THE LOGIC OF OCCURRENCE

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Report 5.86

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June 1986

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Does Tense Logic, in the style of Prior, provide a suitable framework within which to formalize the idea of the occurrence of an event? The answer to this question, it will turn out, is a qualified 'yes', the qualification being that it is necessary first to extend Tense Logic by enriching its syntactic resources.

To see clearly the nature of the problem, let us begin with the observation that not every past-tense sentence can be assigned to one of the logical forms *Pp* and *Hp*. An example which bears out this assertion is the sentence

(1) *Turpin rode from London to York*.

This sentence reports the past occurrence of an event which took time. We cannot represent this as either *Hp* or *Pp*, where *p* is *Turpin rides from London to York*, because it is not the case that (1) ascribes to any past time what *p* ascribes to the present, the reason being that *p* cannot be understood as ascribing anything to the present. Neither can we represent (1) as *Hq* or *Pq*, where *q* is *Turpin is riding from London to York*, for in this case *Pq* would represent the sentence

(2) <u>Turpin</u> <u>was</u> (or <u>has</u> <u>been</u>) <u>riding</u> <u>from</u> <u>London</u> <u>to</u> <u>York</u>.

while <u>Hq</u> would represent the sentence

(3) <u>Turpin</u> <u>has</u> <u>always</u> <u>been</u> <u>riding</u> <u>from</u> <u>London</u> <u>to</u> <u>York</u>.

and neither (2) nor (3) says the same thing as (1): all three sentences have different truth-conditions.

All of this applies equally well to future-tense propositions. We are now faced with a paradox. Consider the sentences

(1) <u>Turpin</u> <u>rode</u> <u>from</u> <u>London</u> <u>to</u> <u>York</u>.

(1') <u>Turpin</u> <u>will</u> <u>ride</u> <u>from</u> <u>London</u> <u>to</u> <u>York</u>.

We cannot represent these in the forms <u>Pp</u> and <u>Fp</u> respectively, for any <u>p</u>, nor in the forms <u>Hp</u> and <u>Gp</u> respectively, and yet they clearly do contain a common element of meaning, namely the idea of Turpin's riding from London to York, which is then modified by the adjunction of the ideas of pastness and futurity respectively. What I have been arguing is that the logical structure underlying this informal description cannot be expressed within the formalism of Tense Logic.

The standard tense operators act on propositions, and while there seems to be a fairly close correspondence between propositions and states of affairs (in a loose way of speaking, which is maddeningly difficult to make precise, propositions may be said to designate or pick out states of affairs), this cannot be immediately exploited in taking account of events, since events cannot be identified with states of affairs: at best, an event might be regarded as something like a succession of states of affairs.

On the other hand, it is possible to associate with any event *E* certain states which specify the temporal incidence of *E*, i.e. when it occurs. These states are

(i) The state which obtains by virtue of *E*'s having occurred in the past;

(ii) The state which obtains by virtue of *E*'s being currently in progress;

(iii) The state which obtains by virtue of *E*'s being destined to occur in the future.

I call these the perfective, progressive, and prospective states associated with the event *E*; they may be represented by the schematic propositions '*E* has occurred', '*E* is occurring' and '*E* will occur' respectively. Together they constitute a trichotomy of temporal incidence for events, which may be set alongside the familiar trichotomy of temporal incidence for states of affairs represented by the schematic propositions 'It has been the case that *p*', 'It is the case that *p*', and 'It will be the case that *p*'. These trichotomies are illustrated in figure 1. In these tables, *p* represents any proposition, and *E* is what I call an *event-radical*, i.e. an expression which designates an event without any implications as to its temporal incidence.

[Figure 1 here]

The items in these boxes are all themselves propositions, and hence candidates for operation on by the tense-logical operators of past, present and future. In particular, we can form the past and future of the progressive, yielding the table illustrated in Figure 1c. Note that '*E* has occurred' and '*E* will occur' are quite different propositions from '*E* has been occurring' and '*E* will be occurring'. In particular, if *E* has begun to occur, but has not yet occurred, then *E* has been occurring, so the past progressive of *E* can be true without the perfective being true; and analogously with the future variants.

What exactly are these event radicals? There are two ways of interpreting them, according as we choose to regard our events as types or tokens. In the former case, an event radical denotes a generic occurrence type, which may in general occur any number of times, or not at all, as for example, John's running a mile. In the latter case, an event radical denotes a specific occurrence, in a particular place at a particular time, which in the nature of things it makes no sense to speak of as occurring more than once, e.g. the mile that John ran at midday on his twentieth birthday.

According to how we interpret our event radicals, we shall end up with rather different event logics. Under the token interpretation, any event must occur at some time (namely the time fixed by the way the event is designated), and hence one of the propositions '*E* has occurred', '*E* is occurring' and '*E* will occur' must be true at each time for every *E*. Furthermore, on this way of looking at things, these three propositions are mutually exclusive, i.e. only one of them can be true for a given *E* at any given time. Under the type interpretation neither of these properties need hold. It is possible for John never to run a mile, in which case none of the three propositions mentioned will ever be true; it is also possible for John to run a mile more than once, in which case there will be times when more than one of them will be true simultaneously.

I think it is fair to say that those who have sought to give an account of the logic of events have mostly opted for the first interpretation, whereby events are regarded as tokens rather than as types. On this view, talk of type events must be analysed as involving quantification over token events. I believe that as an analysis of language this view is mistaken. It is mistaken because, first, it is not possible to identify token events except as instances of some type, and so type events must be conceptually prior to token events; and second, taking type events as basic is closer to the spirit of ordinary language, in which a token event is always specified by mentioning a type event and then adding limiting qualifications to fix a particular occurrence of that type.

From now on, then, I shall use the term _event_ to refer to a type event rather than a token event[1]. This does not mean, though, that I shall have no use in what follows for token events. In fact, what will turn out to be a satisfactory scheme is this: in the object language, which has a tense-logical basis, we take type events as basic and construct token events from them; but in the metalanguage, which has its basis in first-order logic and thus has the resource of quantification at its disposal, we take token events as basic and construct type events from them. The term I shall use to refer to a token event is <u>occurrence</u>. So I shall speak of the occurrences of an event, meaning thereby the tokens of a type.

It is important, incidentally, to distinguish an event which has only one occurrence from that occurrence itself. Some events by their nature cannot occur more than once, e.g. the event of someone's being born, or dying, or indeed doing anything for the first time, for the second time, etc. Such an event may fittingly be called a <u>once-only</u> event; and it is not to be identified with its unique occurrence.

It should be noted that what I present here is a logic of the <u>occurrence</u> of events; no attempt is made to analyse the events themselves, although it is recognized that such analysis is obviously desirable. This kind of simplification is typical of all enterprises which go under the name of logic. In the Propositional Calculus, for example, no attempt is made to analyse propositions beyond their truth-functional structure; indeed, the only feature of a proposition that matters for the Propositional Calculus is its truth value, propositions which agree in truth value counting as equivalent. In Tense Logic, similarly, the feature which matters is the temporal incidence of a proposition, and in this case equivalent propositions will be those with the same temporal incidence. In the same way, the logic I describe below treats events exclusively from the point of view of their temporal incidence, events which occur at the same time counting as equivalent from the point of view of this logic. Hence the model theory matches event radicals with sets of pairs of intervals, this being the simplest way to capture the temporal incidence features required; but this does not mean that in any metaphysical sense events are being identified with sets of pairs of intervals.

One further point, concerning my use of the terms 'perfective' and 'progressive'. These terms are taken, of course, from linguistics, and my use of them is intended to bear some relation to their use in linguistics. This relationship is not, however, very direct. In particular, as regards the progressive, I argued in [2] that the typical use of the progressive in English embodies a modal component, in that, for example, if one says that Turpin is riding to York one thereby imputes to Turpin the intention of getting to York (which is what makes it York that he is riding to rather than anywhere else), and the linguistic expression of this imputation is part of the function of the progressive form of the verb in this sentence. In Event Logic, I abstract from all such modal components in the meaning of the progressive, confining its use to those cases where an actual event, i.e. one which eventually reaches completion, is under way. The motivation underlying this, as explained in [2], is my belief that this chastened form of the progressive (what I call the _broad closed_ sense of the progressive) is basic, all other senses of the progressive being obtainable from this one by the addition of appropriate modal components.

* * *

Let us now embark on the enterprise of formalising the logic of events. First we must specify the language in which event-logical formulae are to be constructed. This consists of a lexicon, rules of formation, and definitions.

The Lexicon consists of the following primitive symbols:

Connectives: $\neg$, &.

Tense Operators: _P_, _F_.

Aspect Operators: _Perf_, _Prog_, _Pros_.

Event Radicals: $\underline{E}_1$, $\underline{E}_2$, $\underline{E}_3$, ...

The Rules of Formation. The well-formed formulae (_wffs_) of Event Language are defined by the rules:

(I) If _E_ is an event-radical then _PerfE_, _ProgE_, and _ProsE_ are all wffs (*atomic* wffs);

(II) If *A* and *B* are wffs then so are $\neg A$, $(A \;\&\; B)$, *FA*, and *PA*.

Definitions. Further symbols are defined, in standard fashion, as follows:

$$A \vee B =_{df} \neg(\neg A \;\&\; \neg B)$$
$$A \rightarrow B =_{df} \neg A \vee B$$
$$A \equiv B =_{df} (A \rightarrow B) \;\&\; (B \rightarrow A)$$
$$GA =_{df} \neg F \neg A$$
$$HA =_{df} \neg P \neg A$$

The intended meanings of wffs of Event Language are formally specified by means of the following model theory. First, we assume that time is linear. We begin with the notion of a *Linear Temporal Frame*, which is a structure $\langle T, < \rangle$, where *T* is any set (the elements of *T* being the 'times' of the model), and < is a total ordering on *T*, i.e. a relation with the following properties:

Irreflexivity: For no *t* in *T* does $t<t$ hold.

Transitivity: For every *t*, *t'*, *t"* in *T*, if $t<t'$ and $t'<t''$ then $t<t''$.

Linearity: For every *t*, *t'* in *T*, either $t<t'$, $t=t'$, or $t'<t$.

The symbol '<' should be read 'precedes'.

Next, we identify what elements of the structure are to represent events. Since, as we have remarked, we do not expect our treatment to extend beyond the facts of temporal incidence of events, it will be enough to identify each occurrence by the time at which it occurs. For reasons that will emerge, it turns out that the most satisfactory choice is to represent an occurrence by an ordered pair $\langle B,A \rangle$, where *B* is the set of times before the occurrence, and *A* is the set of times after it. The formal definition is:

> A *formal occurrence* in $\langle T,< \rangle$ is a pair $\langle B,A \rangle$, where *B* and *A* are non-empty subsets of *T* such that:
>
> (F01) Every element of *B* precedes every element of *A*.
>
> (F02) Every element of *T* which precedes an element of *B* is itself an element of *B*.
>
> (F03) Every element of *T* which is preceded by an element of *A* is itself an element of *A*.

The net effect of this definition is that *B* is an initial segment of *T* under the precedence relation, while *A* is a final segment; and *B* and *A* are disjoint.

Note that the definition of a formal occurrence leaves open the possibility that *B* and *A* jointly exhaust *T*, i.e. that $B \cup A = T$. In such a case we would have a formal occurrence with the property that at any time it either has occurred already or is yet to occur. Such an occurrence is at no time in the process of occurring; rather, it marks the boundary between two states of affairs, e.g. a body's starting to move marks the boundary between a stretch of time throughout which it is at rest and a stretch of time throughout which it is in motion, there being no third possible state to mediate between these two. This kind of occurrence takes no time, and being therefore point-like may fittingly be designated *punctual*. An occurrence which is not punctual has duration, and may accordingly be called *durative*.

An event may now be represented by the collection of all its occurrences, so we define a *formal event* in $\langle \underline{T},<\rangle$ to be nothing other than a set of formal occurrences in $\langle \underline{T},<\rangle$. There is no requirement that this set be non-empty; and indeed the empty formal event must correspond to an event which has no occurrences, just as in the standard model theory for first-order logic the empty set corresponds to a predicate which is not true of anything. The occurrences of a formal event may be all punctual, all durative, or a mixture of both. We may speak of the formal event itself in these cases as punctual, durative, or mixed.

An *event-logical model* can now be defined as a triple $\langle \underline{T},<,\underline{I}\rangle$, where $\langle \underline{T},<\rangle$ is a linear temporal frame and $\underline{I}$ is a mapping from the set of event radicals of the language into the set of formal events in $\langle \underline{T},<\rangle$. We shall usually denote such models as M, M', M", etc.; where necessary, we distinguish the $\underline{T}$, $<$, and $\underline{I}$ pertaining to a particular model M as $\underline{T}_M$, $<_M$, and $\underline{I}_M$.

The truth-definition for event language can now be stated as follows:

Atomic wffs

$M|=\underline{PerfE}[\underline{t}]$ iff $\underline{t}\in\underline{A}$ for some $\langle\underline{B},\underline{A}\rangle\in\underline{I}_M(\underline{E})$.

$M|=\underline{ProsE}[\underline{t}]$ iff $\underline{t}\in\underline{B}$ for some $\langle\underline{B},\underline{A}\rangle\in\underline{I}_M(\underline{E})$.

$M|=\underline{ProgE}[\underline{t}]$ iff $\underline{t}\in\underline{B}'\cap\underline{A}'$ for some $\langle\underline{B},\underline{A}\rangle\in\underline{I}_M(\underline{E})$.

Compound wffs

$M|= \underline{p}[\underline{t}]$ iff not $M|=\underline{p}[\underline{t}]$.

$M|=\underline{p}\&\underline{q}[\underline{t}]$ iff $M|=\underline{p}[\underline{t}]$ and $M|=\underline{q}[\underline{t}]$.

$M|=\underline{Pp}[\underline{t}]$ iff $M|=\underline{p}[\underline{t'}]$ for some $\underline{t'}\in\underline{T}_M$ such that $\underline{t'}<_M\underline{t}$.

$M|=\underline{Fp}[\underline{t}]$ iff $M|=\underline{p}[\underline{t'}]$ for some $\underline{t'}\in\underline{T}_M$ such that $\underline{t}<_M\underline{t'}$.

From the definitions of v, $\rightarrow$, $\equiv$, $\underline{H}$, and $\underline{G}$ we can deduce that:

$M|=\underline{p}v\underline{q}[\underline{t}]$ iff either $M|=\underline{p}[\underline{t}]$ or $M|=\underline{q}[\underline{t}]$.

$M|=\underline{p}\rightarrow\underline{q}[\underline{t}]$ iff $M|=\underline{q}[\underline{t}]$ if $M|=\underline{p}[\underline{t}]$.

$M|=\underline{p}\equiv\underline{q}[\underline{t}]$ iff $M|=\underline{p}[\underline{t}]$ iff $M|=\underline{q}[\underline{t}]$.

$M|=\underline{Hp}[\underline{t}]$ iff $M|=\underline{p}[\underline{t'}]$ for every $\underline{t'}\in\underline{T}_M$ such that $\underline{t'}<_M\underline{t}$.

$M|=\underline{Gp}[\underline{t}]$ iff $M|=\underline{p}[\underline{t'}]$ for every $\underline{t'}\in\underline{T}_M$ such that $\underline{t}<_M\underline{t'}$.

Note also that from the truth-definition for atomic wffs it is a simple deduction that

$$\{\underline{t}\in\underline{T}_M \mid M|=\underline{\text{PerfE}}[\underline{t}]\} = \cup\{\underline{A} \mid \langle\underline{B},\underline{A}\rangle\in\underline{I}_M(\underline{E})\}.$$
$$\{\underline{t}\in\underline{T}_M \mid M|=\underline{\text{ProsE}}[\underline{t}]\} = \cup\{\underline{B} \mid \langle\underline{B},\underline{A}\rangle\in\underline{I}_M(\underline{E})\}.$$
$$\{\underline{t}\in\underline{T}_M \mid M|=\underline{\text{ProgE}}[\underline{t}]\} = \cup\{\underline{B}'\cap\underline{A}' \mid \langle\underline{B},\underline{A}\rangle\in\underline{I}_M(\underline{E})\}.$$

We may describe a model M as punctual (durative) if $\underline{I}_M(\underline{E})$ is punctual (durative) for every $\underline{E}$. It is once-only if $\underline{I}_M(\underline{E})$ contains at most one member for every $\underline{E}$.

We shall use the symbol $\underset{\sim}{M}$ with different subscripts to denote classes of models. In particular we shall use:

$\underset{\sim}{M}_e$ to denote the class of all event-logical models;

$\underset{\sim}{M}_{ep}$ to denote the class of all punctual event-logical models;

$\underset{\sim}{M}_{eo}$ to denote the class of all once-only event-logical models;

$\underset{\sim}{M}_{ed}$ to denote the class of all durative event-logical models.

Obviously, $\underset{\sim}{M}_{ep}$, $\underset{\sim}{M}_{eo}$, and $\underset{\sim}{M}_{ed}$ are all subsets of $\underset{\sim}{M}_e$.

For any class of models M we write $|=_{\underset{\sim}{M}} \underline{p}$ to mean that $M|=\underline{p}[\underline{t}]$ for every $M\in\underset{\sim}{M}$ and $\underline{t}\in\underline{T}_M$. If Σ is a set of wffs we write $\Sigma|=_{\underset{\sim}{M}} \underline{p}$ to mean that $M|=\underline{p}[\underline{t}]$ for every $M\in\underset{\sim}{M}$ and $\underline{t}\in\underline{T}_M$ such that $M|=\underline{s}[\underline{t}]$ for every $\underline{s}\in\Sigma$.

If we ignore the internal structure of the atomic wffs, event language becomes the ordinary language of tense logic. We can in this way regard a model for event language as giving rise to a model for tense language. Such a model is, in general, a triple $\langle T,<,V\rangle$, where V is a valuation function which assigns a truth-value (say 0 or 1) to each of the atomic wffs of the language at each time in T. The truth-definition for a model of this kind is given for atomic wffs by means of the condition $M\models p[t]$ iff $V(p,t)=1$, and for the compound wffs by the same set of rules as was given for the compound wffs of event language above. We can say that an event-logical model $M=\langle T,<,I\rangle$ *generates* the tense-logical model $gen(M)=\langle T,<,V_I\rangle$ by way of the following identifications:

$V_I(\mathit{PerfE},t)=1$ iff $t\in A$ for some $\langle B,A\rangle\in I(E)$.

$V_I(\mathit{ProsE},t)=1$ iff $t\in B$ for some $\langle B,A\rangle\in I(E)$.

$V_I(\mathit{ProgE},t)=1$ iff $t\in B'\cap A'$ for some $\langle B,A\rangle\in I(E)$.

It is a straightforward induction proof from these stipulations to the result that $gen(M)\models p[t]$ iff $M\models p[t]$, for every wff p and time $t\in T_M$.

The importance of this manoeuvre is that it links up the model theory for event logic with that of tense logic; and the latter is a well-researched area with plenty of definitive results for us to draw upon.

In particular, since for an event-logical model $M=\langle \underline{T},<,\underline{I}\rangle$ the temporal frame $\langle \underline{T},<\rangle$ is always linear, the tense-logical model $\langle \underline{T},<,\underline{V}_{\underline{I}}\rangle$ generated by M will also always be linear. Now it is well-known that the class L of linear tense-logical models characterises the axiomatic tense-logical system CL (due to N. Cocchiarella, 1965[2]) defined by the axiom schemata:

(T1) Any tautology of Propositional Calculus

(T2) $\underline{G}(\underline{p}\rightarrow\underline{q})\rightarrow(\underline{Gp}\rightarrow\underline{Gq})$

(T3) $\underline{H}(\underline{p}\rightarrow\underline{q})\rightarrow(\underline{Hp}\rightarrow\underline{Hq})$

(T4) $\underline{PGp}\rightarrow\underline{p}$

(T5) $\underline{FHp}\rightarrow\underline{p}$

(T6) $\underline{Gp}$, where $\underline{p}$ is any axiom

(T7) $\underline{Hp}$, where $\underline{p}$ is any axiom

(T8) $\underline{FFp}\rightarrow\underline{Fp}$

(T9) $\underline{PFp}\rightarrow[\underline{Pp} \vee (\underline{p} \vee \underline{Fp})]$

(T10) $\underline{FPp}\rightarrow[\underline{Pp} \vee (\underline{p} \vee \underline{Fp})]$

and the rule of inference Modus Ponens: from $\underline{p}\rightarrow\underline{q}$ and $\underline{p}$, infer $\underline{q}$. For this reason, we shall base our Proof Theory for Event Logic on the system CL of Linear Tense Logic.

All the event logics we shall be considering, then, will include the axioms (T1)-(T10) and the rule Modus Ponens in their proof theories. In addition, they will contain the following axioms, in which '$\underline{E}$' stands for any event radical:

(E1) $\underline{\text{ProsE}} \rightarrow \underline{\text{HProsE}}$

(E2) $\underline{\text{PerfE}} \rightarrow \underline{\text{GPerfE}}$

(E3) $\underline{\text{ProsE}} \rightarrow \underline{\text{FPerfE}}$

(E4) $\underline{\text{PerfE}} \rightarrow \underline{\text{PProsE}}$

(E5) $\underline{\text{ProgE}} \rightarrow \underline{\text{PProsE}}$

(E6) $\underline{\text{ProgE}} \rightarrow \underline{\text{FPerfE}}$

(E7) $\underline{\text{ProgE}} \rightarrow \underline{\text{H}}(\underline{\text{ProsE}} \vee \underline{\text{ProgE}})$

(E8) $\underline{\text{ProgE}} \rightarrow \underline{\text{G}}(\underline{\text{PerfE}} \vee \underline{\text{ProgE}})$

(E9) $\underline{\text{PProsE}} \rightarrow [\underline{\text{PerfE}} \vee (\underline{\text{ProgE}} \vee \underline{\text{ProsE}})]$

The system defined by the axioms (E1)-(E9) is called *Minimal Event Logic*, denoted E. Note that the temporal mirror-image of (E9), namely

$$\underline{\text{FPerfE}} \rightarrow [(\underline{\text{PerfE}} \vee \underline{\text{ProgE}}) \vee \underline{\text{ProsE}}]$$

need not be given as an axiom; it is already a theorem of E, the proof, somewhat abbreviated, being as follows:

1.	$\underline{\text{FPerfE}} \rightarrow \underline{\text{FPProsE}}$	(E4,RF)
2.	$\underline{\text{FPProsE}} \rightarrow [\underline{\text{PProsE}} \vee (\underline{\text{ProsE}} \vee \underline{\text{FProsE}})]$	(T10)
3.	$\underline{\text{PProsE}} \rightarrow [\underline{\text{PerfE}} \vee (\underline{\text{ProgE}} \vee \underline{\text{ProsE}})]$	(E9)
4.	$\underline{\text{FProsE}} \rightarrow \underline{\text{FHProsE}}$	(E1,RF)
5.	$\underline{\text{FHProsE}} \rightarrow \underline{\text{ProsE}}$	(T5)
6.	$\underline{\text{FProsE}} \rightarrow \underline{\text{ProsE}}$	(4,5,Tautology)
7.	$\underline{\text{FPerfE}} \rightarrow [(\underline{\text{PerfE}} \vee \underline{\text{ProgE}}) \vee \underline{\text{ProsE}}]$	(1,2,3,6,Tautologies)

Here RF is the CL-metatheorem that from '$\underline{p} \to \underline{q}$' we can infer '$\underline{Fp} \to \underline{Fq}$'. A past-tense version RP of this metatheorem also exists, namely that from '$\underline{p} \to \underline{q}$' we can infer '$\underline{Pp} \to \underline{Pq}$'.

Extensions of **E** can be obtained by the addition of further axioms. Some examples we shall consider are:

Punctual Event Logic (EP), obtained from E by the addition of the axiom

(P) $\neg$ProgE.

Once-only Event Logic (EO), obtained from E by the addition of the axioms

(O1) $\neg$(PerfE & ProsE)

(O2) $\neg$(PerfE & ProgE)

(O3) $\neg$(ProsE & ProgE).

Durative Event Logic (ED), obtained from E by the addition of the axioms

(D1) ProsE $\to$ FProgE

(D2) PerfE $\to$ PProgE.

Note that the addition of (P) enables one to simplify the complete axiom-set for EP. The axioms (E5)-(E8) are no longer needed, as they are trivial consequences of (P), the antecedent in each case being always false; and (E9) can be simplified by omitting the middle disjunct. It thus becomes possible to define EP in a language which has been impoverished to the extent of having the operator Prog removed. In such a language the axioms of EP can be stated as

(EP1) ProsE → HProsE

(EP2) PerfE → GPerfE

(EP3) ProsE → FPerfE

(EP4) PerfE → PProsE

(EP5) PProsE → (PerfE v ProsE).

This new version of EP agrees with the old one in its evaluation of all Prog-free wffs; it differs from it only in that it provides no evaluation of wffs containing Prog.

A degree of simplification is also possible in the case of ED, for in this system (E3) and (E4) can be proved from the other axioms. The proof of (E3) is as follows:

1.	ProsE → FProgE	(D1)
2.	FProgE → FFPerfE	(E6,RF)
3.	FFPerfE → FPerfE	(T8)
4.	ProsE → FPerfE	(1,2,3,Tautologies)

(E9) is also redundant in ED, since we have the derivation:

1. FPerfE → FPProgE (D2,RF)

2. FPProgE → [PProgE v (ProgE v FProgE)] (T10)

3. PProgE → PG(PerfE v ProgE) (E8,RP)

4. PG(PerfE v ProgE) → (PerfE v ProgE) (T4)

5. FProgE → FH(ProsE v ProgE) (E7,RF)

6. FH(ProsE v ProgE) → (ProsE v ProgE) (T5)

7. FPerfE → [PerfE v (ProgE v ProsE)]

(1,2,3,4,5,6,Tautologies)

The axioms of ED can thus be reformulated as follows:

(ED1) ProsE → HProsE

(ED2) PerfE → GPerfE

(ED3) ProsE → FProgE

(ED4) PerfE → PProgE

(ED5) ProgE → PProsE

(ED6) ProgE → FPerfE

(ED7) ProgE → H(ProsE v ProgE)

(ED8) ProgE → G(PerfE v ProgE).

For any proof system S we shall write $\vdash_S p$ to mean, as usual, that *p* is provable as a theorem in the system S, and $\Sigma \vdash_S p$ to mean that *p* is deducible from Σ in S. The formal semantics of Event Logic connects the proof theories and the model theories by showing that the classes of models $\underset{\sim}{M}_e$, $\underset{\sim}{M}_{ep}$, $\underset{\sim}{M}_{eo}$, and $\underset{\sim}{M}_{ed}$ characterise the theories E, EP, EO and ED respectively, where as usual to say that a class of models $\underset{\sim}{M}$ <u>characterises</u> a proof system S is to say that for every Σ, p in the language under consideration, $\Sigma \vdash_{S} p$ iff $\Sigma \models_{\mathbf{M}} p$. The remainder of this paper is devoted to proving these results.

* * *

We treat the two directions of the characterisation equivalence separately, namely as:

(1) Soundness. If $\Sigma \vdash_{S} p$ then $\Sigma \models_{\mathbf{M}} p$.

(2) Completeness. If $\Sigma \models_{\mathbf{M}} p$ then $\Sigma \vdash_{S} p$.

(1) Soundness. We begin with Minimal Event Logic, taking our start from the result already mentioned, that Linear Tense Logic CL is sound with respect to the class $\mathbf{L}$ of all linear tense-logical models. This means, *a fortiori*, that CL is sound with respect to the class of tense-logical models generated by event-logical models, i.e. the class $\mathbf{L}_e = \{\underline{gen}(M) \mid M \in \mathbf{M}_e\}$. This is because all such models are linear, i.e. $\mathbf{L}_e$ $\mathbf{L}$. We thus have

$$\models_{\mathbf{L}e} p \text{ iff } M \models p[t] \text{ for every } M \in L_e \text{ and } t \in T_M$$

$$\text{iff } \underline{gen}(M) \models p[t] \text{ for every } M \in \mathbf{M}_e \text{ and } t \in T_M$$

$$\text{iff } M \models p[t] \text{ for every } M \in \mathbf{M}_e \text{ and } t \in T_M$$

$$\text{iff } \models_{\mathbf{M}e} p$$

Since CL is sound with respect to $\underset{\sim}{L}_e$, the axioms and rules of inference of CL are all valid in $\underset{\sim}{L}_e$, and hence, by the above, in $\underset{\sim}{M}_e$. It therefore only remains to show that the event-logical axioms (E1)-(E8) of E are also all valid in $\underset{\sim}{M}_e$. We need only consider the odd-numbered axioms, as the proof of validity for each even-numbered axiom is exactly parallel, mutatis mutandis, to the proof for the immediately preceding odd-numbered axiom. 'Mutatis mutandis' here means 'by application of the "Mirror-Image Rule", i.e. by swapping P, H, Perf, B, and x<y with F, G, Pros, A, and y<x, respectively'.

(E1) ProsE→HProsE. It suffices to show that M|=HProsE[t] for every M in $\underset{\sim}{M}_e$ such that M|=ProsE[t]. Suppose M|=ProsE[t]. This means that t∈B for some <B,A>∈I_M (E). Let t'<t. Then t'∈B by (FO2). Hence M|=ProsE[t']. This holds for any t'<t, so M|=HProsE[t], as required.

(E3) ProsE→FPerfE. We must show that if M|=ProsE[t], where M∈$\underset{\sim}{M}_e$, then M|=FPerfE[t]. Suppose M|=ProsE[t]. Then t∈B for some <B,A> in I_M (E). Since A is non-empty, from the definition of a formal occurrence, there exists some t'∈A, so M|=PerfE[t']. Since t∈B and t'∈A, t<t' by (FO1). Hence M|=FPerfE[t'], as required.

(E5) ProgE → PProsE. Suppose $M \models$ ProgE[t]. Then $t \in B' \cap A'$ for some $\langle B, A \rangle$ in $I_M(E)$. Since B is non-empty, let $t' \in B$. Then $M \models$ ProsE[t']. Also, $t \neq t'$, since $t \in B'$ and $t' \in B$, and $t \nless t'$ (else $t \in B$ by (FO2)). Hence $t' < t$, by linearity, and since $M \models$ ProsE[t'], we have $M \models$ PProsE[t], as required.

(E7) ProgE → H(ProsE v ProgE). Suppose $M \models$ ProgE[t] for some $M \in \underset{\sim}{M}_e$. Then $t \in B' \cap A'$ for some $\langle B, A \rangle$ $I_M(E)$. Let $t' \in T_M$ be such that $t' < t$ (such must exist, since any member of B, which is non-empty, must precede t). Then $t' \notin A$, else $t \in A$ by (FO3). Hence either $t' \in B' \cap A'$ or $t' \in B$. In the former case we have $M \models$ ProgE[t'], in the latter case $M \models$ ProsE[t']. In either case, $M \models$ ProsE v ProgE[t'], so since t' is any time preceding t in M, we have $M \models$ H(ProsE v ProgE)[t], as required.

(E9) PProsE → PerfE v (ProgE v ProsE). Suppose $M \models$ PProsE[t]. Then for some $t' < t$, $M \models$ ProsE[t']. This means that $t' \in B$ for some $\langle B, A \rangle \in I_M(E)$. By set theory, either $t \in B$, $t \in B' \cap A'$, or $t \in A$. But these three cases give $M \models$ ProsE[t], $M \models$ ProgE[t], and $M \models$ PerfE[t] respectively. So in any case we have $M \models$ PerfE v (ProgE v ProsE)[t], as required.

We have now shown that all the axioms and rules of inference of E are valid in $\underset{\sim}{M}_e$, and this is sufficient to show that E is sound with respect to

$\underset{\sim}{M}_e$. It is now a straightforward matter to extend this result to the other systems under consideration, as follows.

Punctual Event Logic has the additional axiom (P) $\neg$*ProgE*. This is not valid in $\underset{\sim}{M}_e$, but it is valid in $\underset{\sim}{M}_{ep}$, since the latter class contains only punctual models. In a punctual model, every occurrence $\langle \underline{B},\underline{A}\rangle$ of every event is such that $\underline{B}\cup\underline{A}=\underline{T}$, i.e. $\underline{B}'\cap\underline{A}'=\emptyset$. Hence we always have that $\underline{t}\notin\underline{B}'\cap\underline{A}'$, i.e. $M\models\neg\underline{ProgE}[\underline{t}]$.

Once-only Event Logic has the further axioms (O1)-(O3), which are again not valid in $\underset{\sim}{M}_e$, but they are valid in $\underset{\sim}{M}_{eo}$, which contains only once-only models. In such a model, $\underline{I}(\underline{E})$ is at most one-membered for every $\underline{E}$. Suppose $M\models\underline{PerfE}[\underline{t}]$, where $M\in\underset{\sim}{M}_{eo}$. Then $\underline{t}\in\underline{A}$ for the unique $\langle \underline{B},\underline{A}\rangle\in\underline{I}(\underline{E})$; hence, since $\underline{B}\cap\underline{A}=\emptyset$, $\underline{t}\notin\underline{B}$ for any $\langle \underline{B},\underline{A}\rangle\in\underline{I}(\underline{E})$, i.e. $M\models\neg\underline{ProsE}[\underline{t}]$; and also $\underline{t}\notin\underline{B}'\cap\underline{A}'$ for any $\langle \underline{B},\underline{A}\rangle\in\underline{I}(\underline{E})$, i.e. $M\models\neg\underline{ProgE}[\underline{t}]$. Hence we have $M\models\underline{PerfE}\rightarrow\neg\underline{ProsE}[\underline{t}]$ and $M\models\underline{PerfE}\rightarrow\neg\underline{ProgE}[\underline{t}]$, which are straightforwardly equivalent to $M\models\neg(\underline{PerfE}\ \&\ \underline{ProsE})[\underline{t}]$ and $M\models\neg(\underline{PerfE}\ \&\ \underline{ProgE})[\underline{t}]$ respectively, thereby validating (O1) and (O2). (O3) is validated similarly.

Durative Event Logic has the extra axioms (D1) and (D2). We need only show validity for (D1), the proof for (D2) being *mutatis mutandis* exactly similar. Suppose, then, that $M \models \underline{ProsE}[\underline{t}]$ for some $M \in \underset{\sim}{M}_{ed}$ and $\underline{t} \in \underline{T}_M$. Then $\underline{t} \in \underline{B}$ for some $\langle \underline{B}, \underline{A} \rangle \in \underline{I}_M(\underline{E})$. Since M is a durative model, $\underline{B}' \cap \underline{A}' \neq \emptyset$, so let $\underline{t}' \in \underline{B}' \cap \underline{A}'$. Then $M \models \underline{ProgE}[\underline{t}']$. Since $\underline{t} \in \underline{B}$ and $\underline{t}' \in \underline{B}'$, $\underline{t} \neq \underline{t}'$; also $\underline{t}' \nless \underline{t}$ (else $\underline{t}' \in \underline{B}$ by (FO2)); hence $\underline{t} < \underline{t}'$. Since $M \models \underline{ProgE}[\underline{t}']$, we have $M \models \underline{FProgE}[\underline{t}]$, as required. This completes the soundness proofs.

(2) Completeness. We begin by considering quite generally what is required for a proof that a given proof theory S is complete with respect to a given class of models $\underset{\sim}{M}$. The desired result, namely

$$\text{'If } \Sigma \models_{\underset{\sim}{M}} \underline{p} \text{ then } \vdash_S \underline{p}\text{'}$$

is first rewritten as

$$\text{'If } \Sigma \nvdash_S \underline{p} \text{ then } \Sigma \not\models_{M} \underline{p}\text{'}$$

which in turn is rewritten as

'If $\Sigma \cup \{\neg \underline{p}\}$ is consistent in S then $\Sigma \cup \{\neg \underline{p}\}$ has a model in $\underset{\sim}{M}$'

or, more tidily, as

'Any set of wffs consistent in S has a model in $\underset{\sim}{M}$'.

By a model for a set of wffs Σ, in the case of tense- and event-logical models, is meant a model M such that for some $\underline{t}_0 \in \underline{T}_M$, $M \models \underline{p}[\underline{t}_0]$ for each $\underline{p}$ in Σ.

For Minimal Event Logic, then, we must show there exists a model in $\underset{\sim}{M}_e$ for any set of wffs consistent in E. Let Σ be our consistent set, and extend Σ in the usual way to a maximally consistent set Σ^+, i.e. a consistent extension of Σ to which no further wffs may be added without incurring inconsistency. Now note that if Σ^+ is consistent in E then it is consistent in CL, since every derivation in CL is also a derivation in E, so any proof of inconsistency in CL would also be a proof of inconsistency in E. Since CL is complete with respect to $\underset{\sim}{L}$, any set Σ consistent in E has a model M in $\underset{\sim}{L}$ (for the purposes of this model, wffs of the forms $\underline{PerfE}$, $\underline{ProgE}$, and $\underline{ProsE}$ are considered as unanalysed primitives). We shall show how to derive from this model a model $M' \in \underset{\sim}{M}_e$ such that $\underset{\sim}{M} = \underline{gen}(\underset{\sim}{M}')$. We do this by showing how to specify the value of $\underline{I}(\underline{E})$ for each radical $\underline{E}$ occurring in Σ^+.

Assume, then, that $M \models \underline{p}[\underline{t}_0]$ for each $\underline{p} \in \Sigma^+$. Since Σ^+ is maximally consistent, we can actually assert the stronger result that $M \models \underline{p}[\underline{t}_0]$ iff $\underline{p} \in \Sigma^+$. Then, given a radical $\underline{E}$, define the sets

$$\underset{\sim}{A} = \{\underline{t} \in \underline{T}_M \mid M \models \underline{PerfE}[\underline{t}]\}$$

$$\underset{\sim}{B} = \{\underline{t} \in \underline{T}_M \mid M \models \underline{ProsE}[\underline{t}]\}$$

$$\underset{\sim}{D} = \{\underline{t} \in \underline{T}_M \mid M \models \underline{ProgE}[\underline{t}]\}.$$

Now let p be any theorem of E. Since Σ^+ is maximally consistent in E, $p \in \Sigma^+$, and hence $M \models p[t_0]$. By axioms (T6) and (T7), Gp and Hp are also theorems of E, so $M \models Gp[t_0]$ and $M \models Hp[t_0]$. Since M is a CL-model, these imply that $M \models p[t]$ for all $t > t_0$ and all $t < t_0$ respectively. Hence $M \models p[t]$ for all $t \in T_M$.

In particular, suppose $t \in B$. Then $M \models ProsE[t]$, and since (E1) is an axiom of E, $M \models ProsE \vee HProsE[t]$. Hence $M \models HProsE[t]$, i.e. $M \models ProsE[t']$ for all $t' < t$, i.e $t' \in B$ for all $t' < t$. Hence B is an initial segment of T, i.e. any time which precedes a member of B is itself a member of B. Similarly, it can be shown that A is a final segment of T, the proof in this case making use of axiom (E2).

Again, if $t \in B$, so $M \models ProsE[t]$, then from axiom (E3) $M \models FPerfE[t]$, so for some $t' \in T$, $t < t'$ and $M \models PerfE[t']$, so $t' \in A$. Hence if B is non-empty, so is A. Similarly, (E4) can be used to show that if A is non-empty then so is B. Combining these results gives us that $A = \emptyset$ iff $B = \emptyset$.

If $t \in D$, then $M \models ProgE[t]$, so from axioms (E5) and (E6), $M \models PProsE[t]$ and $M \models FPerfE[t]$. The first of these guarantees the existence of a $t' \in T$ with $M \models ProsE[t']$, so $t' \in B$; and the second likewise gives us a $t'' \in A$. So if $D \neq \emptyset$, then $B \neq \emptyset$ and $A \neq \emptyset$.

Now assume $\mathbf{A}\neq\emptyset$ (so $\mathbf{B}\neq\emptyset$), and let $t\in T$. Then so long as t is not the first member of T, it is preceded by some $t'\in\mathbf{B}$, so $M\models$ PProsE[t]. By axiom (E9), this means that $M\models$ PerfE v (ProgE v ProsE)[t] and hence that $t\in\mathbf{B}\cup\mathbf{D}\cup\mathbf{A}$. If t is not the last member of T, the same result follows using the mirror image of (E9), which we proved as a theorem. Finally, if t is the only member of T, then (E3), (E4) and (E5) respectively imply that $M\models$ ProsE[t], $M\models$ PerfE[t], and $M\models$ ProgE[t], so in this case $\mathbf{B}=\mathbf{A}=\mathbf{D}=\emptyset$. In conclusion, then, we have that if B and A are non-empty, then $\mathbf{B}\cup\mathbf{D}\cup\mathbf{A}=T$.

Now for a given radical E, let us say that a formal occurrence $\langle B,A\rangle$ in $\langle T,<\rangle$ is compatible with E so long as $A\subseteq\mathbf{A}$, $B\subseteq\mathbf{B}$, and $B'\cap A'\subseteq\mathbf{D}$ [3]. We shall show that

$$\mathbf{A} = \cup\{A \mid \langle B,A\rangle \text{ is compatible with } E\}$$

$$\mathbf{B} = \cup\{B \mid \langle B,A\rangle \text{ is compatible with } E\}$$

$$\mathbf{D} = \cup\{B'\ \ A' \mid \langle B,A\rangle \text{ is compatible with } E\}$$

In fact, it follows immediately from the definition of compatibility that the right-hand side of each of these equalities is included in the left-hand side. It remains, therefore, to show that the reverse inclusions also hold.

First, let $t \in \mathbf{A}$. We must show that $t \in A$ for some $\langle B, A\rangle$ compatible with E. We distinguish two cases:

(a) Suppose $B \cup A = T$. In this case, consider the formal occurrence $\langle \mathbf{A}', \mathbf{A}\rangle$. We have $\mathbf{A}' \subseteq \mathbf{B}$, $\mathbf{A} \subseteq \mathbf{A}$, and $(\mathbf{A}')' \cap \mathbf{A}' = \emptyset \subseteq \mathbf{D}$. Hence $\langle \mathbf{A}', \mathbf{A}\rangle$ is compatible with E, and moreover $t \in \mathbf{A}$, as required.

(b) Suppose $\mathbf{B} \cup \mathbf{A} \neq T$. In this case, consider the formal occurrence $\langle \mathbf{B}, \mathbf{A}\rangle$. We have $\mathbf{B} \subseteq \mathbf{B}$, $\mathbf{A} \subseteq \mathbf{A}$, and $\mathbf{B}' \cap \mathbf{A}' \subseteq \mathbf{D}$ (since $\mathbf{B} \cup \mathbf{D} \cup \mathbf{A} = T$). Hence $\langle \mathbf{B}, \mathbf{A}\rangle$ is compatible with E, and $t \in \mathbf{A}$, as required.

This proves the first equality; and the second can be proved exactly similarly, *mutatis mutandis*.

For the third equality, let $t \in \mathbf{D}$. We must show that $t \in B' \cap A'$ for some $\langle B, A\rangle$ compatible with E. Define $B(t) = \{t' \in T \mid t' < t\}$ (the set of times *before* t) and $A(t) = \{t' \in T \mid t < t'\}$ (the set of times *after* t), and consider the formal occurrence $\langle \mathbf{B} \cap B(t), \mathbf{A} \cap A(t)\rangle$. Then $\mathbf{B} \cap B(t) \subseteq \mathbf{B}$, $\mathbf{A} \cap A(t) \subseteq \mathbf{A}$, and

$$
\begin{aligned}
[\mathbf{B} \cap B(t)]' \cap [\mathbf{A} \cap A(t)]' &= [\mathbf{B}' \cup B(t)'] \cap [\mathbf{A}' \cup A(t)'] \\
&= [\mathbf{B}' \cup A(t) \cup \{t\}] \cap [\mathbf{A}' \cup B(t) \cup \{t\}] \\
&= (\mathbf{B}' \cap \mathbf{A}') \cup (\mathbf{B}' \cap B(t)) \cup (\mathbf{A}' \cap A(t)) \cup \{t\}.
\end{aligned}
$$

Now $\mathbf{B}' \cap \mathbf{A}' \subseteq \mathbf{D}$ (as in (b) above), and $\{t\} \subseteq \mathbf{D}$. Also, by axiom (E7), since $M \models \textit{ProgE}[t]$, we have $M \models \textit{ProsE} \vee \textit{ProgE}[t']$ for all $t' \in B(t)$, i.e. $B(t) \subseteq \mathbf{B} \cup \mathbf{D}$.

Hence $\mathbf{B}'\cap B(t)\subseteq D$. Similarly, using axiom (E8), $\mathbf{A}'\cap A(t)\subseteq \mathbf{D}$. But a union of subsets of $\mathbf{D}$ is again a subset of $\mathbf{D}$, so $[\mathbf{B}\cap B(t)]'\cap[\mathbf{A}\cap A(t)]'\subseteq \mathbf{D}$. Hence $\langle \mathbf{B}\cap B(t), \mathbf{A}\cap A(t)\rangle$ is compatible with E, and $t\in[\mathbf{B}\cap B(t)]'\cap[\mathbf{A}\cap A(t)]'$, as required.

Let us now for each radical E define

$$I(E) = \{\langle B,A\rangle \mid \langle B,A\rangle \text{ is compatible with } E\}.$$

Define an event-logical model $M'=\langle T,<,I\rangle$. We then have that

$$\begin{aligned}
M' \models \underline{\text{Perf}E}[t] \quad &\text{iff} \quad t\in A \text{ for some } \langle B,A\rangle\in I(E)\\
&\text{iff} \quad t\in A \text{ for some } \langle B,A\rangle \text{ compatible with } E\\
&\text{iff} \quad t\in \mathbf{A}\\
&\text{iff} \quad M \models \underline{\text{Perf}E}[t]
\end{aligned}$$

and similarly

$$M' \models \underline{\text{Pros}E}[t] \quad \text{iff} \quad M \models \underline{\text{Pros}E}[t]$$

and

$$M' \models \underline{\text{Prog}E}[t] \quad \text{iff} \quad M \models \underline{\text{Pros}E}[t].$$

Since M and M' agree on all the atomic wffs, and have the same recursive component in their truth-definitions, it follows that they have the same theory (in fact $M'=\underline{\text{gen}}(M)$). Hence, since $M \models p[t_0]$ for each $p\in\Sigma^+$, $M'\models p[t_0]$ for each $p\in\Sigma^+$ also, so M' is a model for Σ, as required.

We have now proved that E is complete with respect to $\underset{\sim}{M}_{e}$. The extension of this result to the other event logics we have considered is straightforward, as follows.

Punctual Event Logic. We have, for every E and t, $M \models \neg \mathit{Prog}E[t]$. Hence $\underset{\sim}{D}=\emptyset$. This means that the formal occurrences $\langle B,A\rangle$ which are compatible with E all have $B' \cap A' = \emptyset$, i.e. they are all punctual. Hence the model M' constructed in the previous proof is automatically punctual in this case, i.e belongs to $\underset{\sim}{M}_{ep}$. Hence EP is complete with respect to $\underset{\sim}{M}_{ep}$.

Once-only Event Logic. For every E and t we have $M \models (\mathit{Perf}E \;\&\; \mathit{Pros}E)[t]$, $M \models (\mathit{Perf}E \;\&\; \mathit{Prog}E)[t]$, and $M \models (\mathit{Pros}E \;\&\; \mathit{Prog}E)[t]$. These imply that $\underset{\sim}{B} \cap \underset{\sim}{A} = \underset{\sim}{A} \cap \underset{\sim}{D} = \underset{\sim}{B} \cap \underset{\sim}{D} = \emptyset$; hence the only formal occurrence compatible with E is $\langle \underset{\sim}{B},\underset{\sim}{A}\rangle$. So $I(E)$ must be $\{\langle \underset{\sim}{B},\underset{\sim}{A}\rangle\}$, and M' is therefore once-only, i.e. belongs to $\underset{\sim}{M}_{eo}$. Hence EO is complete with respect to $\underset{\sim}{M}_{eo}$.

Durative Event Logic. We cannot show that the occurrences compatible with E are all durative; they need not be. Instead, we show that if we put

$$I(E)=\{\langle B,A\rangle \mid \langle B,A\rangle \text{ is durative and compatible with } E\}$$

then everything goes through as required. What is needed is to show that, in

ED, we have for each E

$$\mathbf{A} = \bigcup\{A \mid \langle B,A\rangle \text{ is durative and compatible with } E\}$$

$$\mathbf{B} = \bigcup\{B \mid \langle B,A\rangle \text{ is durative and compatible with } E\}$$

$$\mathbf{D} = \bigcup\{B' \;\; A' \mid \langle B,A\rangle \text{ is durative and compatible with } E\}.$$

As before, the inclusion of the right-hand side in the left-hand side is immediate, so we need only consider the reverse inclusion.

Let $t \in \mathbf{A}$, so $M \models \mathit{PerfE}[t]$. By axiom (D2), $M \models \mathit{PProgE}[t]$, so there is a time $t' < t$ such that $M \models \mathit{ProgE}[t']$. By axiom (E7), for any time t'' preceding t', either $M \models \mathit{ProgE}[t'']$ or $M \models \mathit{ProsE}[t'']$. Hence $B(t') \subseteq \mathbf{B} \cup \mathbf{D}$, so $\mathbf{B}' \cap B(t') \subseteq \mathbf{D}$. Let $B_0 = \mathbf{B} \cap B(t')$, so $B_0 \subseteq \mathbf{B}$. Similarly, by axiom (E8), $A(t') \subseteq \mathbf{A} \cup \mathbf{D}$, so $\mathbf{A}' \cap A(t') \subseteq \mathbf{D}$. Let $A_0 = \mathbf{A} \cap A(t')$, so $A_0 \subseteq \mathbf{A}$ and $t \in A_0$. Now

$$\begin{aligned} B_0' \cap A_0' &= [\mathbf{B}' \cup B(t')'] \quad [\mathbf{A}' \cup A(t')'] \\ &= [\mathbf{B}' \cup A(t') \cup \{t'\}] \cap [\mathbf{A}' \cup B(t') \cup \{t'\}] \\ &= [\mathbf{B}' \cap \mathbf{A}'] \cup [\mathbf{B}' \cap B(t')] \cup [\mathbf{A}' \cap A(t')] \cup \{t'\} \\ &\subseteq \mathbf{D}. \end{aligned}$$

Hence $\langle B_0, A_0\rangle$ is compatible with E and since $t' \in B_0' \cap A_0'$, $\langle B_0, A_0\rangle$ is durative. But $t \in A_0$, so $t \in \bigcup\{A \mid \langle B,A\rangle$ is durative and compatible with $E\}$, as required.

The second inclusion can be proved in an exactly similar way, *mutatis mutandis*. The third inclusion follows immediately from the corresponding inclusion in the general case, since we have

$$\bigcup\{\underline{B}'\cap\underline{A}' \mid \langle\underline{B},\underline{A}\rangle \text{ is compatible with } \underline{E}\}$$
$$= \bigcup\{\underline{B}' \ \underline{A}' \mid \langle\underline{B},\underline{A}\rangle \text{ is durative and compatible with } \underline{E}\}.$$

This is because in the left-hand side of this equation, the only pairs $\langle\underline{B},\underline{A}\rangle$ which contribute to the union are the durative ones, since by definition a $\langle\underline{B},\underline{A}\rangle$ for which $\underline{B}'\cap\underline{A}'\neq\emptyset$ is durative.

We have now proved that ED is complete with respect to $\underset{\sim}{M}_{ed}$, and all the promised soundness and completeness proofs have been duly delivered.

NOTES

1. For a detailed account of the philosophical and linguistic motivation for this way of treating events, see [2] .

2. For the system CL, see [1]. More widely available accounts may be found in [4] and [3]. The completeness proof for CL is outlined in the latter work.

3. I am indebted to Kit Fine for suggesting the idea of compatible occurrences, thereby enabling me to simplify the proof of completeness considerably.

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P A S T	P R E S E N T	F U T U R E
It has been the case that p	It is the case that p	It will be the case that p

(a) The tense-logical trichotomy

PERFECTIVE	PROGRESSIVE	PROSPECTIVE
E has occurred	E is occurring	E will occur

(b) The event-logical trichotomy

P A S T PROGRESSIVE	P R E S E N T PROGRESSIVE	F U T U R E PROGRESSIVE
E has been occurring	E is occurring	E will be occurring

(c) One possible interaction between the trichotomies

Figure 1. Three trichotomies of temporal incidence