# The Erdős Graph and the Beast

*Paul M.B. Vitanyi*

[1]

C.W.I. and University of Amsterdam

**Abstract**

Let him that hath understanding count the number of the beast: for it is the number of a man; and his number is six hundred three score and six. *St. John, Book of Revelations.*

## 1 Introduction

The number of the beast has been interpreted to stand for Pope Leo X (by Michael Stifel (1486 - 1567)). He considered the name LEO DECIMVS X, added the values of the constituent letters L,D,C,I,V,X in their meaning of Roman numerals, discarded the other letters because they do not denote a Roman numeral and also the Roman numeral 'M' because its meaning 'Mystery' eliminates its value. Apart from Stifel's invention, 666 has been variously interpreted to symbolize the Pope of Rome in general (according to Napier); Martin Luther (according to Napier's contemporary the Jesuit Father Bongus S.J.); Kaiser Wilhelm (during World War 1); Adolf Hitler (during World War 2); and earlier the Roman Emperor Nero (because 666 is claimed to spell 'Nero' when expressed in the letter symbols of the Aramaic language, in which the Book of Revelations was originally written; cf. [3].)

The Erdős graph has been the subject of some papers [1,2] and certainly of many conversations, usually enlivened by the intake of appropriate liquids. In [2] it is conjectured that as time goes by the scrutinized graph will contain arbitrary large cliques. The inspiration for an example in [4], or more to the point, the incentive to write that paper at all, was the emergence of the Beast from the Graph. Recently I found this old note back: it considers the matter in some more detail, viz., to show how the assumption of arbitrary large cliques in the Erdős graph evokes the number of the Beast, that is 666, from the graph. It may be of some interest for people who like this sort of thing. The paper continues the age old tradition of finding contrived interpretations for the Number of the Beast.

[1]CWI, Kruislaan 413, 1098SJ Amsterdam, The Netherlands. Email: paulv@cwi.nl

# 2 Evoking the Beast from the Graph

Assume that we consider undirected graphs without multiple edges and self loops. The Erdős graph has the mathematicians (nowadays perhaps mathematicians and computer scientists) for its vertices. In its basic form, two vertices are joined by an edge if the corresponding mathematicians have coauthored a paper. A *clique* in a graph is a subgraph in which each pair of vertices is connected by an edge. A clique, in the Erdős graph, with more than two vertices must contain at least one edge joining equal sexed vertices. Let $(p_1, p_2)$ denote a *partition* of the $p$ vertices of a clique $K_p$ in $p_1$ *male* and $p_2$ *female* mathematicians. Let $\alpha(p_1, p_2)$ denote the *ratio* between the number of edges joining vertices of equal sex and the number of edges joining vertices of opposite sex, in the partition $(p_1, p_2)$ of $K_p$. Let $M(K_p)$ be the *minimum* of $\alpha(p_1, p_2)$ , for all partitions of vertices of $K_p$. The following equation is easily proven.

$$M(K_p) \;=\; \frac{p^2 - 2p + \text{remainder}(p/2)}{p^2 - \text{remainder}(p/2)} \quad . \tag{$*$}$$

It follows that, for all $p > 1$, we have $M(K_p) < 1$. Moreover, that $M(K_p)$ is monotone increasing and $\lim_{p\to\infty} M(K_p) \;=\; 1$.

If we replace the arbitrary large clique assumption by the more pleasant one of arbitrary large cliques and all possible male-female ratios, for every $p > 1$ there eventually will appear a clique in the graph in which the male-female ratio realizes $M(K_p)$. According to the equation (*), for every positive constant $\epsilon$ there is a $p_\epsilon$ such that $1 - \epsilon < M(K_p) < 1$. Since $M(K_p) \le \alpha(p_1, p_2) \le \infty$ we can, for every $p$, always find a partition $(p_1, p_2)$ of $p$ such that $\alpha(p_1, p_2)$ is closest to 1. (However, for most arguments, $\alpha(.) \gg 1$.)

It is well known that, while the male/female ratio amongst mathematicians is vastly greater than 1, in the general human populace this ratio differs for various age groups, ethnic groups, and at different times for the same group, but is usually somewhere in between 100/100 and 112/100.

We consider cliques with $\alpha(p_1, p_2) \;=\; 1$. Calculation shows that this is the case for $p_1 - p_2 = \sqrt{p_1 + p_2}$. Setting the ratio male/female to [2] circa 4% more males than females, that is, 108.062/100, we obtain $p_1 - p_2 \;=\; 25.803$ and $p \;=\; 666$. This necessitates dividing one vertex into a partly male and partly female mathematician. We would like to avoid this. [3] Hence we allow $\alpha(p_1, p_2)$ to depart slightly from 1. It follows that

$$\alpha(p_1, p_2) \;=\; \frac{\binom{p_1}{2} + \binom{p_2}{2}}{\binom{p_1 + p_2}{2} - \binom{p_1}{2} - \binom{p_2}{2}} \quad ,$$

which yields

$$p_1^2 - 2\alpha(p_1, p_2)p_1p_2 + p_2^2 - p_1 - p_2 = 0.$$

Using $p_1 = 1.08125p_2$ we have that

$$\alpha(p_1, p_2) = 1.00305 - 0.96242/p_2.$$

[2] This is a realistic figure of birth rate.

[3] The case is not too far-fetched, and, anyway, the clique only needs one such case.

Since $p_1/p_2 = 160/173$ and 173 is prime, $p_2$ must be a multiple of 160 to avoid mathematicians with sex chromosome deficiencies. So $p_2 = 160$ yields $\alpha(p_1, p_2) < 0.997$; $p_2 = 320$ yields $\alpha(p_1, p_2) \approx 1.0000425$ and $p_2 = 480$ yields $\alpha(p_1, p_2) > 1.001$. Since $\alpha(.)$ is strictly increasing with $p_2$, the nearest we can get $\alpha(p_1, p_2)$ to 1 is by taking $p_2 = 320$ and $p_1 = 346$, again yielding the ominous clique $K_{666}$.

## REFERENCES


1. Coffman, C., And what is your Erdős number? *American Mathematical Monthly* **79** (1969) 791.

2. Erdős, P., On the fundamental problem of mathematics, *American Mathematical Monthly* **79** (1972) 149 - 150.

3. Eves, H., *An Introduction to the History of Mathematics.* Holt, Rinehart and Winston, New York, 1967.

4. Vitanyi, P.M.B., How well can a graph be $n$-colored? *Discrete Mathematics,* **34** (1981), 69 - 80.