TECHNICAL REPORT

Converting a Regression Coefficient into a Correlation Coefficient

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Theorem. Let t be the value of Student's statistic under the hypothesis that a particular regression coefficient is equal to zero. Then the squared value of the corresponding correlation coefficient is $t^2/(t^2 + f)$, where f is the number of degrees of freedom.

Proof. Denote by $A = (a_{ij})$ the $p \times p$ Grammatrix corresponding to the $n \times p$ datamatrix with variable 1 as dependent variable and variables 2,...,p as independent variables. Then $f = n - p$. To be specific, consider $r = r_{12:34...p}$ and $b = b_{12:34...p}$. We shall prove

$$r^2 = \frac{t^2}{t^2 + f} \quad ,$$

where

$$t^2 = \frac{b^2}{\text{est.Var } b} \quad .$$

Let D_{ij} be the minor corresponding to a_{ij}, and partition A as follows

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \quad ,$$

where

$$A_{11} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad .$$

Now, the following formulae are valid

$$b = \frac{D_{12}}{D_{11}} \quad ,$$

$$r = \frac{D_{12}}{\sqrt{D_{11}\, D_{22}}} \quad ,$$

$$\text{est.Var } b = \left(\frac{1}{f}\,\frac{\det A}{D_{11}}\right)\cdot\left(\frac{\det A_{22}}{D_{11}}\right) \quad .$$

Hence

$$\frac{t^2}{t^2 + f} = \frac{D_{12}^2}{D_{12}^2 + \det A \cdot \det A_{22}} \quad .$$

The result follows by verifying that $\det A \cdot \det A_{22} = D_{11}D_{22} - D_{12}^2$.

Remark. The assertion of the theorem is just a rewriting of the formula

$$t = \frac{r}{\sqrt{1 - r^2}}\ \sqrt{n - p}\ , \qquad p \geqq 2 \ .$$

References.

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