Quantum Computing Exercises

September 22, 2000

1. Nielsen-Chuang Exercise 4.4:
   Construct the Hadamard gate from R_x and R_z rotations and a global phase $e^{i\phi}$.

2. Nielsen-Chuang Exercise 4.17:
   Construct a CNOT from two Hadamard gates and one controlled-Z (which maps $|11\rangle \to -|11\rangle$ and is identity on the other basis states).

3. Remember that the Deutsch-Jozsa algorithm builds the $n+1$-qubit state

   $$(H^{\otimes n+1})U_F(H^{\otimes n+1})|0\ldots01\rangle.$$

   It then measures this state and outputs 'constant' if it observed $|0\ldots01\rangle$, and outputs 'balanced' otherwise. Here U_F maps $|i,b\rangle \to |i,b\oplus F(i)\rangle$, and $F : \{0,1\}^n \to \{0,1\}$ is either balanced or constant.

   Show that the same algorithm can also solve the following *Bernstein-Vazirani* problem:
   Given the promise that there is an $a \in \{0,1\}^n$ such that $F(i) = a \cdot i \mod 2$, find a.

   How many queries (= applications of F) would a classical algorithm need to determine a?