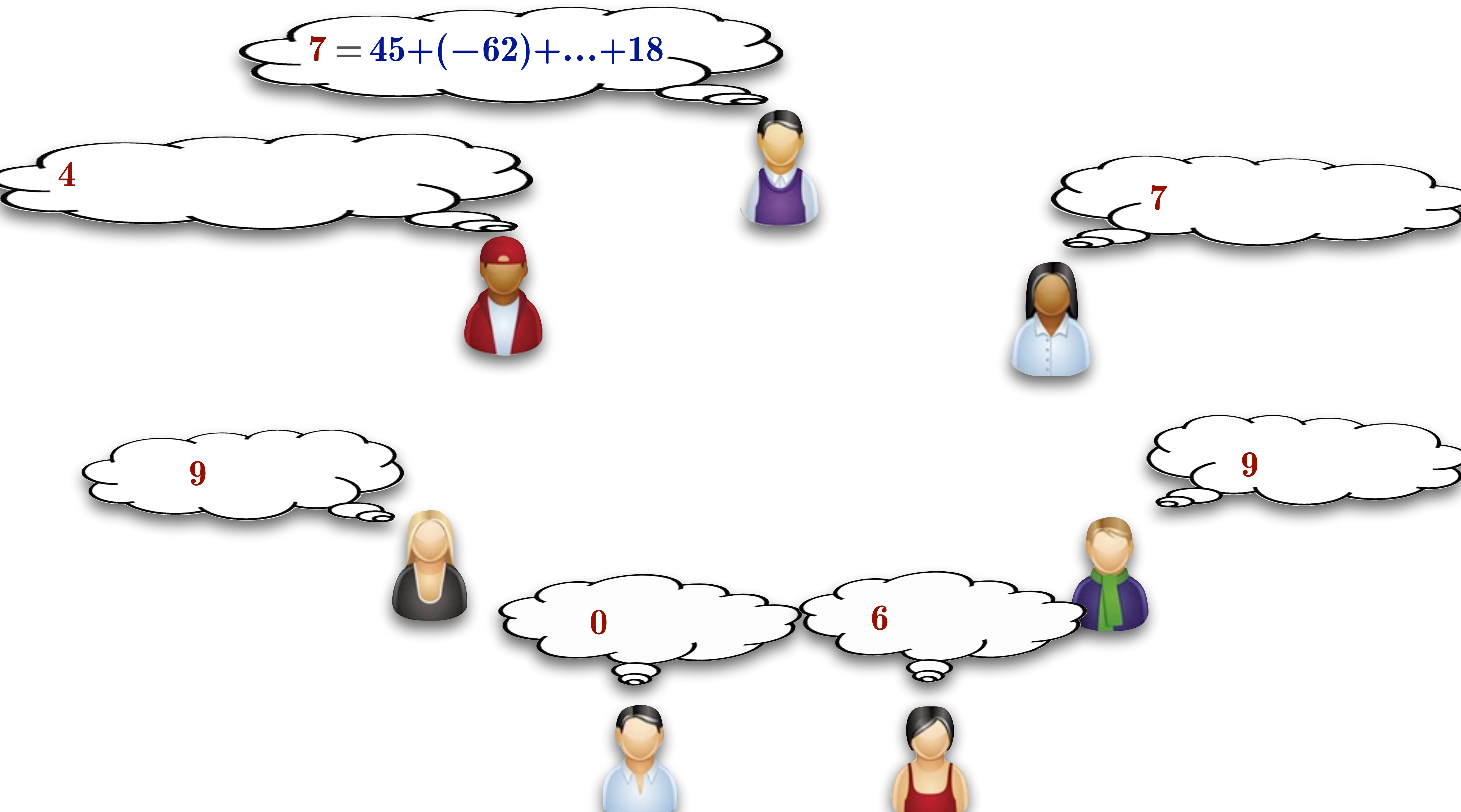

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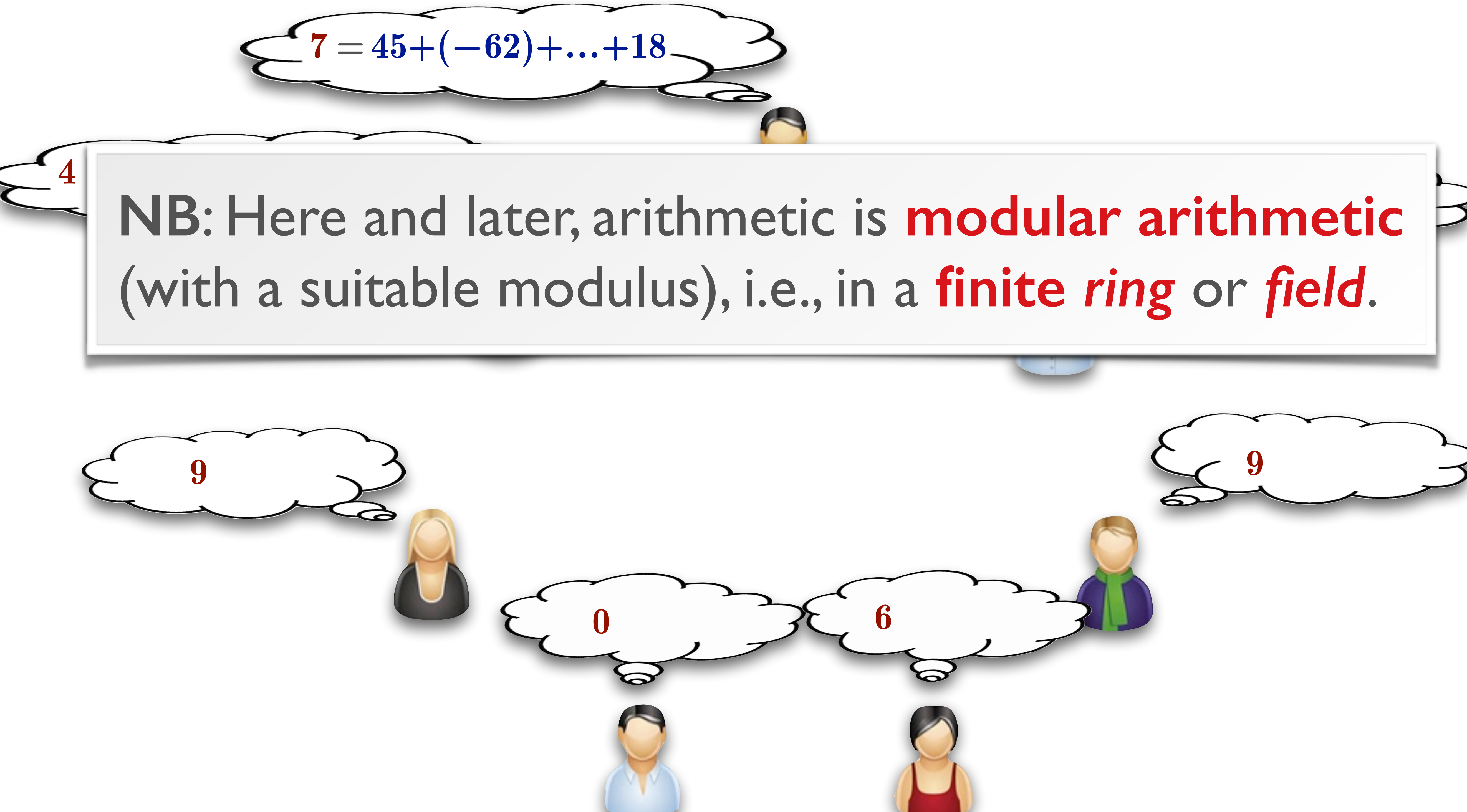

NB: Here and later, arithmetic is **modular arithmetic** (with a suitable modulus), i.e., in a **finite *ring*** or ***field***.

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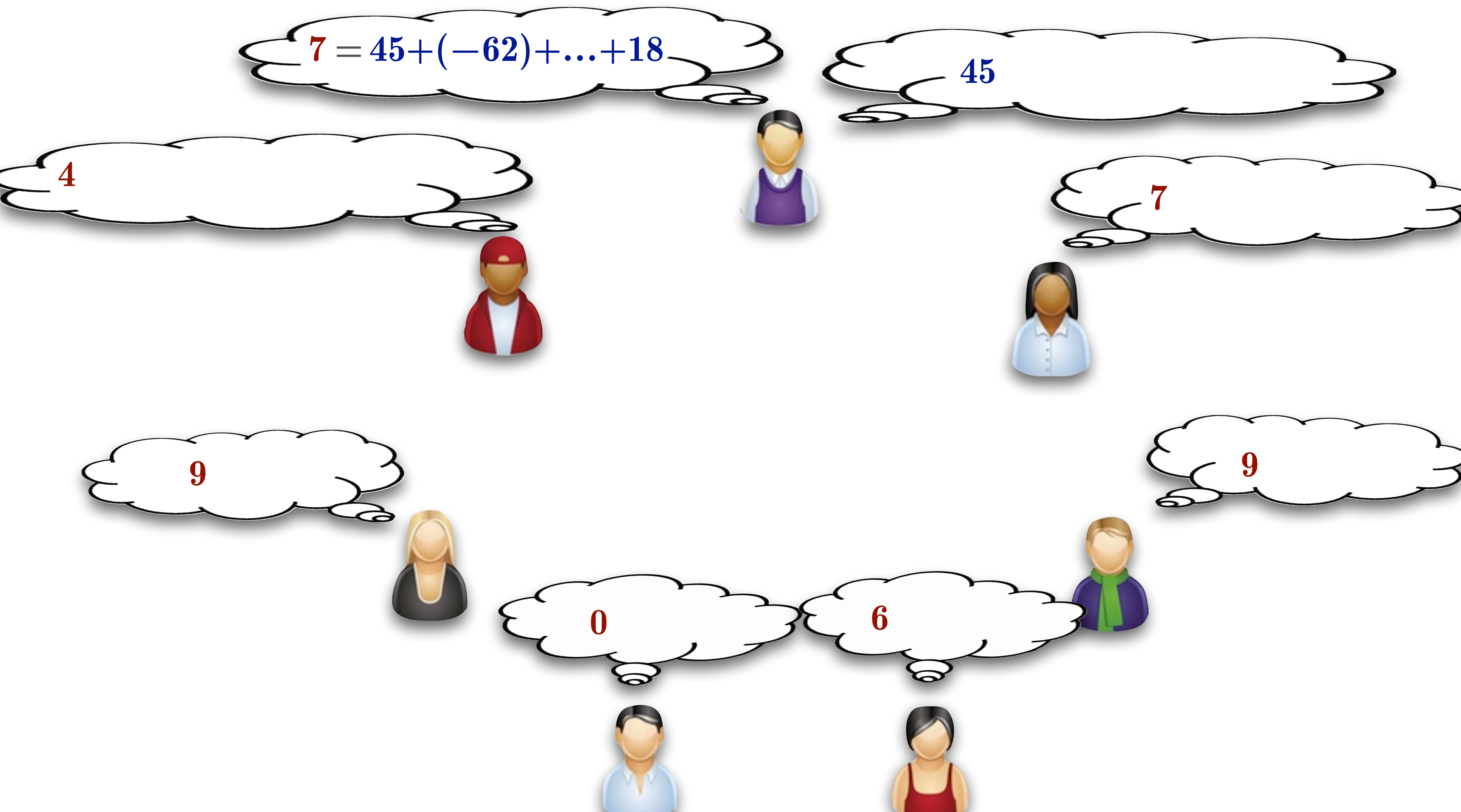

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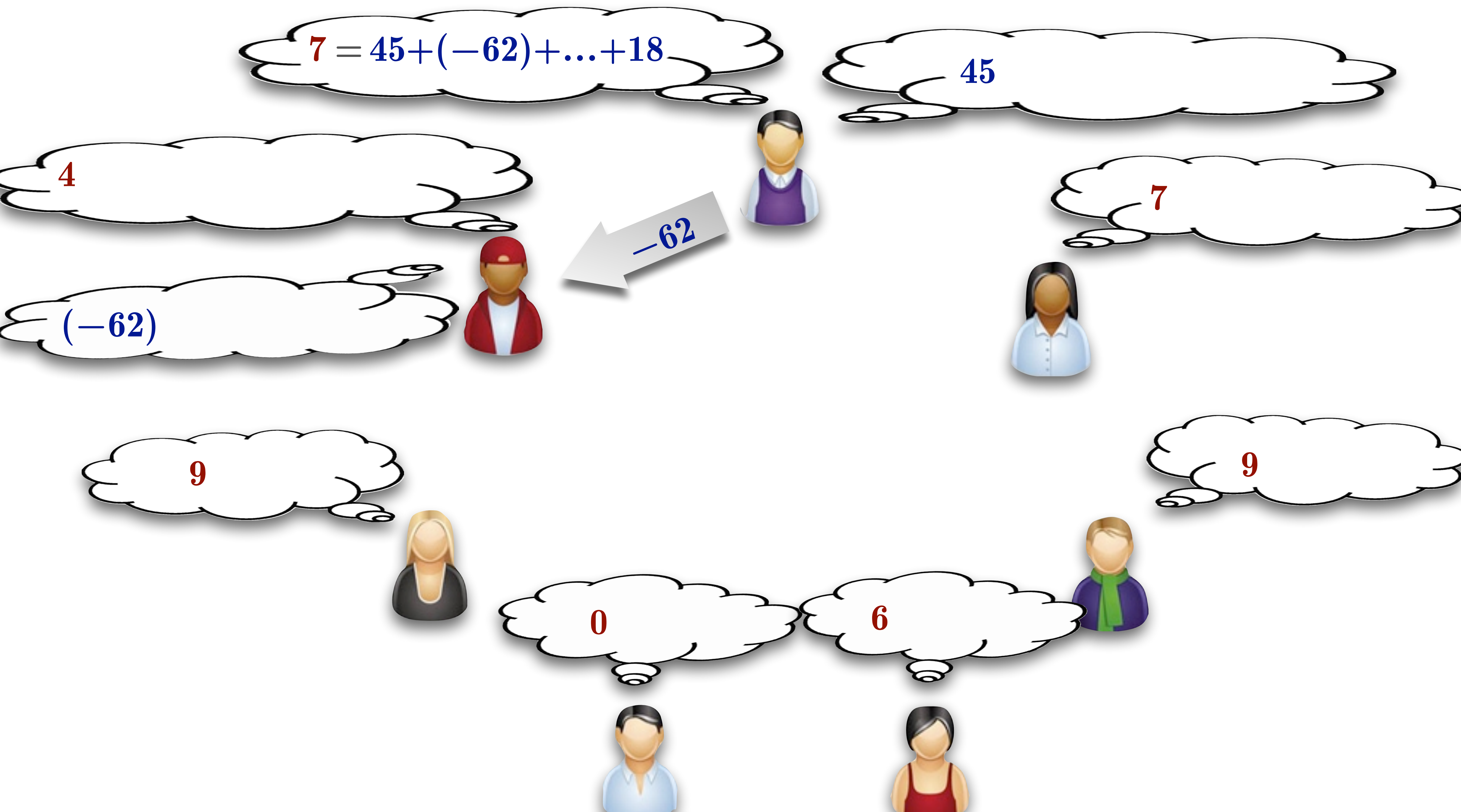

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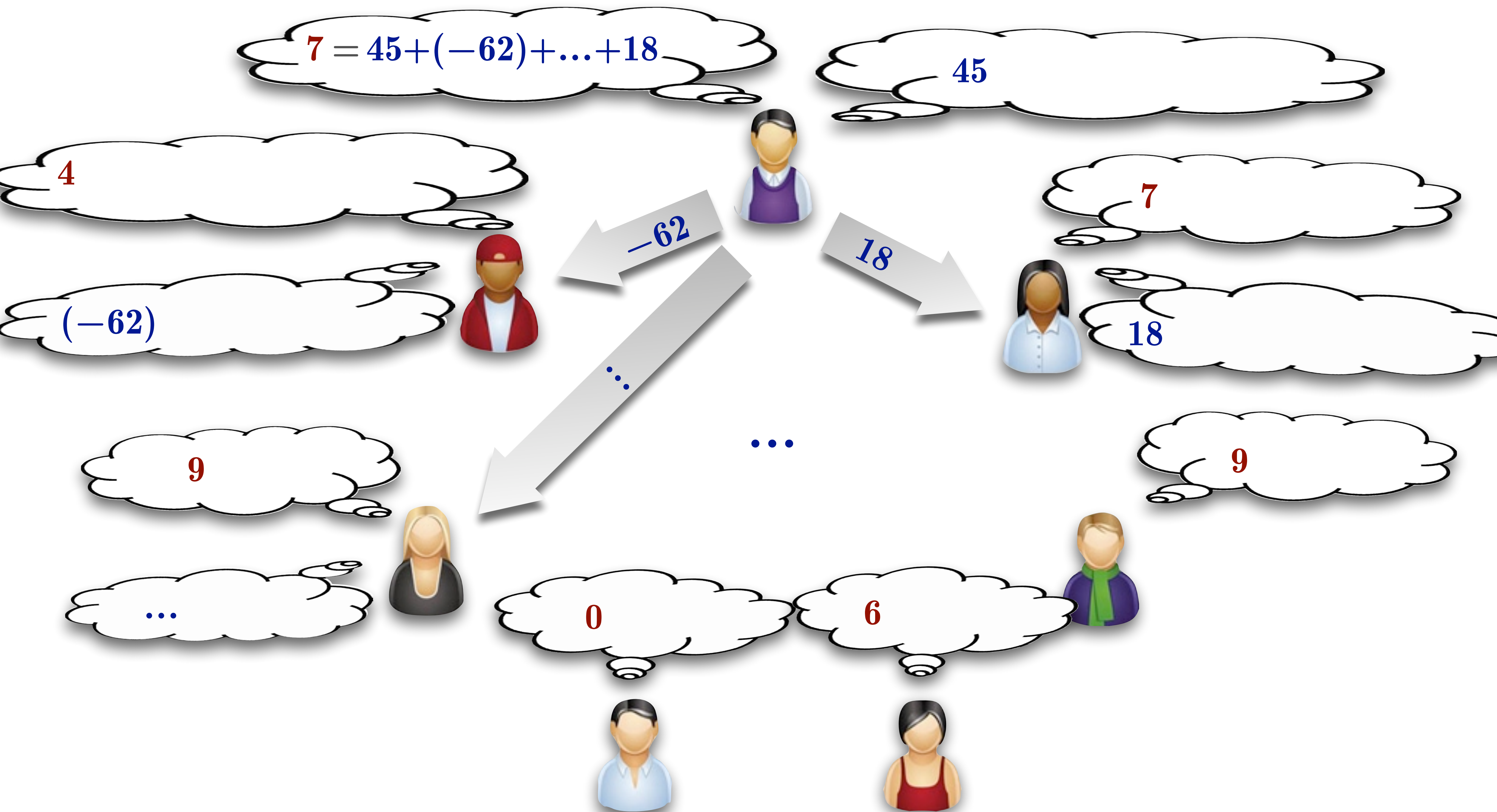

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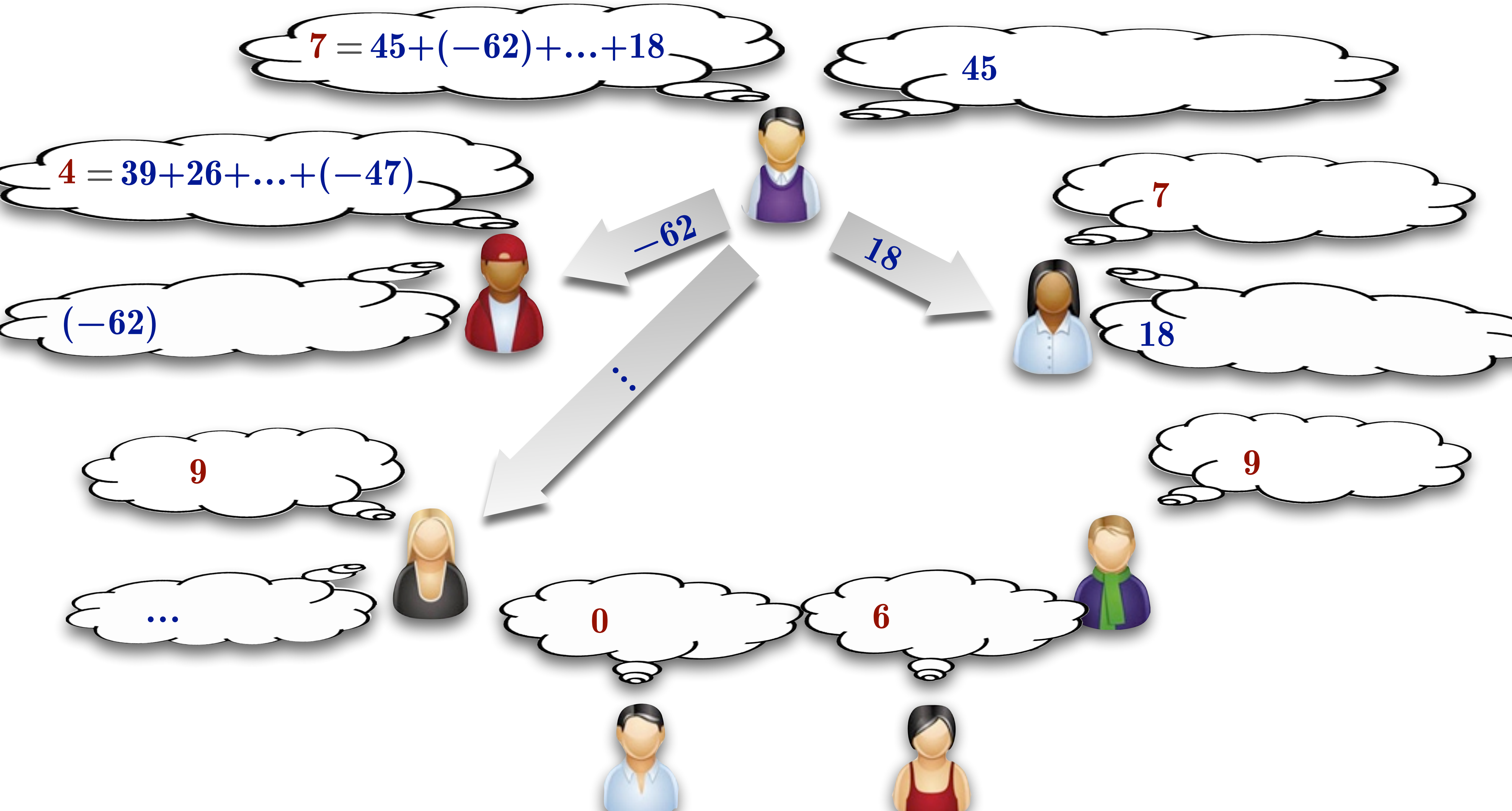

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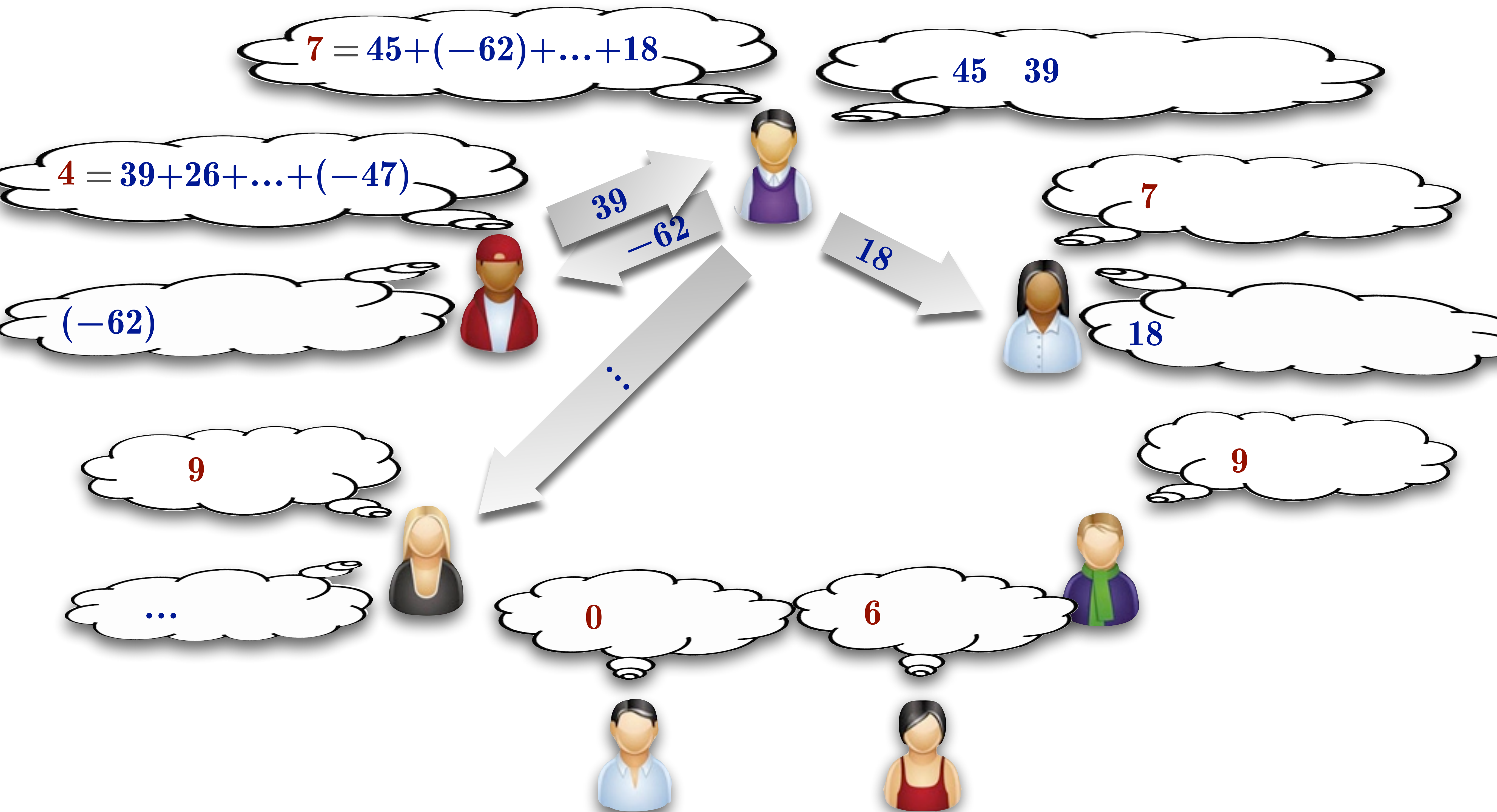

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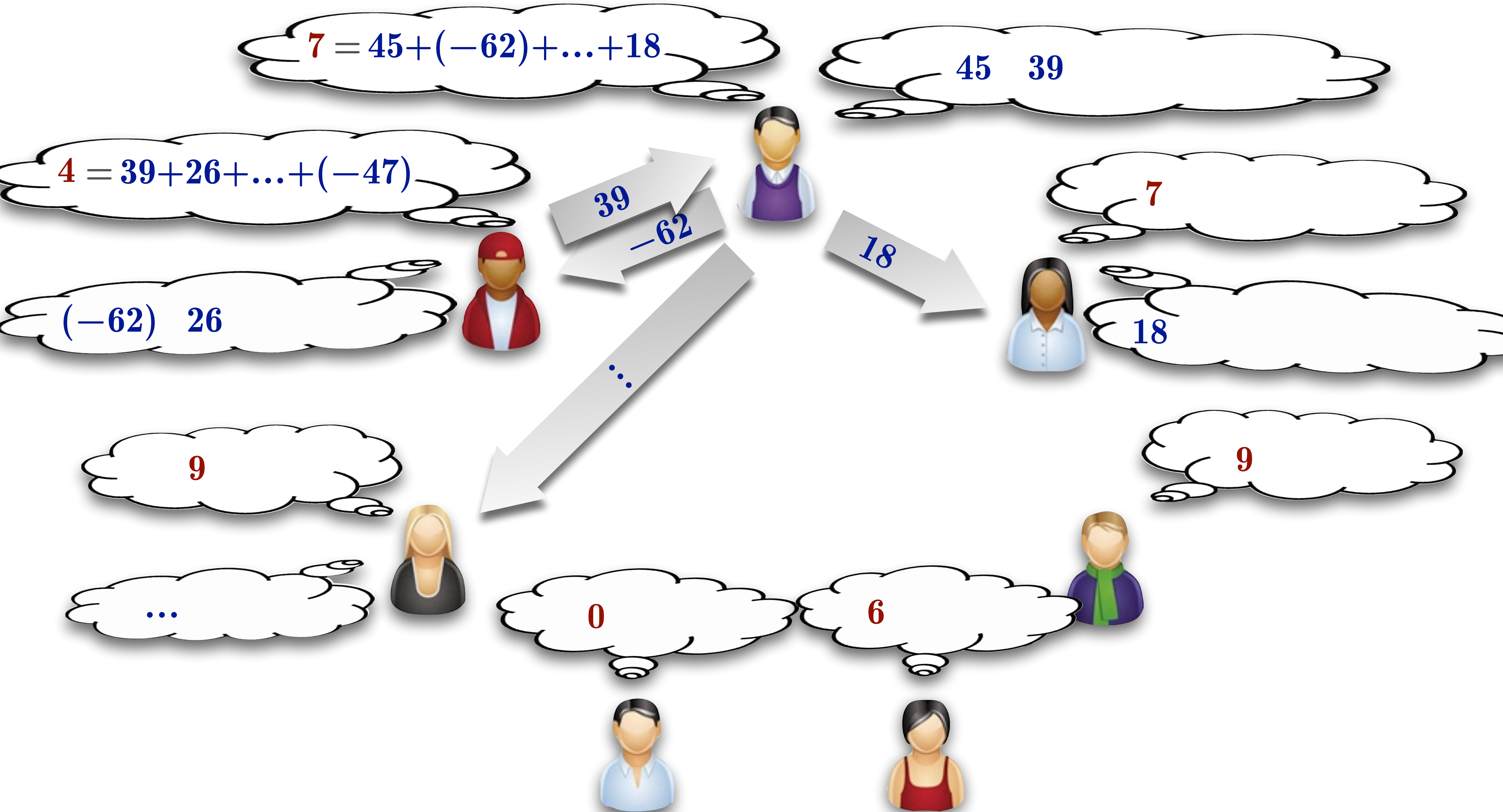

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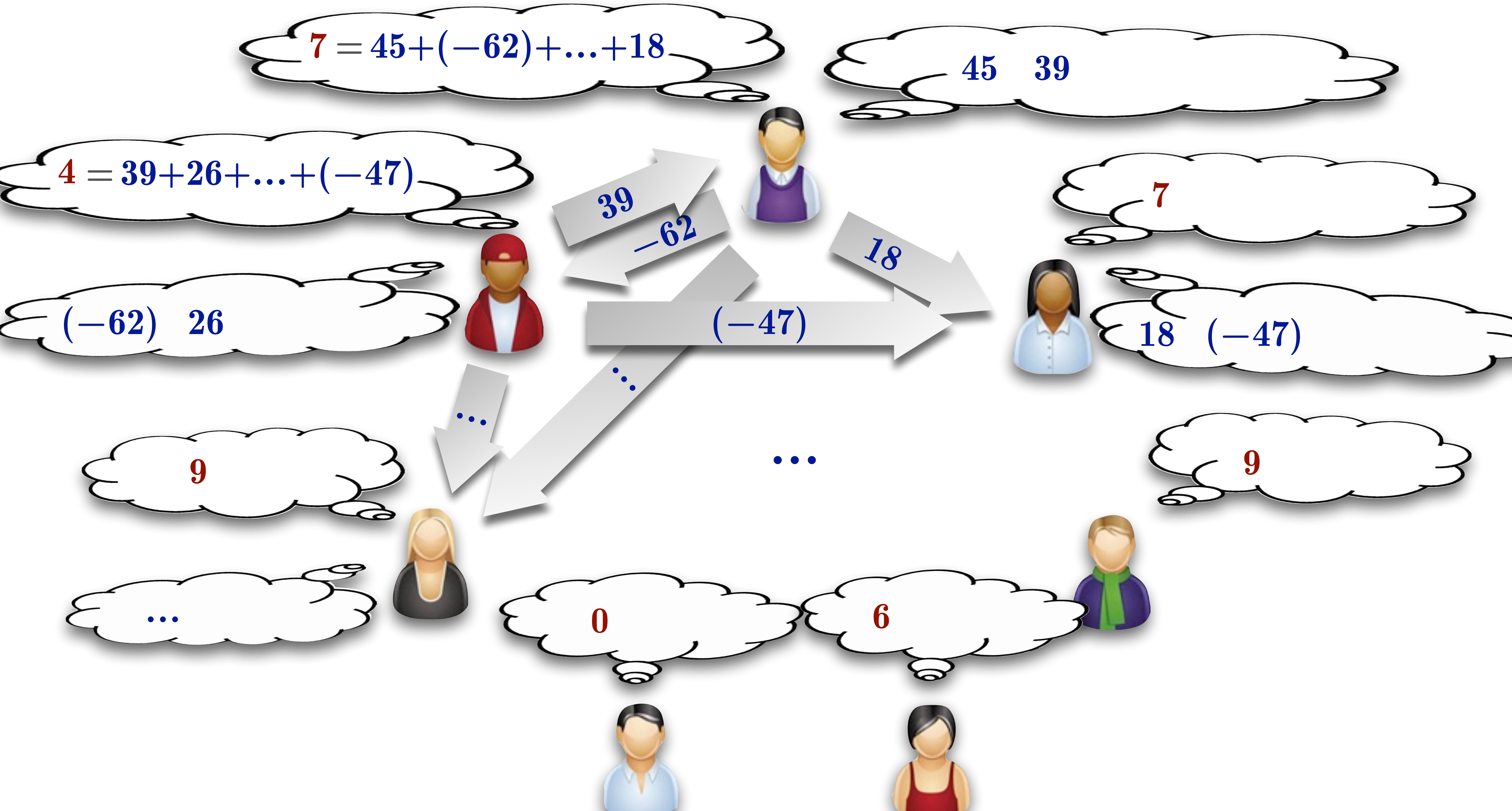

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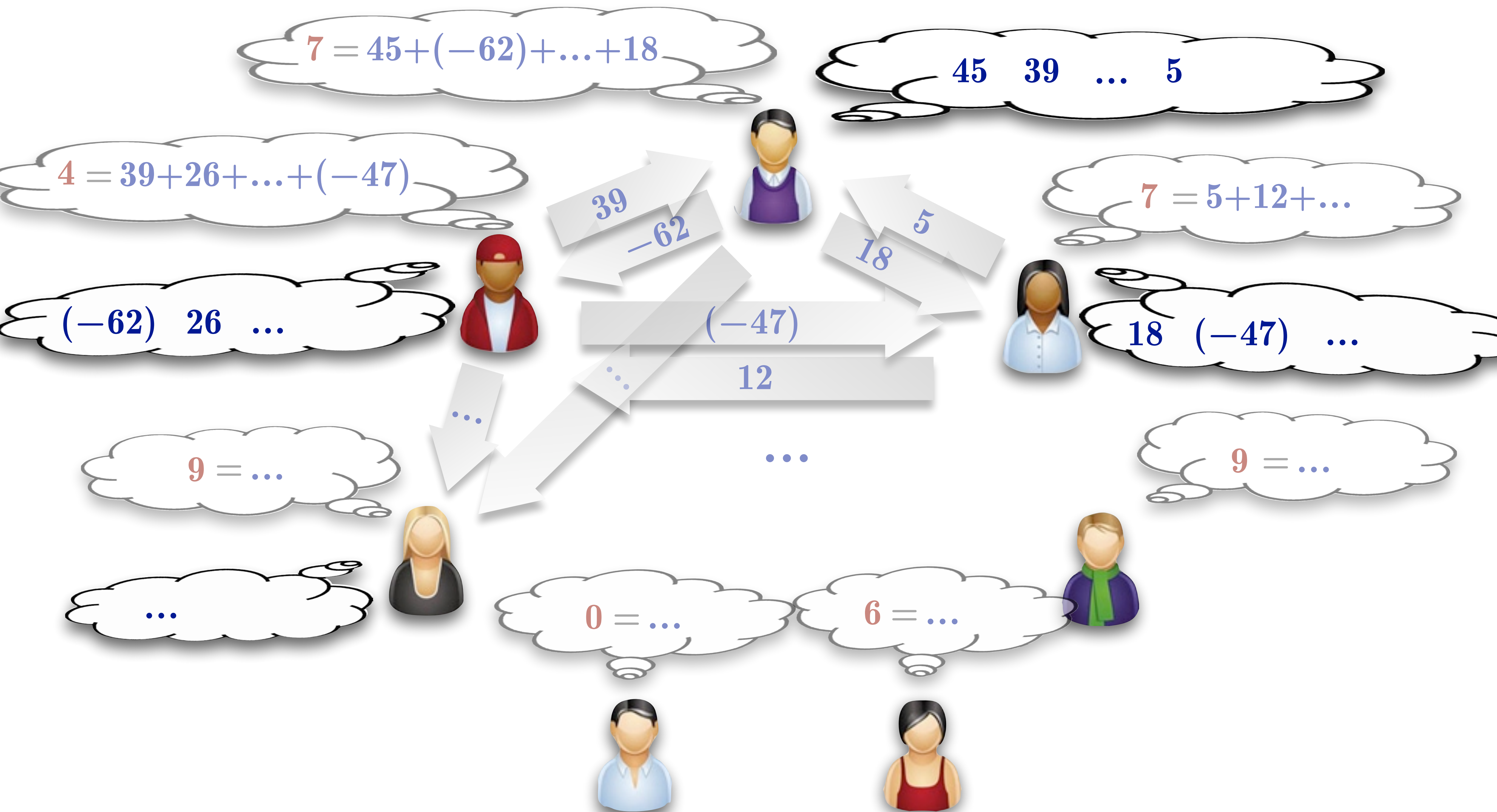

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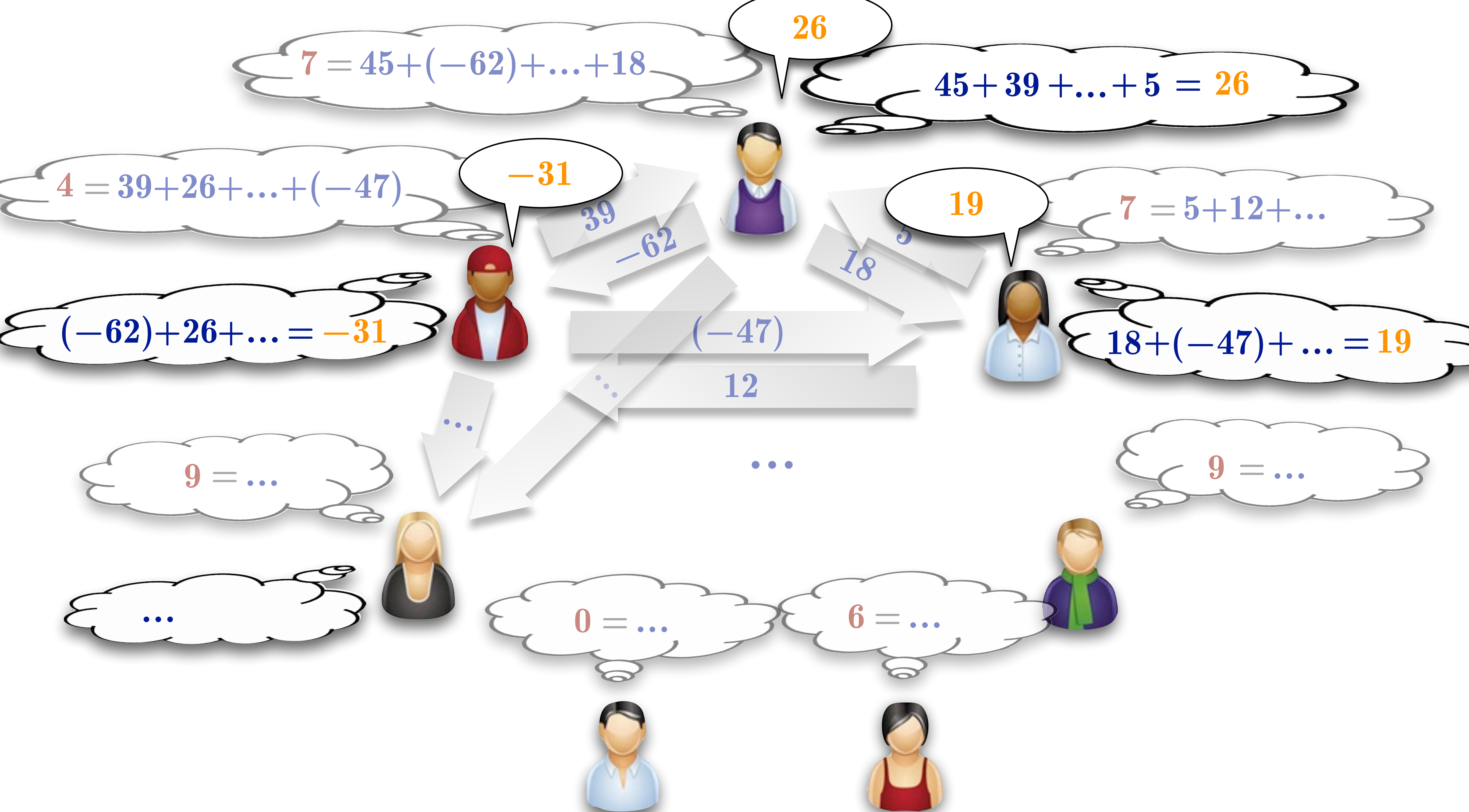

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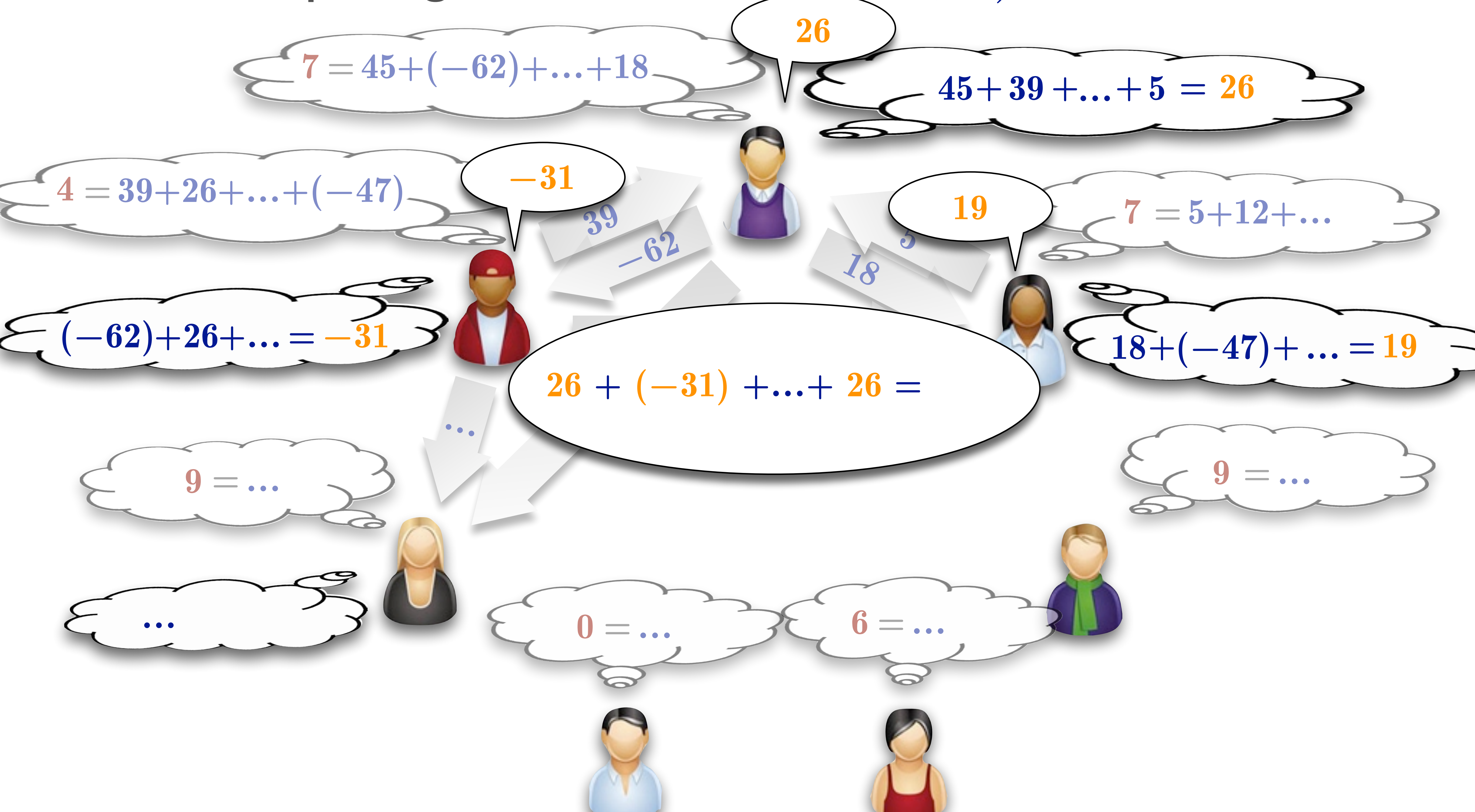

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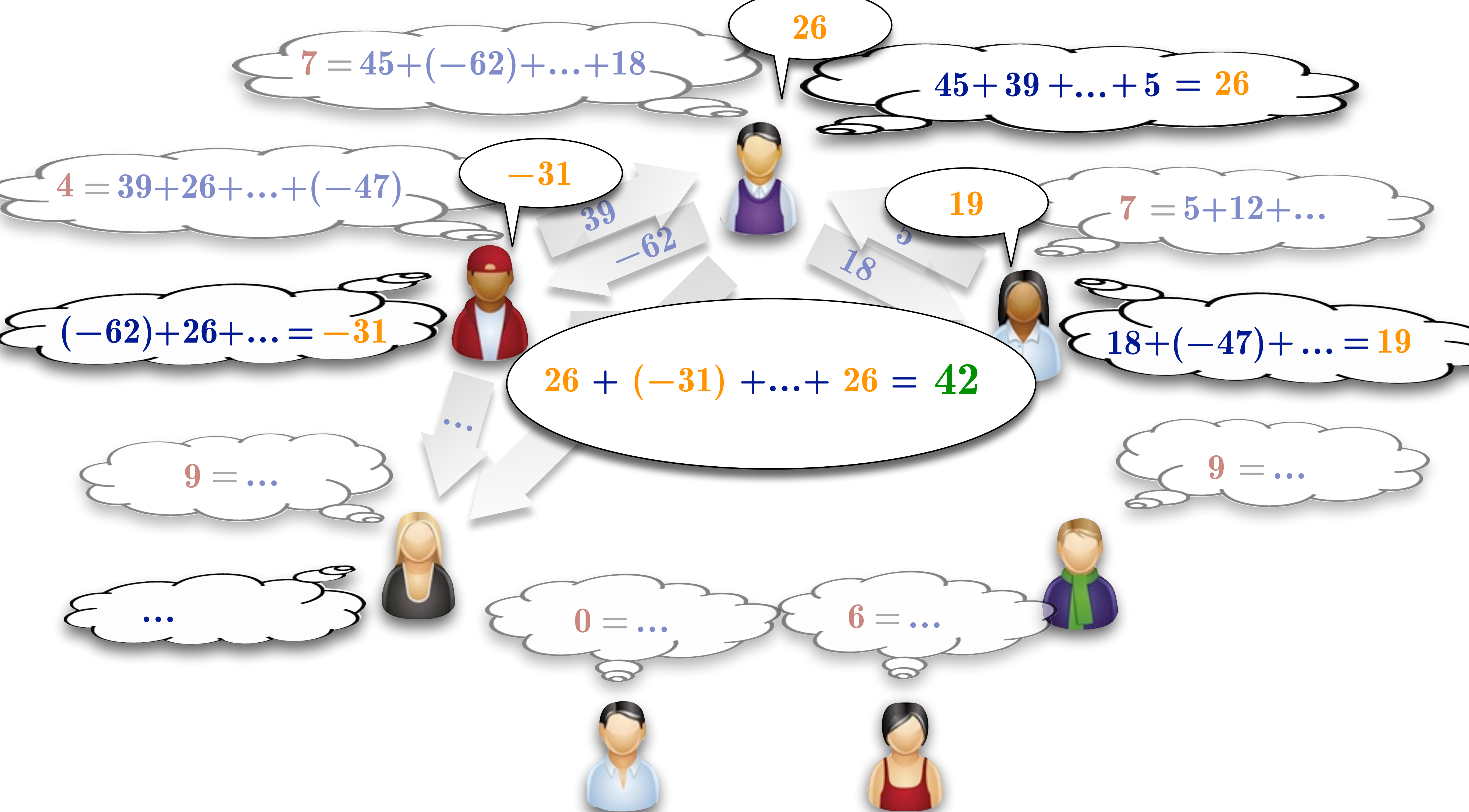
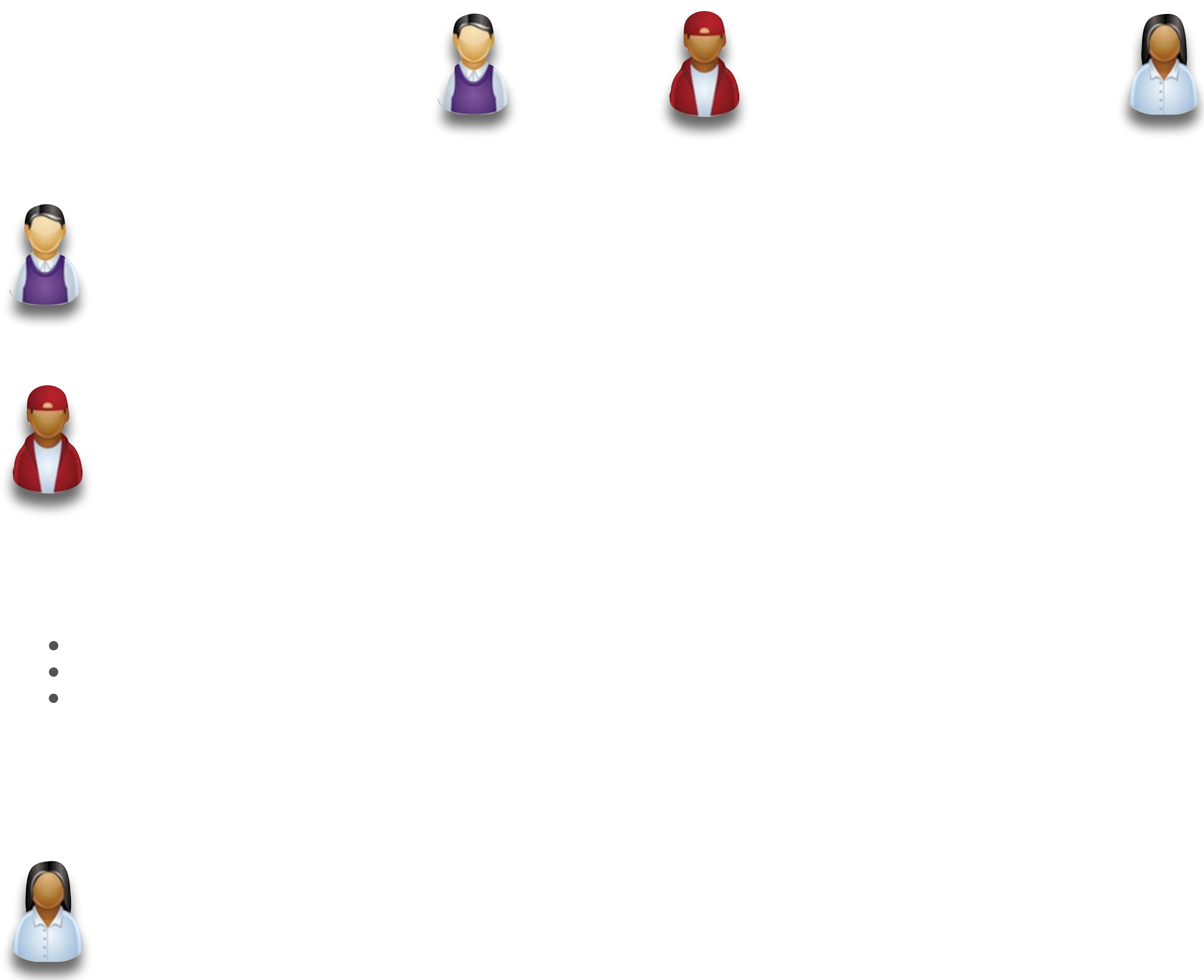


A More Abstract Description



A More Abstract Description

 $$x_1 = x_{11} + x_{12} + \ldots + x_{1n}$$

 $$x_2 = x_{21} + x_{22} + \ldots + x_{2n}$$

$\vdots$ $\qquad$ $\vdots$

 $$x_n = x_{n1} + x_{n2} + \ldots + x_{nn}$$

A More Abstract Description



$$
\begin{array}{lcccccccc}
x_1 & = & x_{11} & + & x_{12} & + & \ldots & + & x_{1n} \\
x_2 & = & x_{21} & + & x_{22} & + & \ldots & + & x_{2n} \\
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 & & & & & & & & \\
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 & & + & & + & & & & + \\
\vdots & & & & & & & & \\
 & & + & & + & & & & + \\
x_n & = & x_{n1} & + & x_{n2} & + & \ldots & + & x_{nn} \\
 & & = & & = & & & & = \\
 & & y_1 & & y_2 & & & & y_n
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$$

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+ & & + & & + & & & & + \\
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+ & & + & & + & & & & + \\
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Offers ***privacy*** of inputs against **arbitrary** coalitions ✔

A More Abstract Description

Parties can lie about their partial result:
→ no ***correctness*** or ***fairness*** guaranteed

$$x_2 = x_{21} + x_{22} + \dots + x_{2n}$$

Offers ***privacy*** of inputs against **arbitrary** coalitions

$$\begin{array}{ccccccccc}
\vdots & & \vdots & & & & & & \\
+ & & + & & + & & & & + \\
x_n & = & x_{n1} & + & x_{n2} & + & \dots & + & x_{nn} \\
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Any party can **stall** the procedure → **not *abort-free***

$$\begin{array}{ccccccccc} x_n & = & x_{n1} & + & x_{n2} & + & \dots & + & x_{nn} \\ \| & & \| & & \| & & & & \| \\ y & = & y_1 & + & y_2 & + & \dots & + & y_n \end{array}$$

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Offers ***privacy*** of inputs against **arbitrary coalitions**

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Approach/solution **limited** to linear functions

$y = y_1 + y_2 + \ldots + y_n$

A Useful Tool: (Linear) Secret Sharing

At the core is a cryptographic primitive for

distributing ("***sharing***") a secret input s

by means of

preparing ***shares*** $s_1, s_2, \ldots, s_n$ and giving s_i to party P_i,

so that:

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- if $s_1, \ldots, s_n$ is a sharing of s, and $s'_1, \ldots, s'_n$ of s' then

  $s_1 + s'_1, \ldots, s_n + s'_n$ is a sharing of $s + s'$. *linearity*

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Prime example:

$s_1, \ldots, s_n$ random subject to $s = s_1 + \ldots + s_n \pmod{p}$

reconstructability

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linearity

A Paradigm for Doing MPC

Sharing phase:

Computation phase:

Reconstruction phase:

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Every party P_i **shares** his input x_i .

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Still some issues about dishonest parties lying.

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reconstructuctability

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so that for some t

reconstructability

- from ~~**all** n~~ **any** $t+1$ shares, the secret s can be recovered

- given ~~**less** than n~~ **at most** t shares, no info on s is revealed *privacy*

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Example: Shamir Secret Sharing

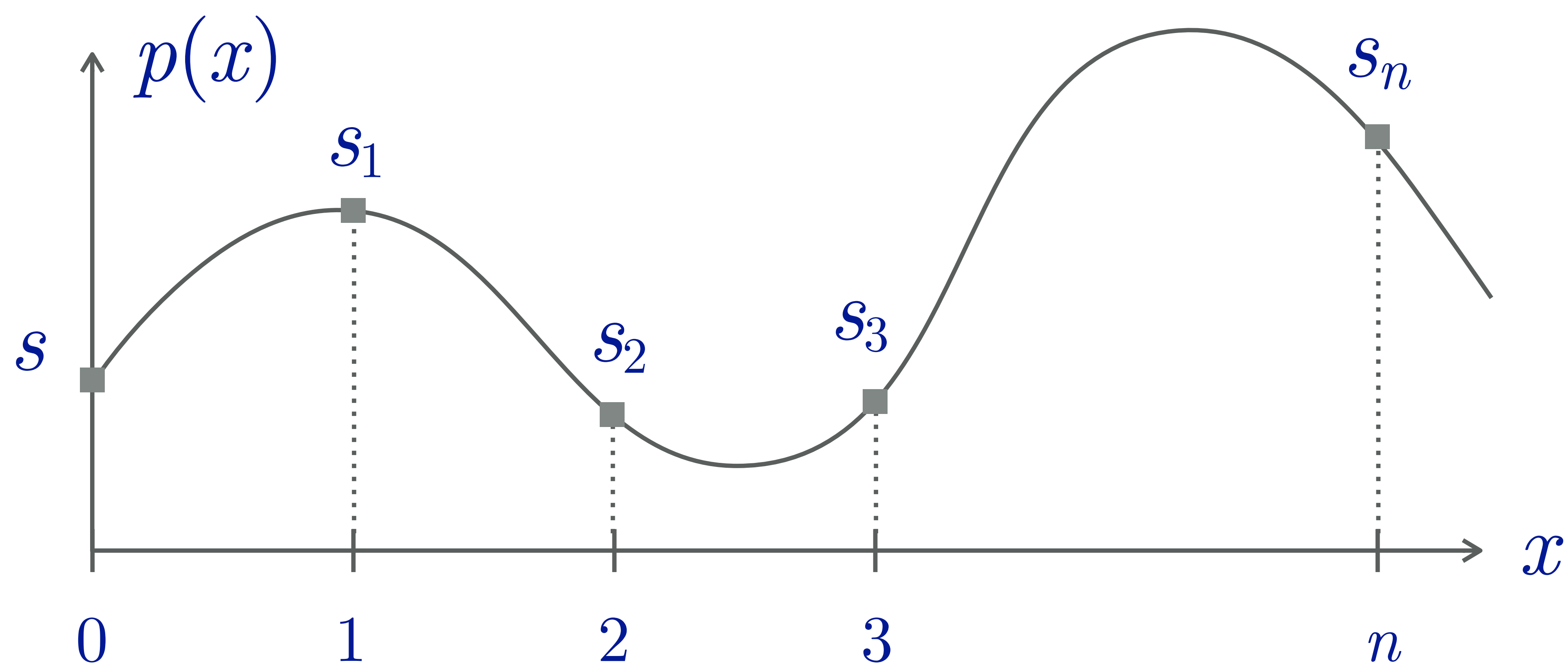
To share s: choose a polynomial

$$p(x) = s + a_1 x + \ldots + a_t x^t$$

with ***random*** $a_1, \ldots, a_t$ and constant coefficient s, and set

$$s_i = p(i)$$

for $i = 1, \ldots, n$.



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As for linearity: if

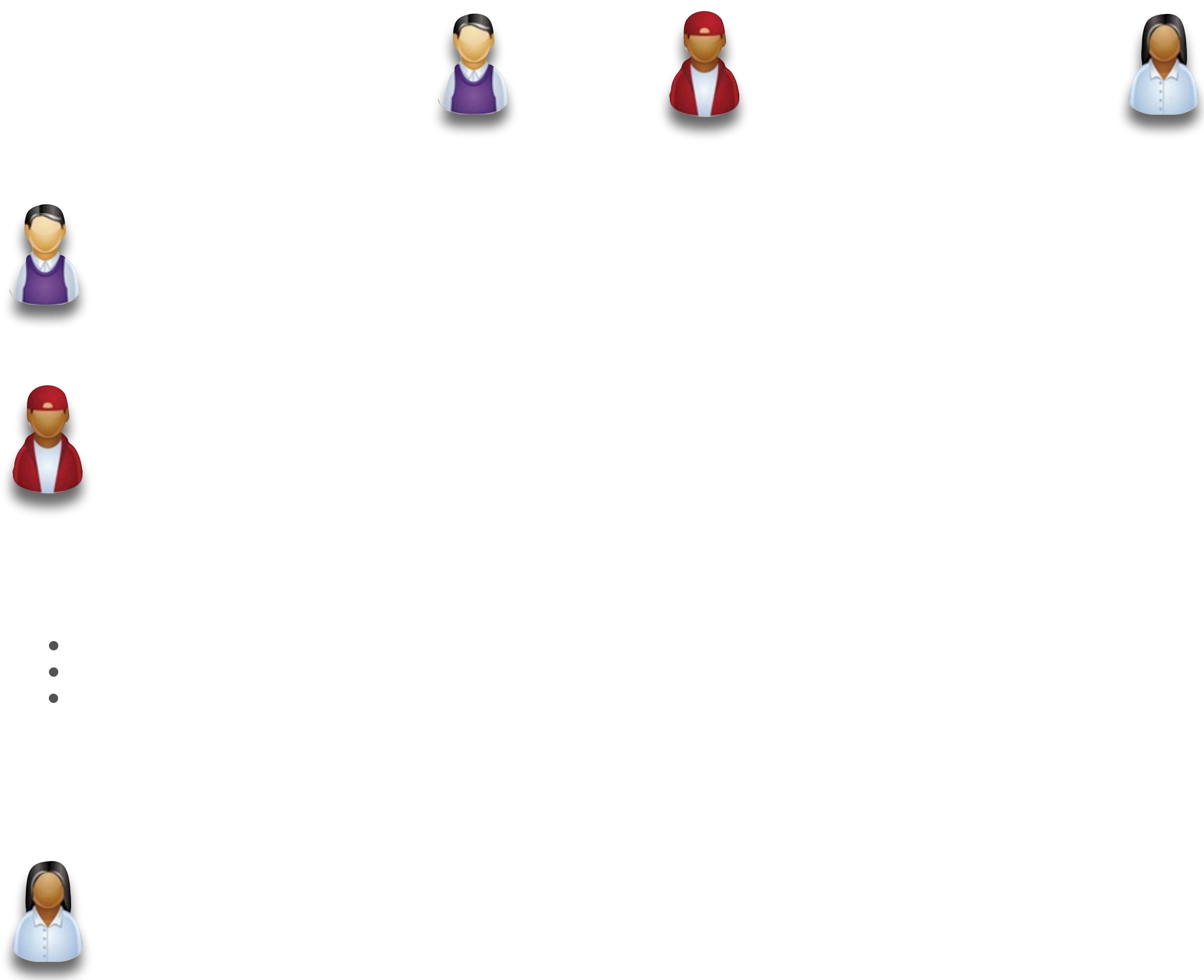
$$s_i = p(i) \text{ for } p(x) = s + a_1 x + \ldots + a_t x^t$$

$$s'_i = p'(i) \text{ for } p'(x) = s' + a'_1 x + \ldots + a'_t x^t$$

then

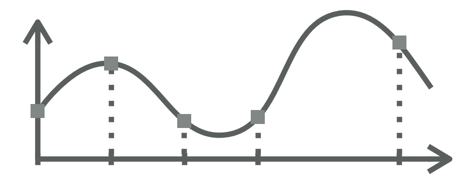
$$s_i + s'_i = p''(i) \text{ for } p''(x) = p(x) + p'(x) = (s + s') + \ldots .$$

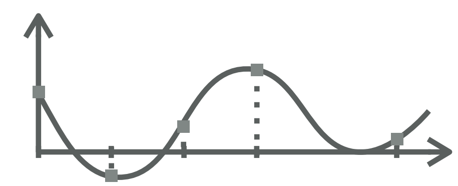
Using Shamir's Secret Sharing Scheme



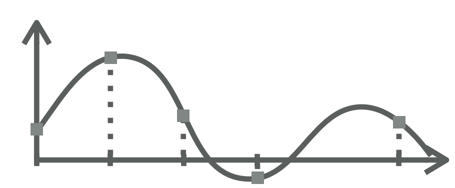
Using Shamir's Secret Sharing Scheme

 $x_1 \longrightarrow x_{11} \quad x_{12} \quad \ldots \quad x_{1n}$ 

 $x_2 \longrightarrow x_{21} \quad x_{22} \quad \ldots \quad x_{2n}$ 

$\vdots \qquad \vdots$

 $x_n \longrightarrow x_{n1} \quad x_{n2} \quad \ldots \quad x_{nn}$ 

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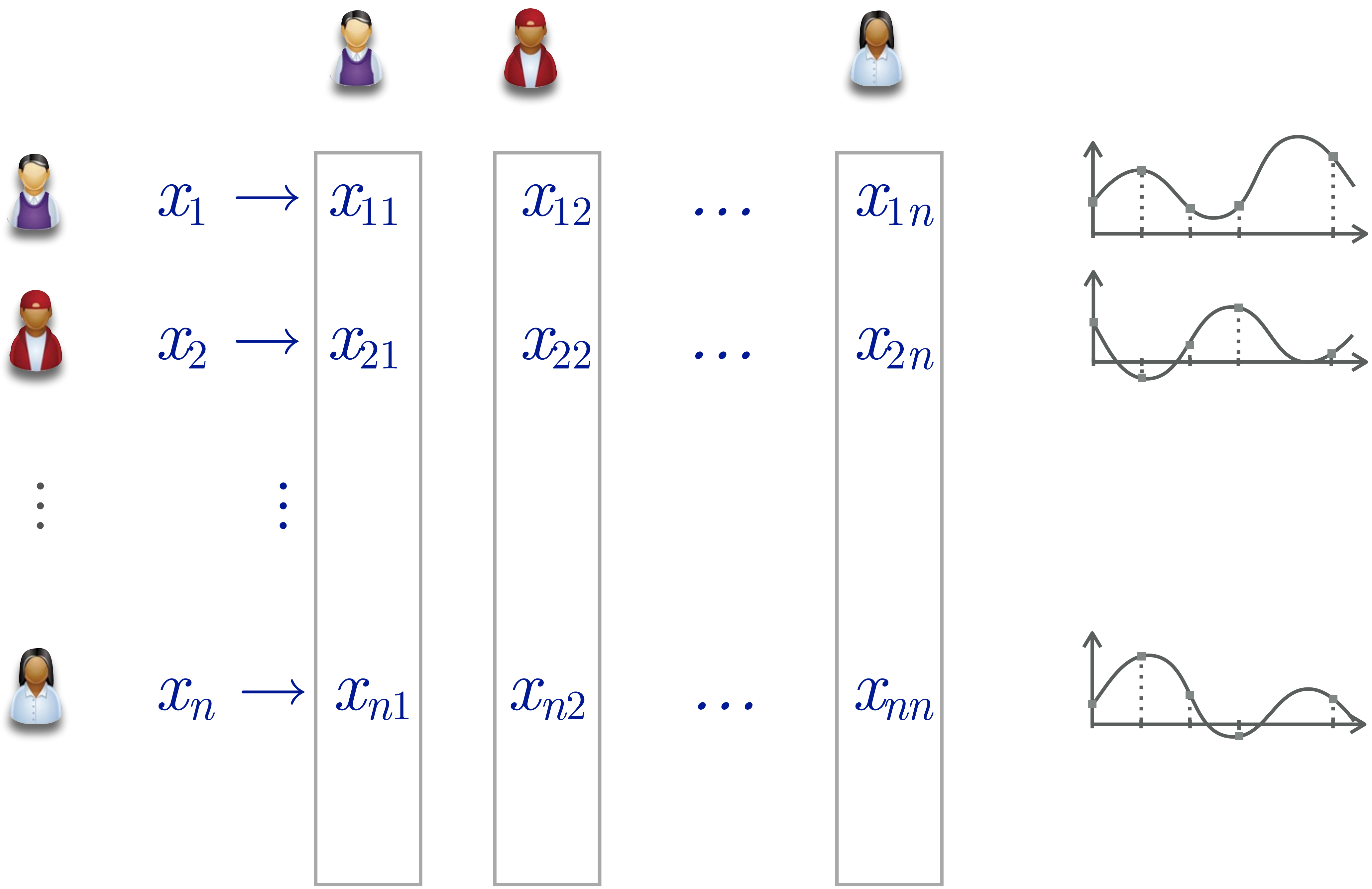

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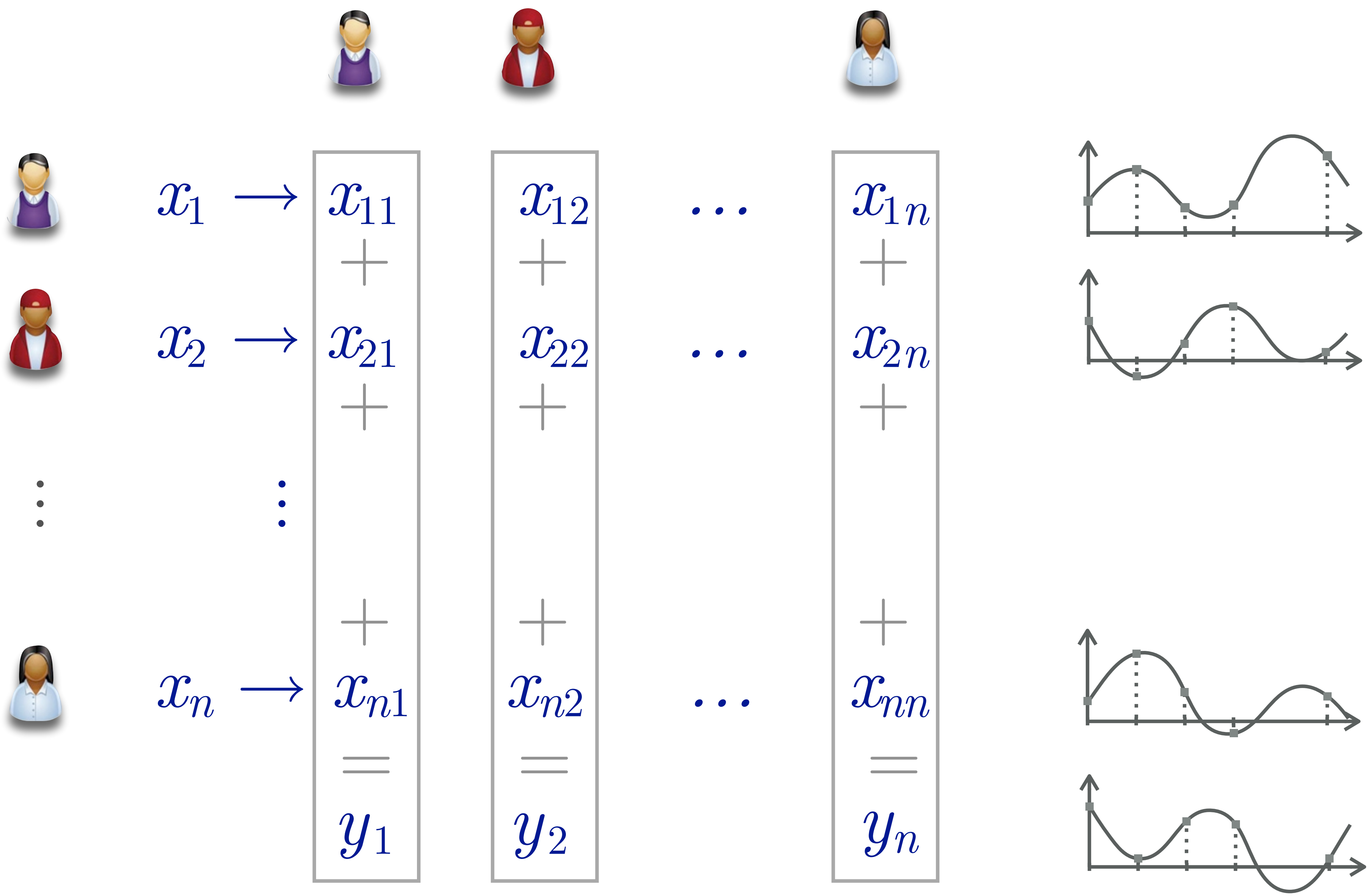

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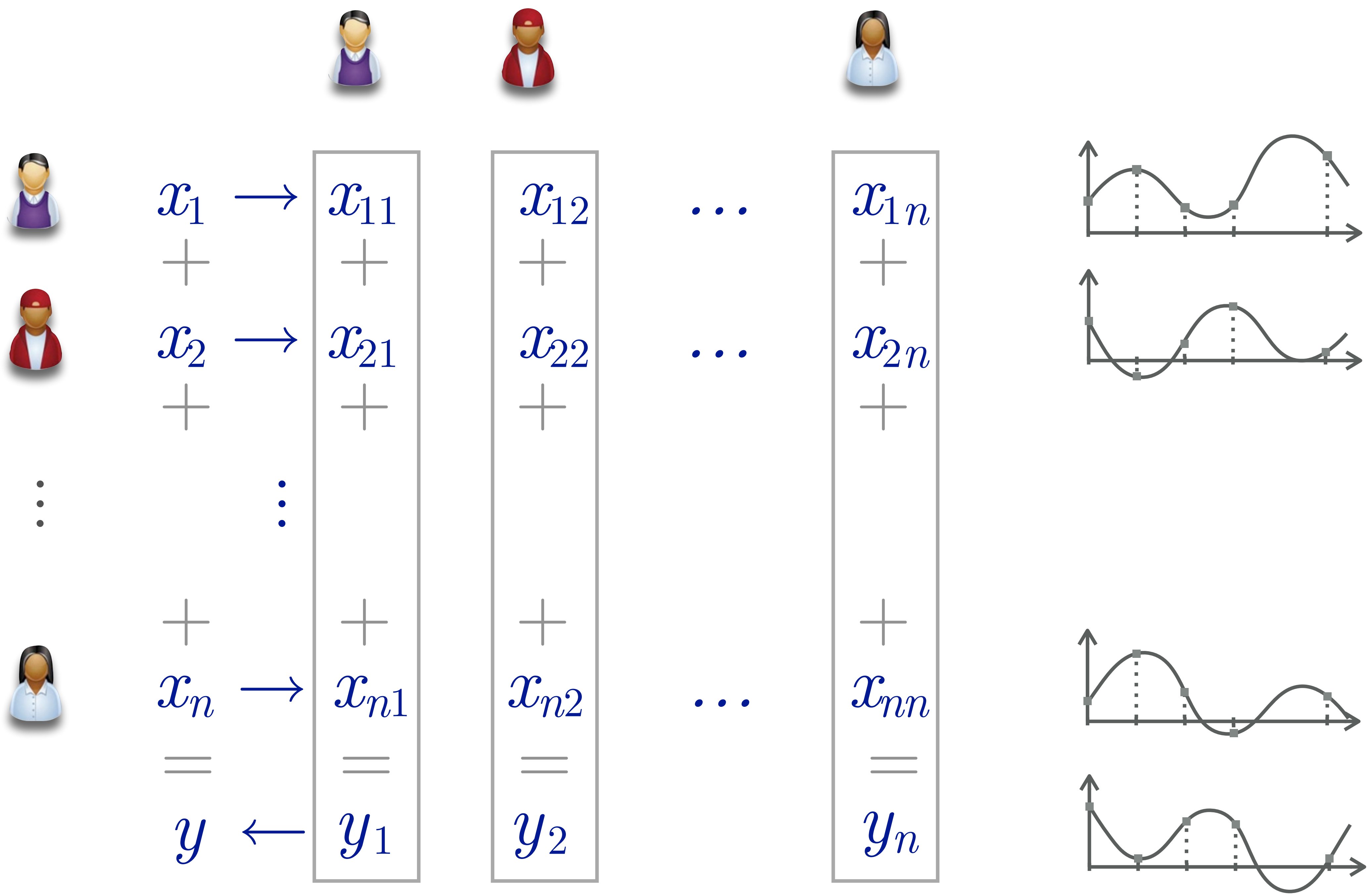

Using Shamir's Secret Sharing Scheme

Offers *privacy* of inputs against t dishonest parties ✔

$$
\begin{array}{ccccccccc}
x_1 & \longrightarrow & x_{11} & & x_{12} & & \cdots & & x_{1n} \\
+ & & + & & + & & & & + \\
x_2 & \longrightarrow & x_{21} & & x_{22} & & \cdots & & x_{2n} \\
+ & & + & & + & & & & + \\
\vdots & & \vdots & & & & & & \\
+ & & + & & + & & & & + \\
x_n & \longrightarrow & x_{n1} & & x_{n2} & & \cdots & & x_{nn} \\
= & & = & & = & & & & = \\
y & \longleftarrow & y_1 & & y_2 & & & & y_n
\end{array}
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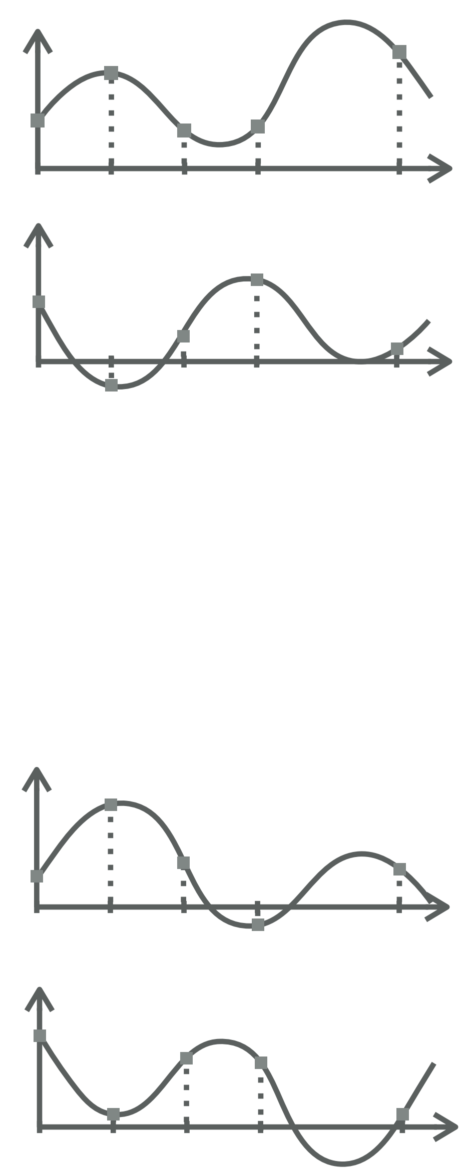

Using Shamir's Secret Sharing Scheme

Offers ***privacy*** of inputs **against** t **dishonest parties**

Redundancy in shares ($y_1, \ldots, y_n$ must lie on deg-t poly):

→ cheating will be detected

→ *correctness* (but not *abort-free* nor *fair*)

$$
\begin{array}{ccccccc}
x_n & \longrightarrow & x_{n1} & x_{n2} & \cdots & x_{nn} \\
= & & = & = & & = \\
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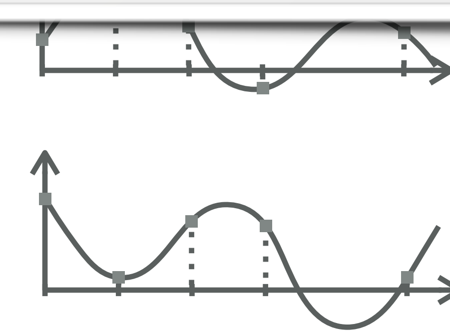
$$



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If we can enforce **consistent sharings** (**we can!**) of x_i's, set $t < n/3$, and use Reed-Solomon error correction:

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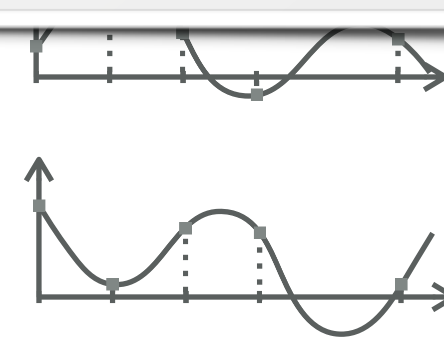
$$\begin{array}{ccccccc} x_n & \to & x_{n1} & x_{n2} & \cdots & x_{nn} \\ = & & = & = & & = \\ y & \leftarrow & y_1 & y_2 & & y_n \end{array}$$



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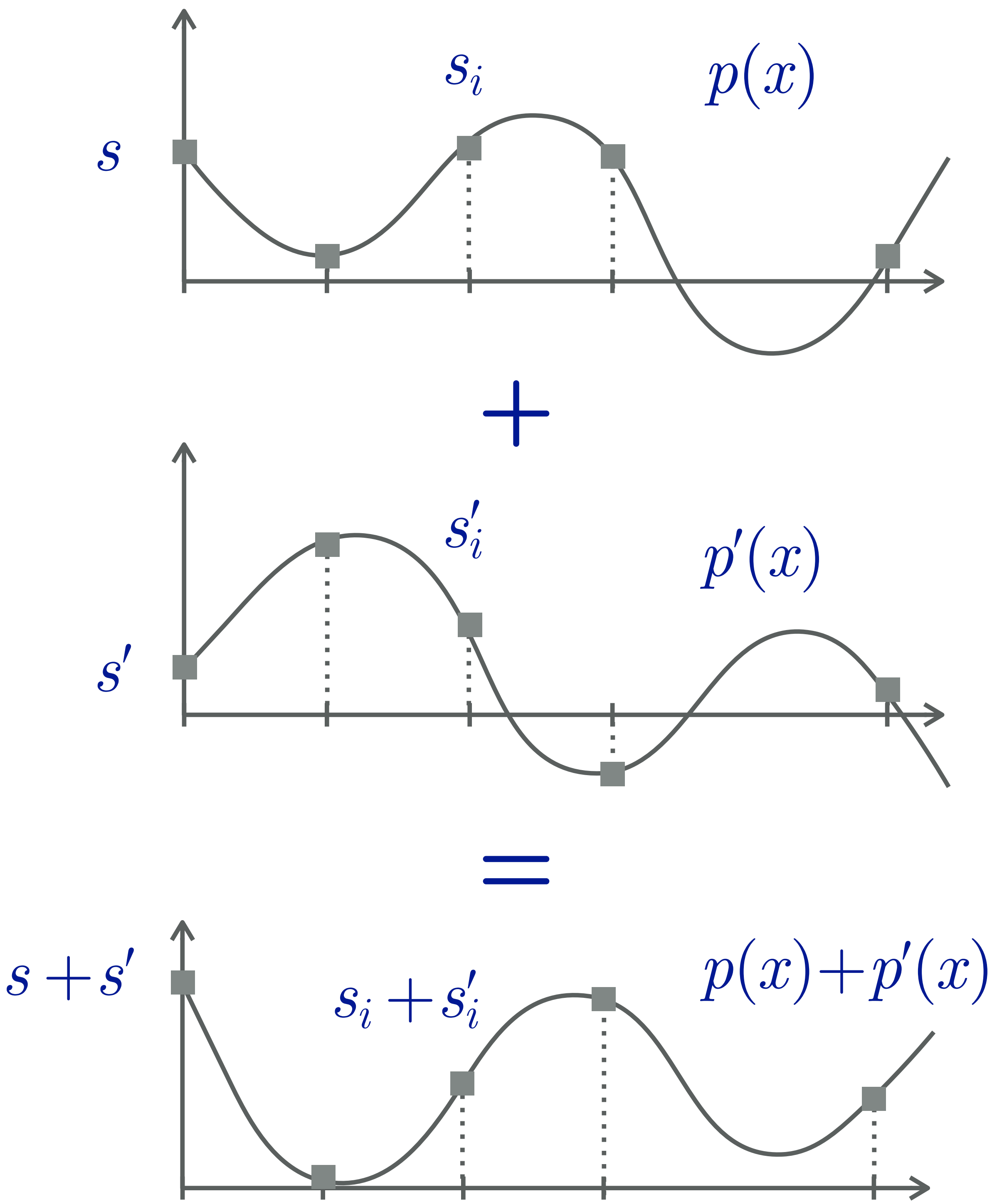
⇒ Works for ***addition*** / ***linear function evaluation*** only

Towards Secure Multiplications

For addition, exploited:



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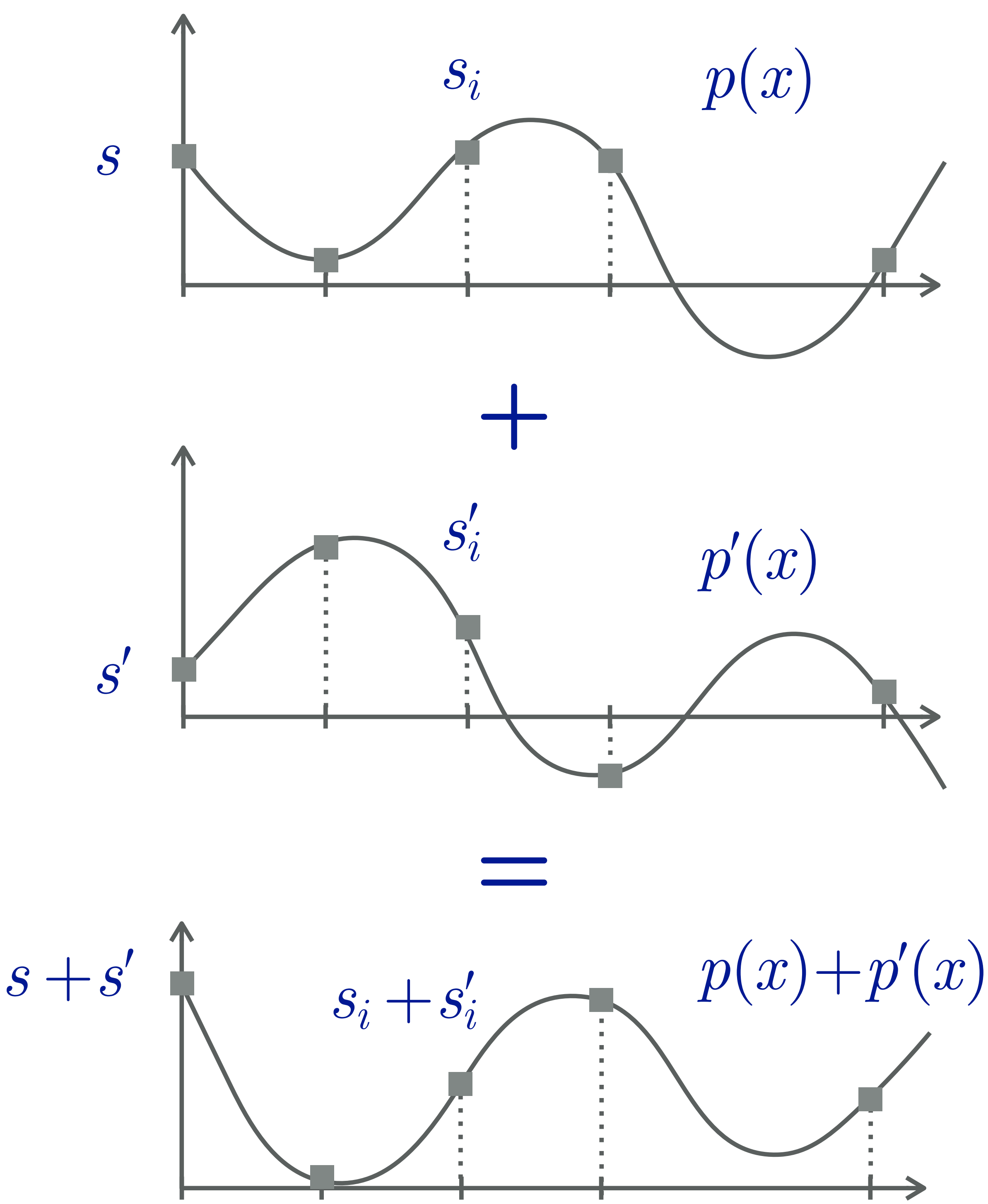
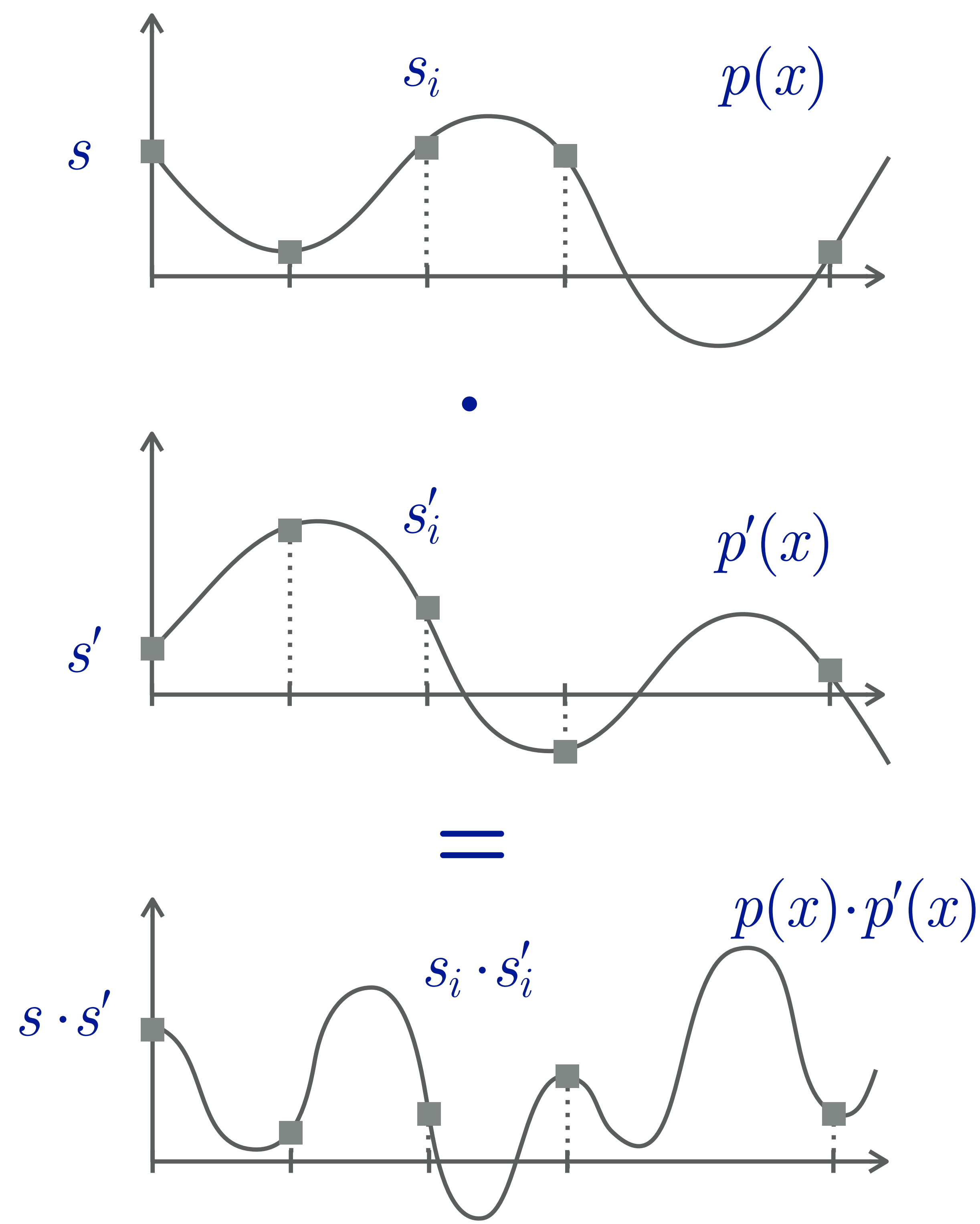


Similarly, for multiplication:



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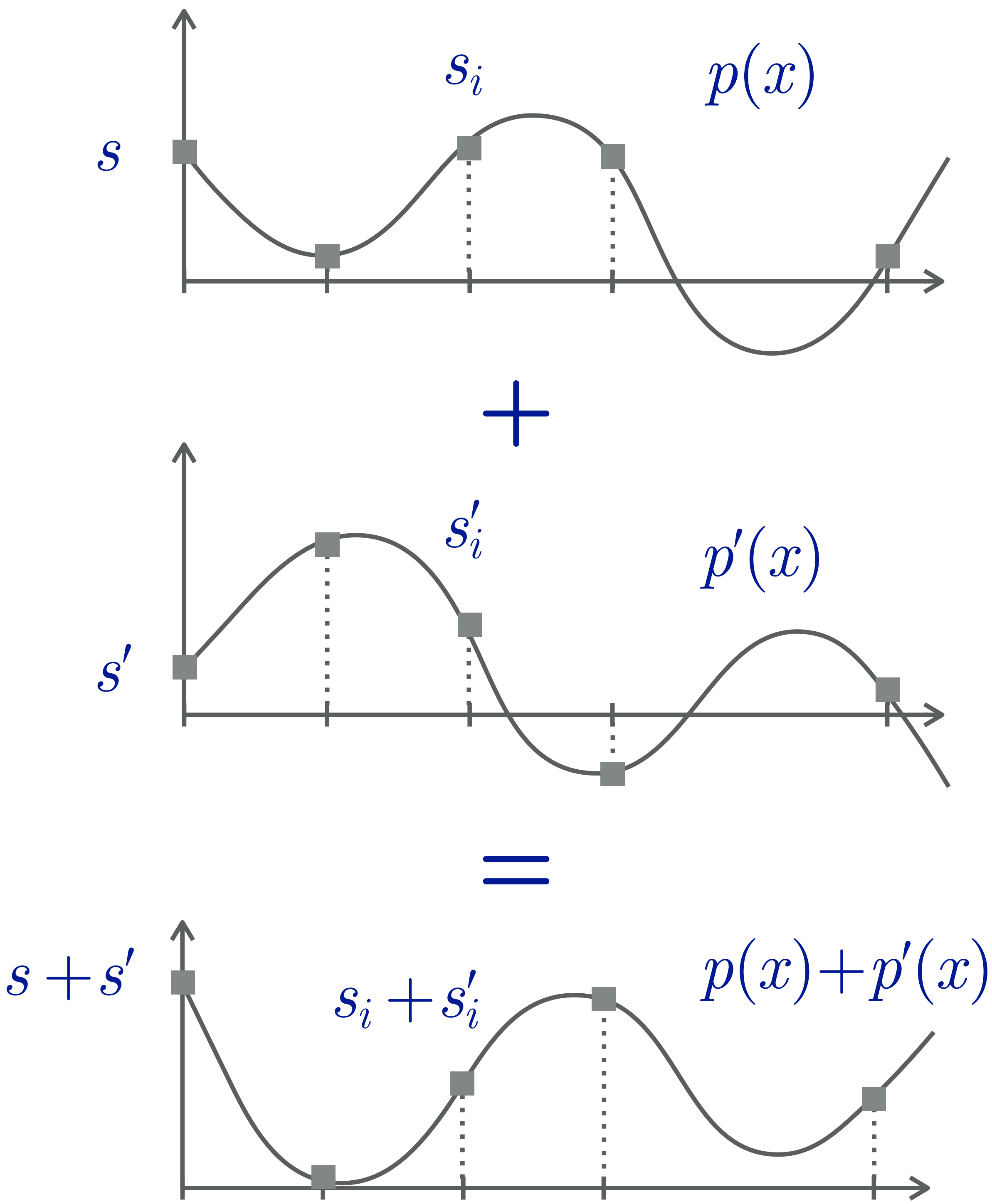

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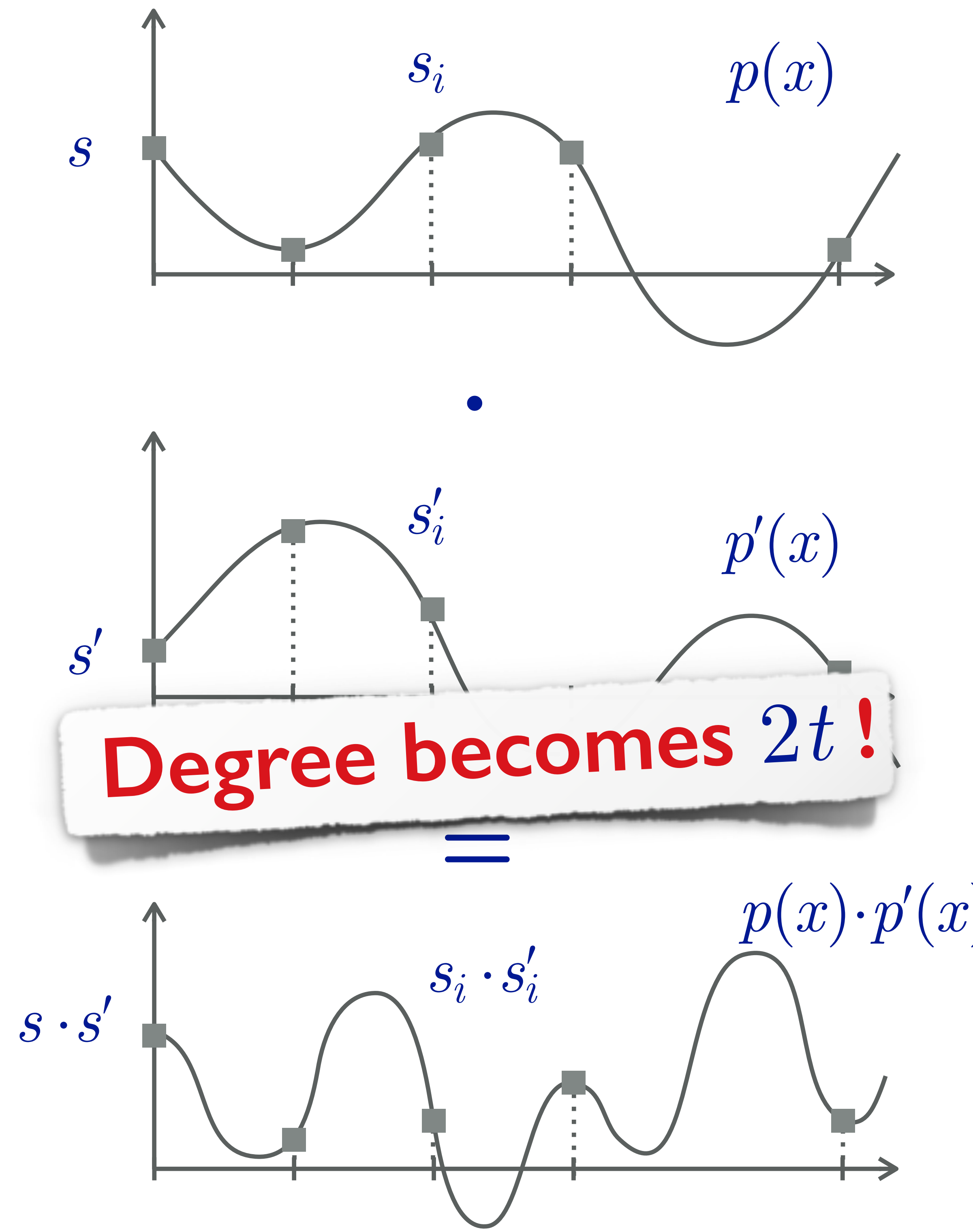
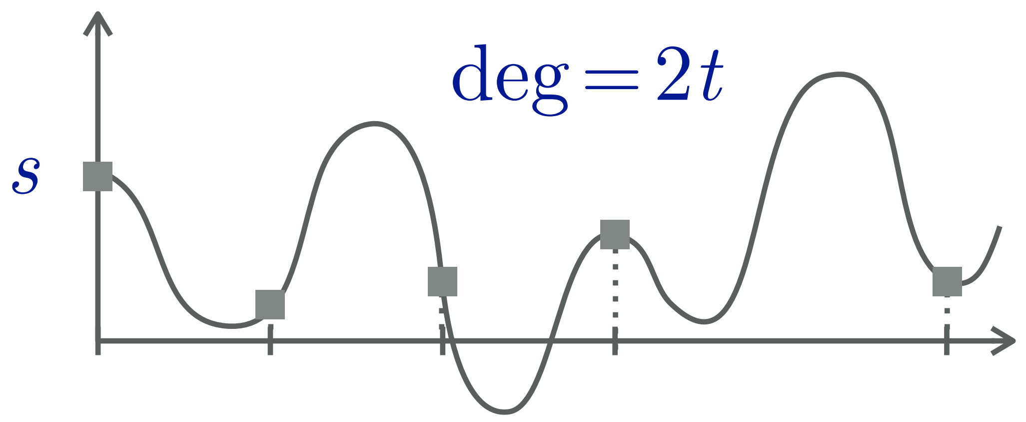


Degree Reduction

Due to [Chaum et al. 88], reinvented again in 2007.



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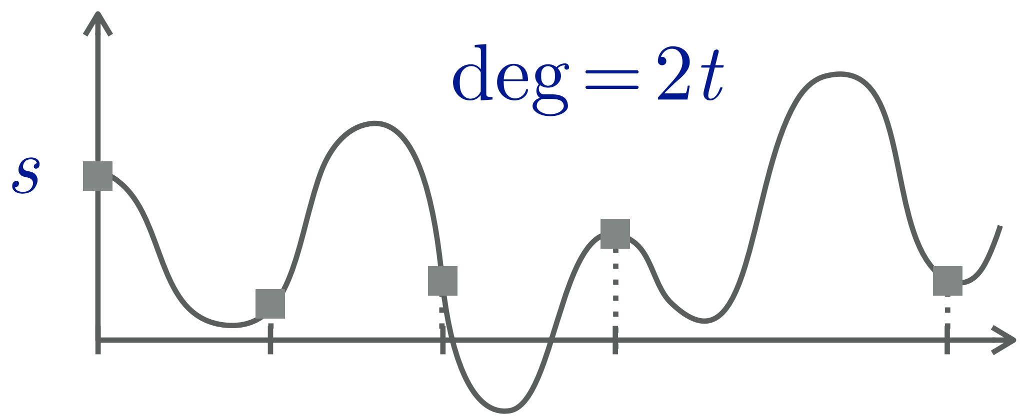
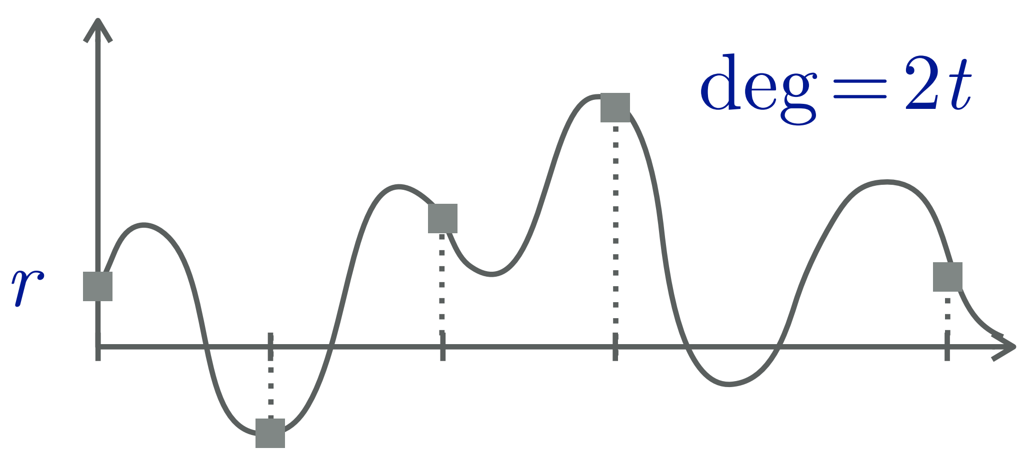
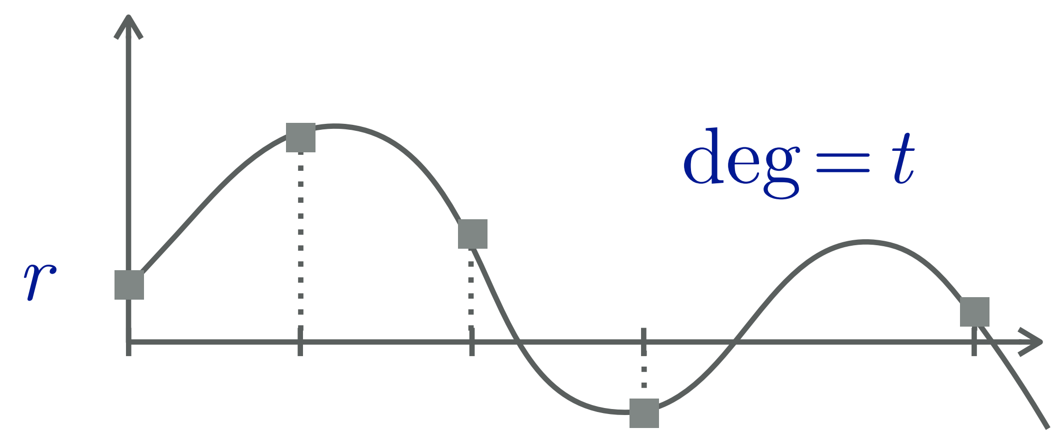
Produce a deg-$2t$ and a deg-t sharing of random **unknown** r.







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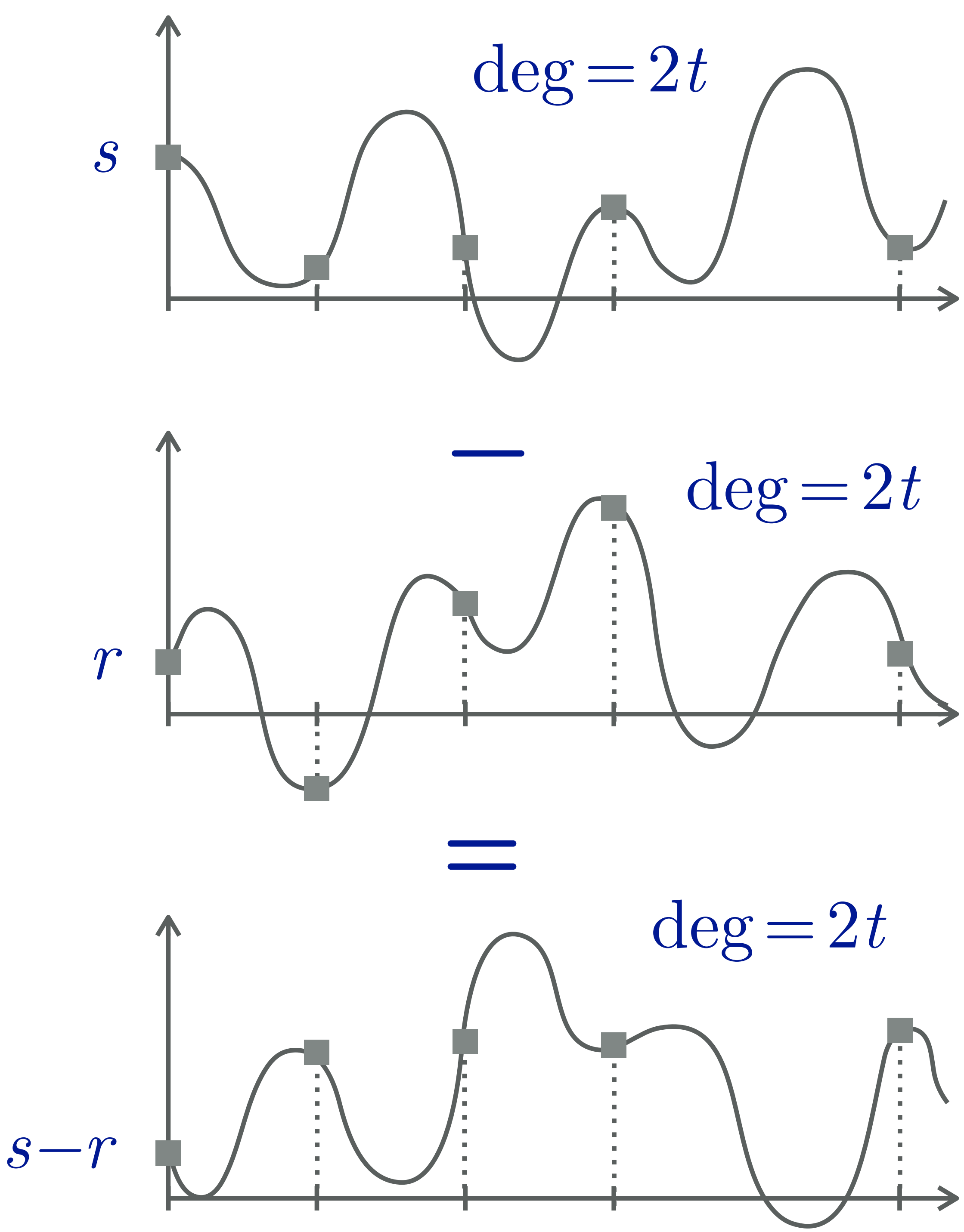
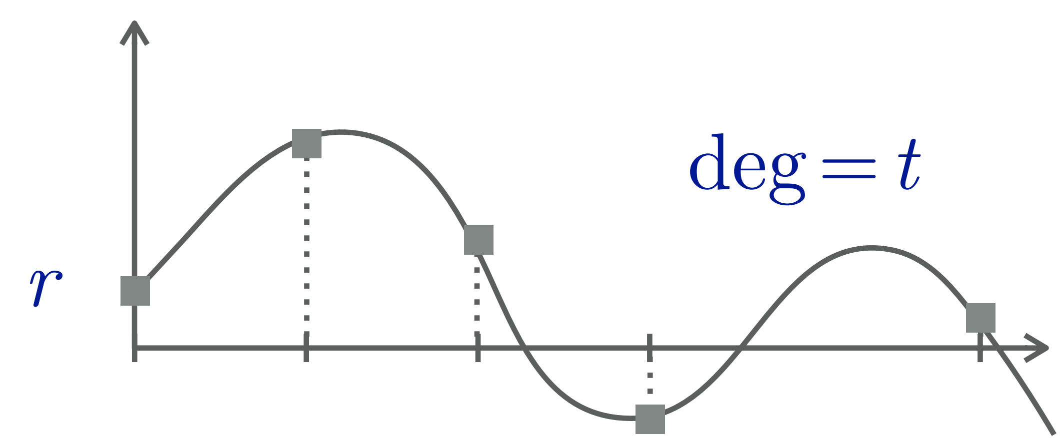


Locally compute the deg-$2t$ sharing of $\delta = s - r$.



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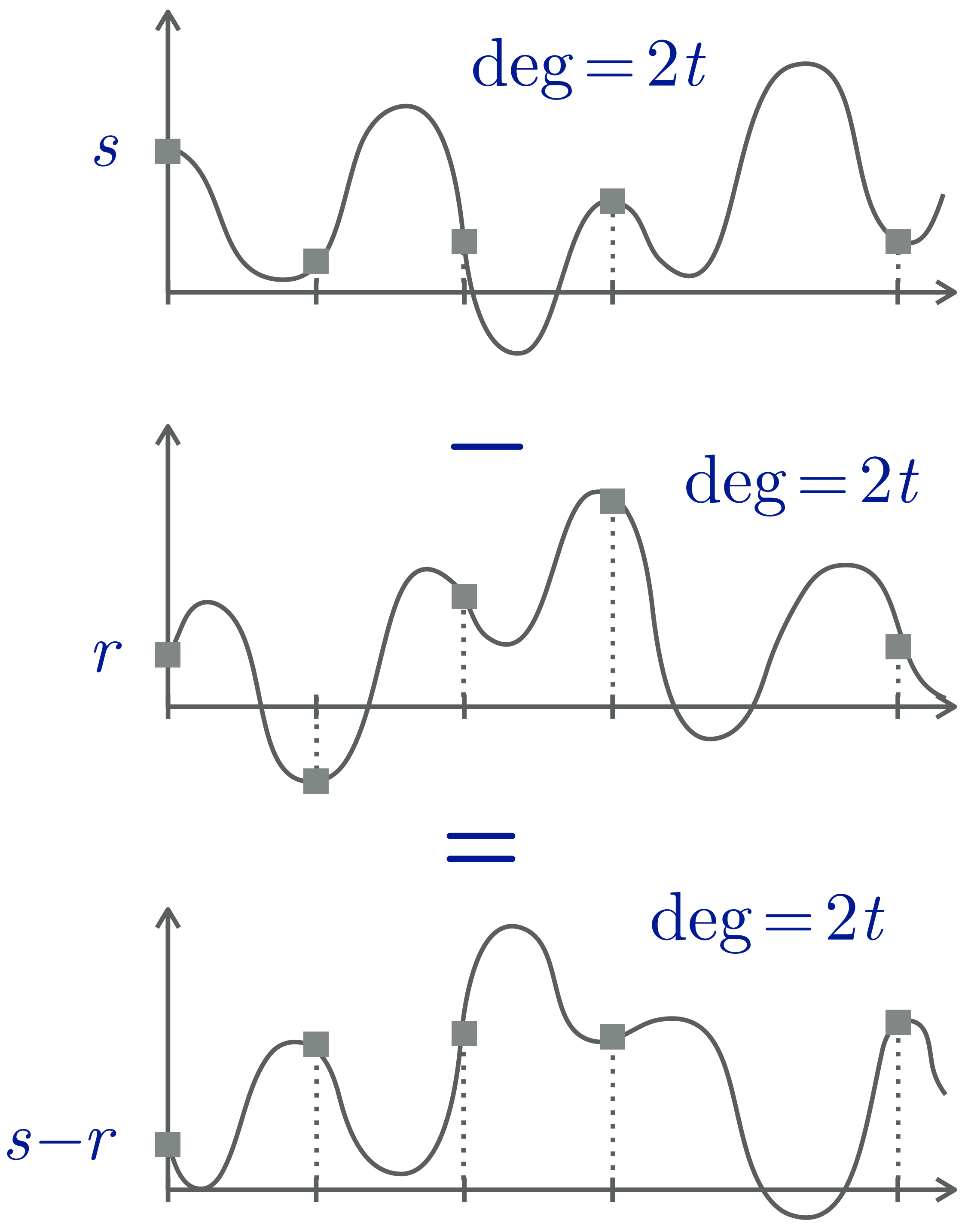
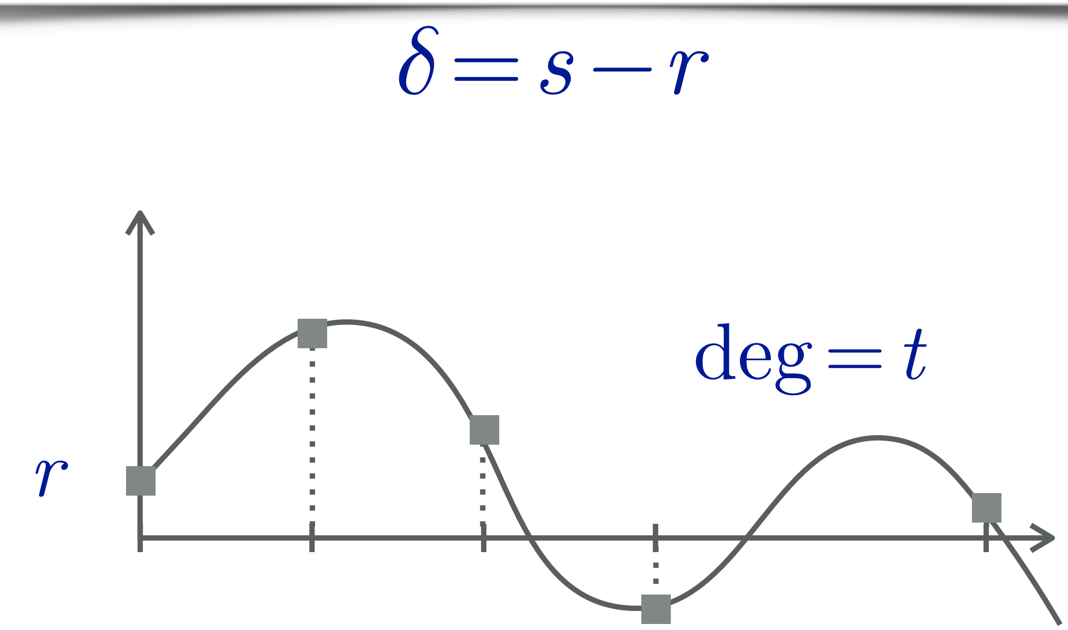


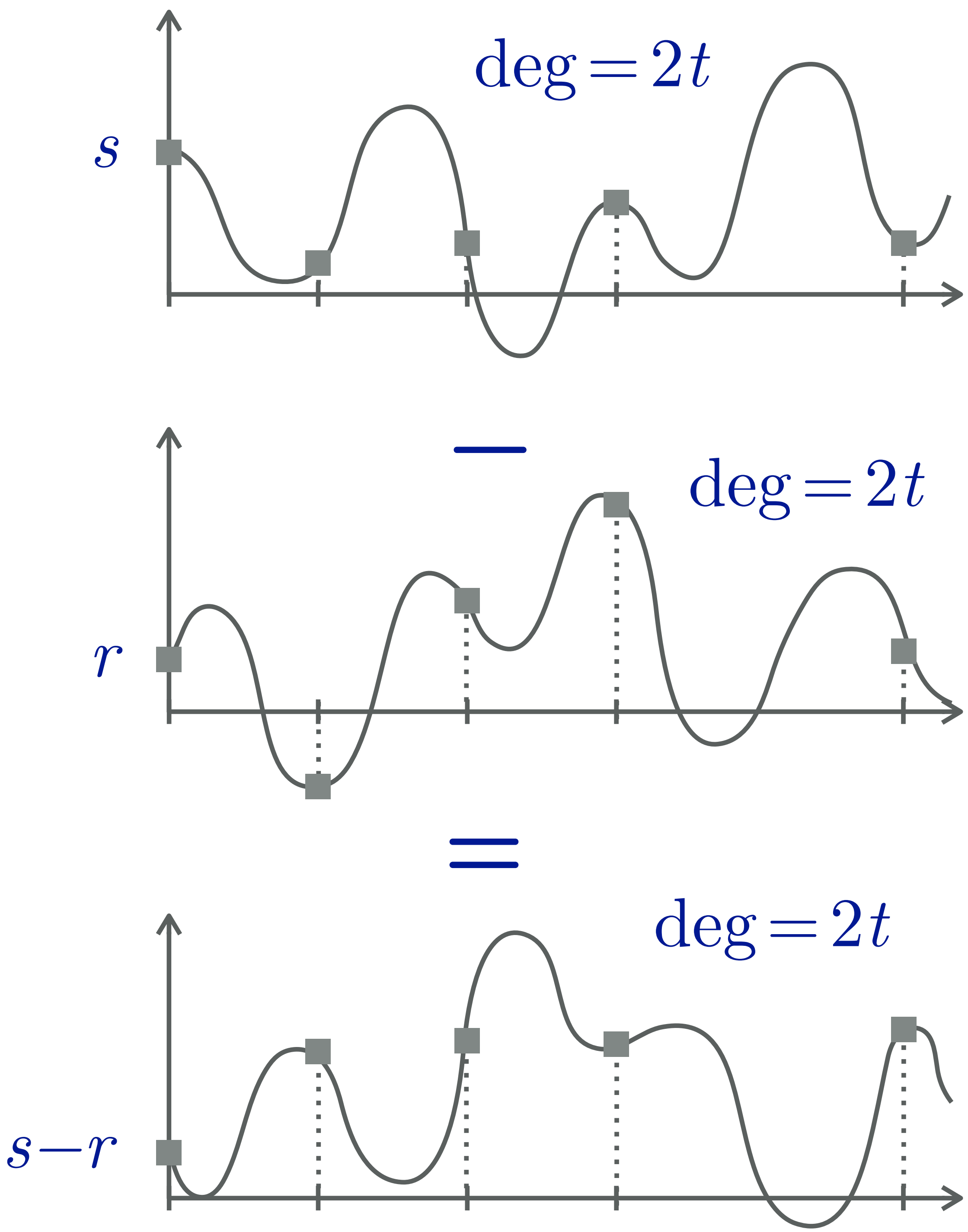
Reconstruct $\delta = s - r$, and



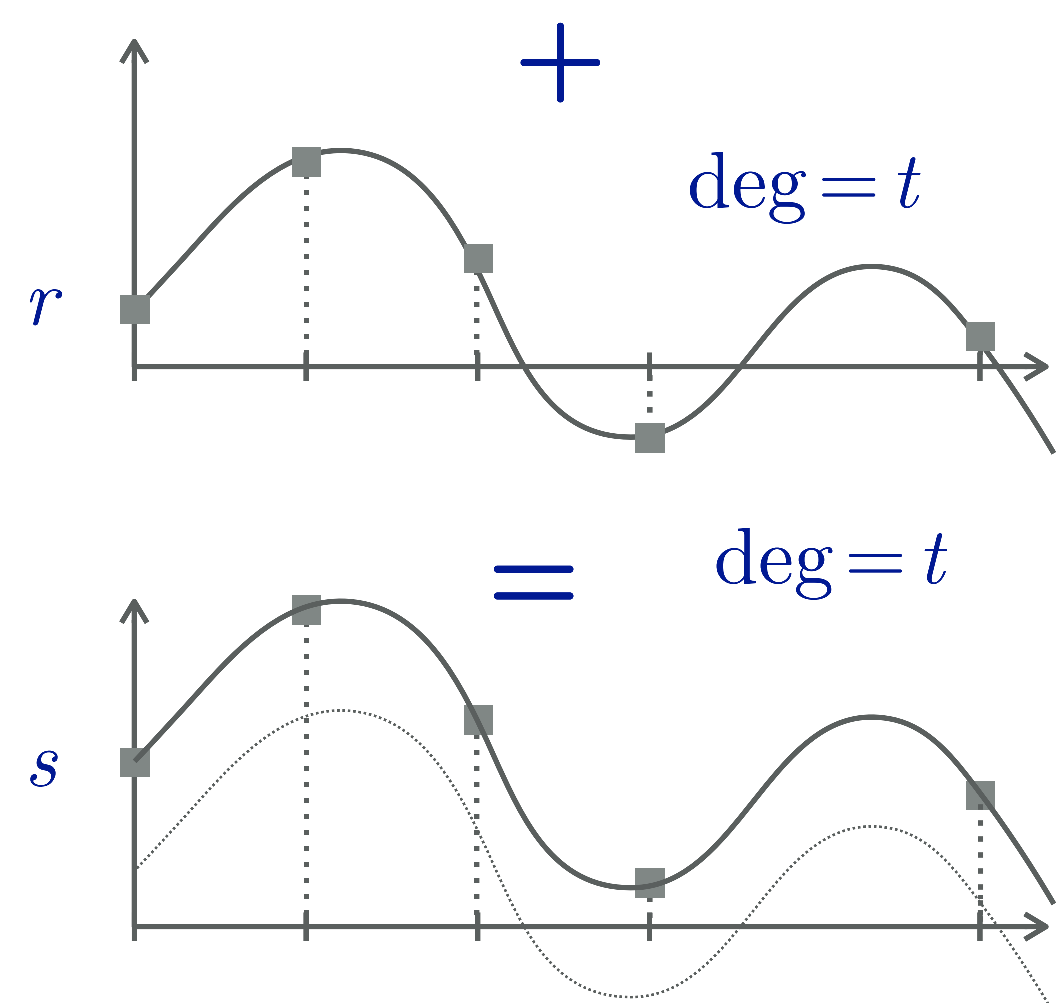
Degree Reduction

Due to [Chaum et al. 88], reinvented again in 2007.

Reconstruct $\delta = s - r$, and
add δ to the deg-t sharing of r.



$\delta = s - r$



Putting Above (And More) Things Together

Techniques for secure ***addition*** & secure ***multiplication***

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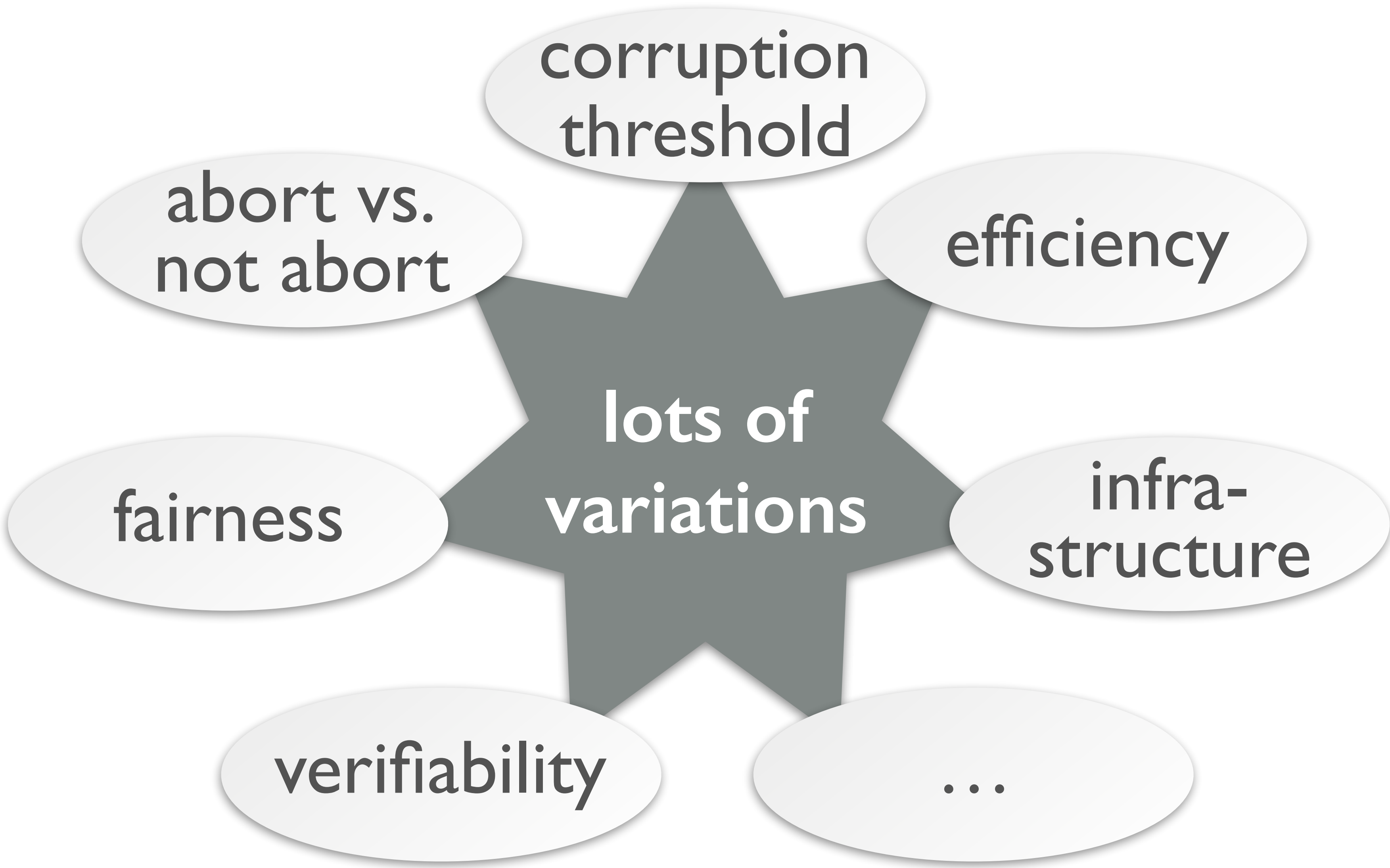
Together with basic result from theory of computation:
"*any computation can be put as an arithmetic computation*"

$\Downarrow$

Every computation can be done **securely**, i.e., so that

- everyone learns the correct result,
- yet nothing more than than,
- even if some of the parties are dishonest.

Various Relations & Dependencies

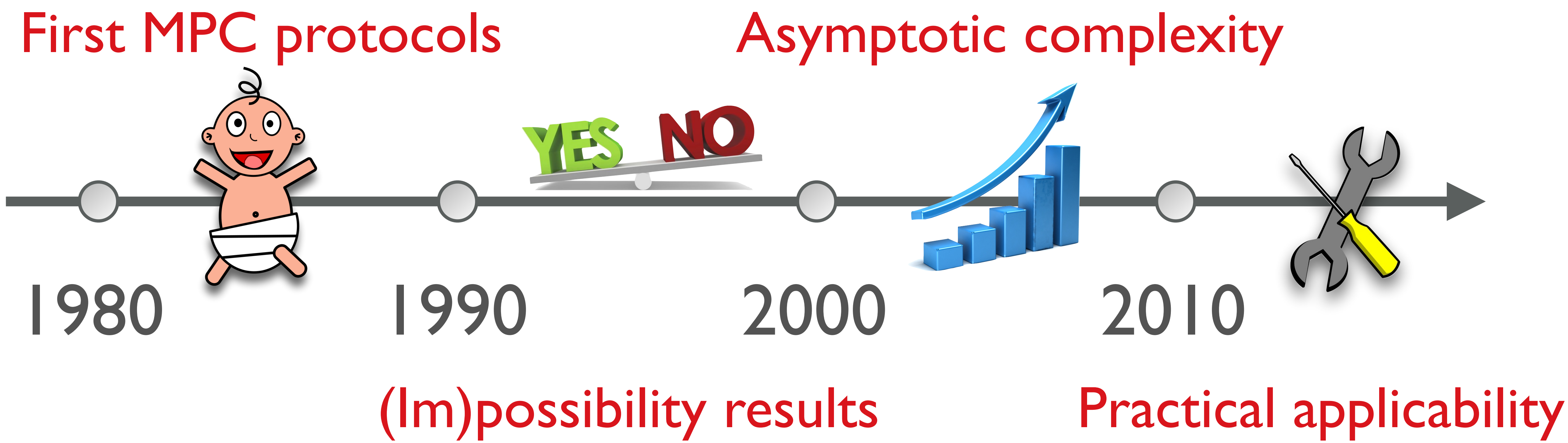


Optimal solution being very much **application dependent**.

Road Map

- WHAT is multiparty computation?
- HOW does multiparty computation work?
- WHERE can/is multiparty computation be/ used?

Timeline from Theory to Practice



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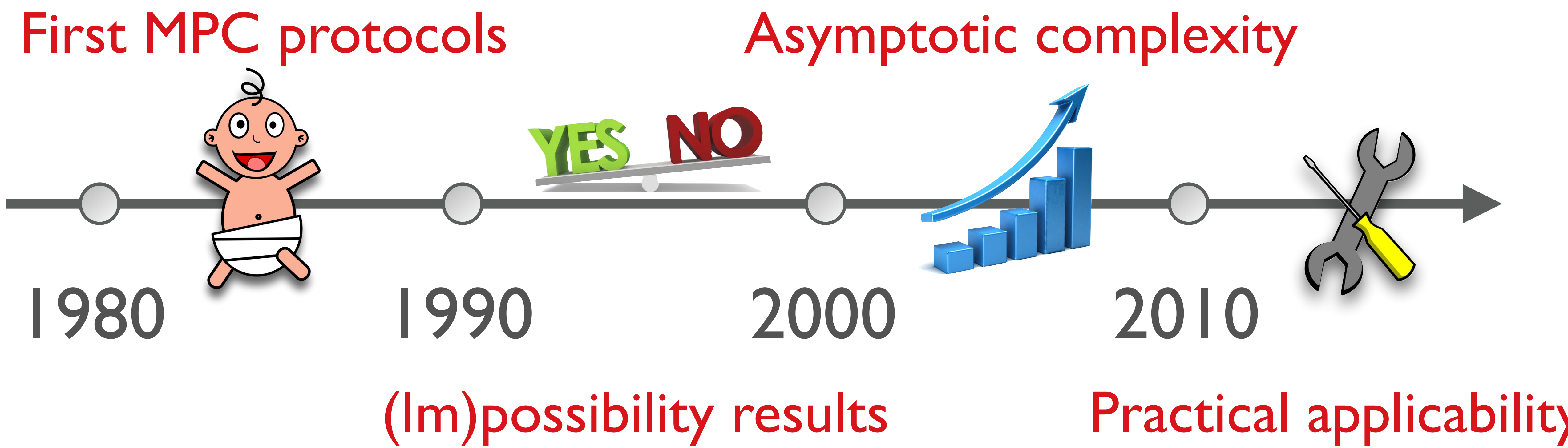

Current state of "practical MPC":

- ∃ companies that offer MPC solutions
- ∃ software libraries that facilitate "writing MPC code"
- ∃ isolated cases of real-life MPC deployment

But: **no** plug'n'play solution (seems to be inherent)

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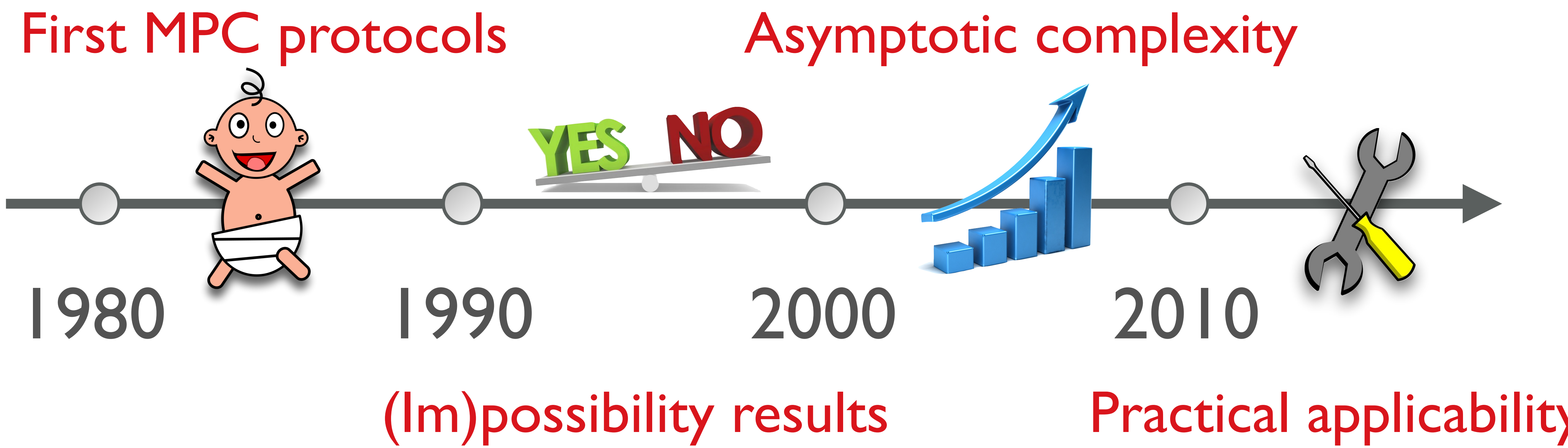

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Application scenario:

- Farmers in Demark wish to trade sugar beet contracts, giving them rights to produce/sell to a certain price.
- Danisco (buying the beets) needs to be involved as well.

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Solution: **Use MPC**

- Since 2008, auction runs as a 3-party computation.
- Market clearing price computed in a secure way, i.e., **without** revealing individual bids.

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Application scenario:

- Researchers in Estonia wanted to study the correlation between *working during university* and *failing to graduate*.
- Required: linking databases from *Estonian Tax & Customs Board* and from *Ministry of Education & Research*.

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Solution: **Use MPC**

- Statistical analysis was done by a 3-party computation, **without** revealing the data bases.

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Application scenario:

- Have every user name & password you enter on a site checked against credentials that are known to be unsafe.

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- Google offers a *Password Checkup* extension for Chrome, which uses a 2-party computation to check your credentials, **without** Google learning your credentials.

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Potential Future Real-life MPC Example

[Joint work with CWI Crypto, TNO, UvA - **Demonstrator only**]

Reference: A New Approach to Privacy-Preserving Clinical Decision Support Systems for HIV Treatment (arXiv:1810.01.01107)

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Application scenario:

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- Effectiveness of a drug is related to genotype of HIV virus.
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Solution: **Use MPC**

- We built a MPC prototype for a "experience database" with support for *time-to-treatment-failure* queries.

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- No plug'n'play: need for tailor-made solution is inherent

CWI Centrum Wiskunde & Informatica

Universiteit Leiden

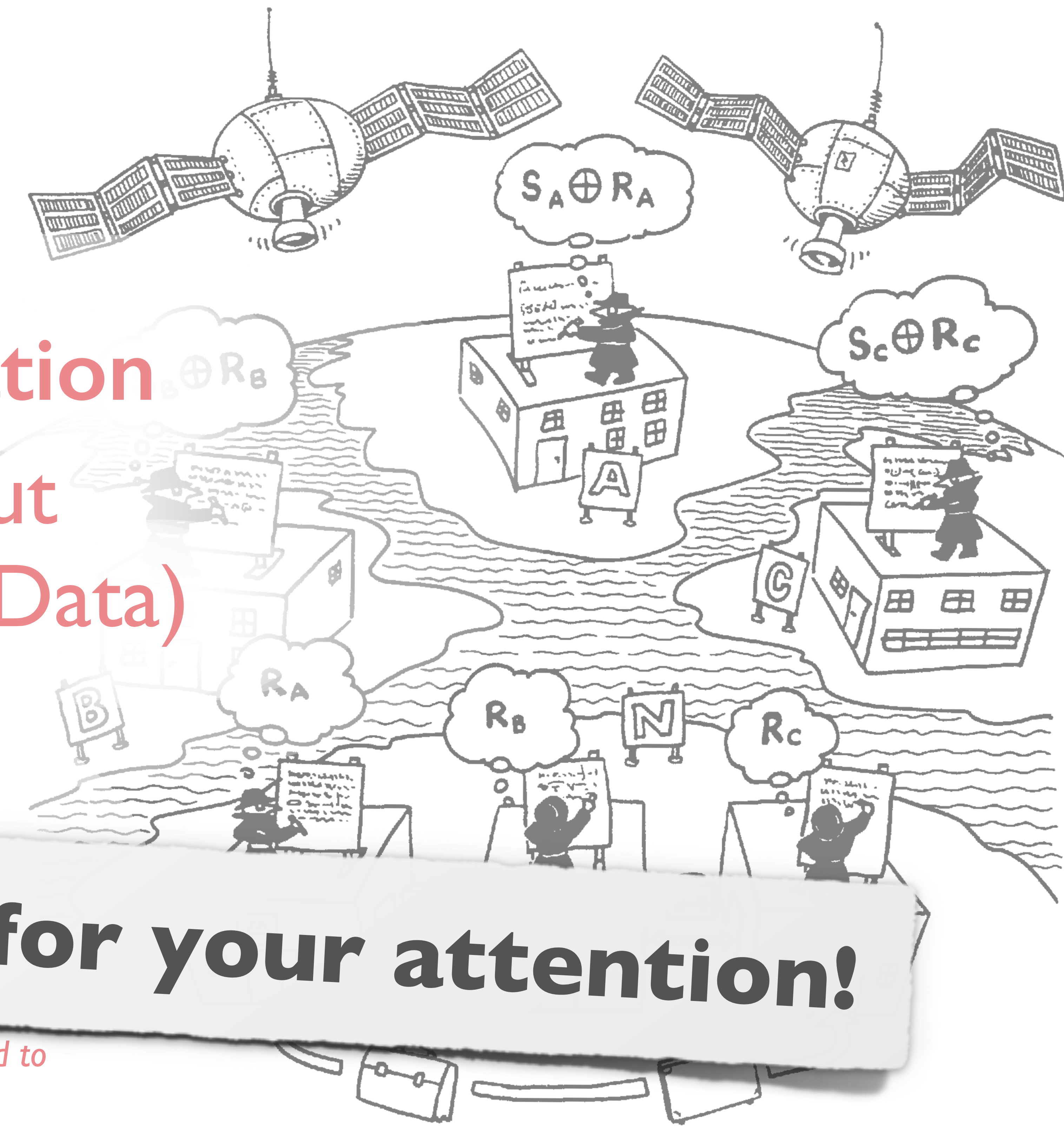
CWI Lectures
November 21 & 22, 2019

Multiparty Computation
Collaborate Without Compromise(ing Your Data)

Serge Fehr

Centrum Wiskunde & Informatica (CWI)

Mathem



Thank you for your attention!

On the occasion of the Dijkstra Fellowship being awarded to David Chaum