

Preface

This book gives an introduction to the classical well-known special functions which play a role in mathematical physics, in particular in boundary value problems. Usually we call a function "special" when the function, just as the logarithm, the exponential and trigonometric functions (the elementary transcendental functions), belongs to the toolbox of the applied mathematician, the physicist or engineer. Usually there is a particular notation, and a number of properties of the function are known. This branch of mathematics has a respectable history with great names such as Gauss, Euler, Fourier, Legendre, Bessel and Riemann. They all have spent much time to this subject. A great part of their work was inspired by physics and the resulting differential equations. About 70 years ago these activities culminated in the standard work *A Course of Modern Analysis* by Whittaker and Watson, which has had great influence and is still important.

This book has been written with students of mathematics, physics and engineering in mind, and also researchers in these areas who meet special functions in their work, and for whom the results are too scattered in the general literature. Calculus and complex function theory are the basis for all this: integrals, series, residue calculus, contour integration in the complex plane, etc.

The selection of topics is based on my own preferences, and of course on what in general is needed for working with special functions in applied mathematics, physics and engineering. This book gives more than a selection of formulas. In the many exercises hints for solutions are often given. In order to keep the book to a modest size, no attention is paid to special functions which are solutions of periodic differential equations such as Mathieu and Lamé functions; these functions are only mentioned when separating the wave equation. The current interest in q -hypergeometric functions would justify an extensive treatment of this topic. It falls outside the scope of the present work, but a short introduction is nevertheless given.

Present students and researchers have computers with formula processors at their disposal. For instance, Maple, Matlab and Mathematica are powerful packages, with possibilities of computing and manipulating special functions. It is very useful to exploit this software, but often extra analysis and knowledge of special functions is needed to obtain optimal results.

At several occasions in the book I have paid attention to the asymptotic and numerical aspects of special functions. When this becomes of a too spe-

cialistic nature the references to recent literature are given. In a separate chapter the stability aspects of recurrence relations for several special functions are discussed. It is explained that a given recurrence relation cannot always be used for computations. Much of this information is available in the literature, but it is difficult for beginners to locate.

Part of the material for this book is collected from well-known books, such as from Hochstadt, Lebedev, Olver, Rainville, Szegö and Whittaker & Watson. In addition to these I have used Dutch university lecture notes, in particular those by Prof. H.A. Lauwerier (University of Amsterdam) and Prof. J. Boersma (Technical University Eindhoven).

The enriching and supporting comments of Dick Askey, Johannes Boersma, Tom Koornwinder, Adri Olde Daalhuis, Frank Olver and Richard Paris on earlier versions of the manuscript and Kees van der Laan's help with several TeX problems are much appreciated. When there are still errors in the many formulas I have myself to blame. But I hope that the extreme standpoint of Dick Askey, who once advised me: "Never trust a formula from a book or table; it only gives you an idea how the exact result looks like", is not applicable to the set of formulas in the present book. However, this is a useful warning.